1. 지수

[1] 정의

① $\underbrace{a \times a \times \cdots \times a}_{n\text{개}} = a^n$ (a의 n제곱)

② a의 n제곱근 $\iff x^n = a$ 「n은 2이상인 자연수」

$\sqrt[n]{a}$: n제곱근 a

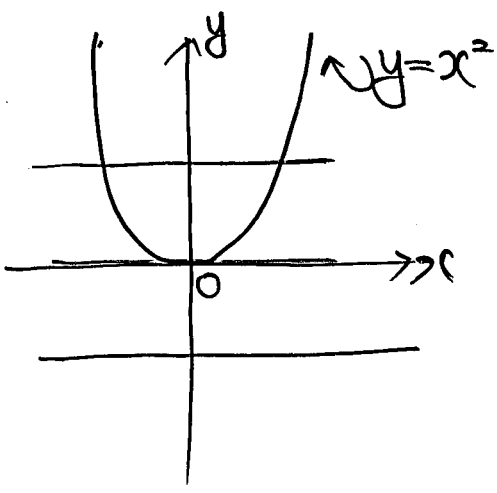
cf) $\sqrt[2]{3} = \sqrt{3}$

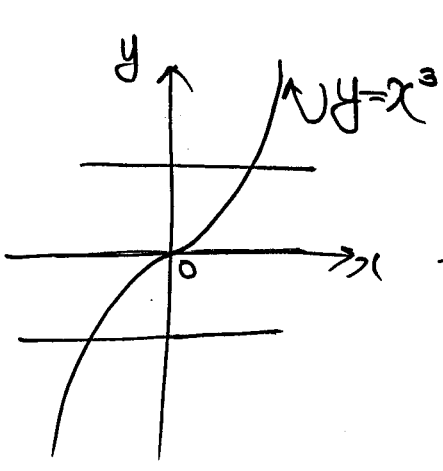
- $x^n = a$ (x는실수, a:실수, n은2이상인자연수)

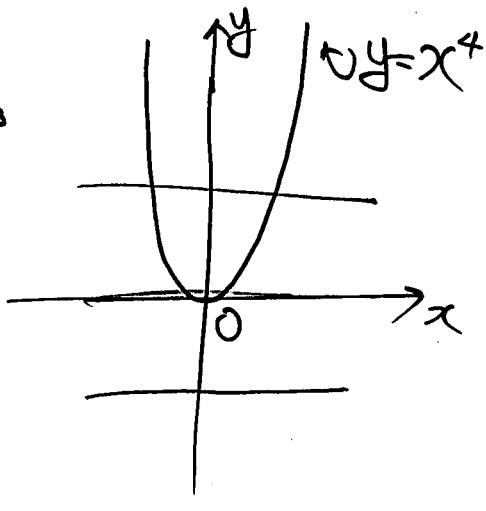
	$a>0$	$a=0$	$a<0$
n:홀수	$\sqrt[n]{a}$	0	$\sqrt[n]{a}$
n:짝수	$\pm\sqrt[n]{a}$	0	정의 X

$\sqrt[3]{-6} = -\sqrt[3]{6}$

$\sqrt[4]{-3}$: 정의 X







$\sqrt{-1} = i$

2 성질

($a>0$, $b>0$, m, n : 2이상인 자연수)

① $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$ ② $\frac{\sqrt[n]{b}}{\sqrt[n]{a}} = \sqrt[n]{\frac{b}{a}}$

③ $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ ④ $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

3 지수의 확장

① $a^0 = 1$ $(a \neq 0)$

② $a^{-n} = \frac{1}{a^n}$ ($a \neq 0$, n은 양의정수)

③ $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ ($a>0$, m은정수, n은 2이상인 자연수)

④ $a>0$, $b>0$, r, s : 유리수

$a^r \times a^s = a^{r+s}$

$a^r \div a^s = a^{r-s}$

$(a^r)^s = a^{rs}$

$(ab)^r = a^r b^r$

cf) $\sqrt[n]{a^n} = \begin{cases} a & (n\text{이홀수}) \\ |a| & (n\text{이짝수}) \end{cases}$ $|a| = \begin{cases} a & (a \geq 0) \\ -a & (a<0) \end{cases}$

P13

EX) 8의 세제곱근 $\Leftrightarrow x^3=8$

$x^3-8=0$

$(x-2)(x^2+2x+4)=0$

$x=2$ 또는 $x=-1\pm\sqrt{3}i$

P15

(1) $\sqrt[3]{-27}=\sqrt[3]{(-3)^3}=-3$

(2) $x^4=16$ (x는실수)

$x=\pm\sqrt[4]{16}=\pm 2$ | $\sqrt[4]{16}=\sqrt[4]{2^4}=2$

P18

(1) $\sqrt[3]{2}\times\sqrt[3]{4}=\sqrt[3]{8}=\sqrt[3]{2^3}=2$

(2) $\frac{\sqrt[4]{32}}{\sqrt[4]{2}}=\sqrt[4]{16}=\sqrt[4]{2^4}=2$

(3) $(-\sqrt[6]{4})^3=-(\sqrt[6]{4})^3=-\sqrt[6]{4^3}=-\sqrt[6]{2^6}=-2$

(4) $\sqrt[3]{\sqrt{64}}=\sqrt[6]{64}=\sqrt[6]{2^6}=2$

P19

11

(1) $x^2=25$ (x는실수) $x=\pm\sqrt{25}=\pm 5$

(2) $x^3=-8$ $x=\sqrt[3]{-8}=\sqrt[3]{(-2)^3}=-2$

(3) $x^4=81$ $x=\pm\sqrt[4]{81}=\pm\sqrt[4]{3^4}=\pm 3$

2)

(1) $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$

(2) $\sqrt[4]{0.0016} = \sqrt[4]{(0.2)^4} = 0.2$

(3) $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$

(4) $\sqrt[6]{64} = \sqrt[6]{2^6} = 2$

3) ③

① 16의 네 제곱근 $\Leftrightarrow x^4 = 16$ ∴ 4개

② -3^6의 여섯제곱근 $\Leftrightarrow x^6 = -729$ ∴ 6개

~~③~~ 세제곱근 -8 : $\sqrt[3]{-8}$

④ $\sqrt[4]{-81}$: 정의 ×

⑤ 27의 세제곱근 $\Leftrightarrow x^3 = 27$ 3개

4)

(1) $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{27} = \sqrt[3]{3^3} = 3$

(2) $\sqrt[3]{27^2} = \sqrt[3]{9^3} = 9$

$27^2 = (3^3)^2 = 3^6$
$= (3^2)^3$

(3) $\frac{\sqrt[3]{64}}{\sqrt[3]{8}} = \sqrt[3]{8} = \sqrt[3]{2^3} = 2$

(4) $\sqrt{\sqrt{16}} = \sqrt[4]{16} = \sqrt[4]{2^4} = 2$

5)

(1) $\sqrt{(-2)^2} = |-2| = 2$ $\sqrt[3]{(-2)^3} = -2$

$\sqrt[4]{(-2)^4} = |-2| = 2$ $\sqrt[5]{(-2)^5} = -2$

(주식) = 0

(2) $\sqrt[3]{54} = \sqrt[3]{27} \times \sqrt[3]{2} = \sqrt[3]{3^3} \cdot \sqrt[3]{2} = 3 \cdot \sqrt[3]{2}$

$\sqrt[3]{16} = \sqrt[3]{8} \times \sqrt[3]{2} = \sqrt[3]{2^3} \cdot \sqrt[3]{2} = 2 \cdot \sqrt[3]{2}$

(준식) = (3 + 2 - 1) $\sqrt[3]{2} = 4 \cdot \sqrt[3]{2}$

P20

EX) (1) $a^2b \times ab^4 = a^3b^5$

(2) $(a^2b^3)^2 = a^4b^6$

(3) $a^4b^3 \times \frac{a^5}{b} = a^9b^2$

P21

EX) (1) $3^0 = 1$ (2) $(-5)^0 = 1$

(3) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

(4) $(-\frac{1}{3})^{-2} = \frac{1}{(-\frac{1}{3})^2} = \frac{1}{\frac{1}{9}} = 9$

P23

EX)

(1) $2^3 \times 2^{-4} = 2^{-1} = \frac{1}{2}$

(2) $6^{-3} \div 6^{-5} = 6^2 = 36$

(3) $(3^{-1})^{-1} = 3^1 = 3$

(4) $(2^{-2})^{-3} = 2^6 = 64$

(5) $(2 \times 3)^{-2} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

P25

EX) (1) $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4$

(2) $9^{-\frac{3}{2}} = (3^2)^{-\frac{3}{2}} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

P26

EX) (1) $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4$

(2) $8^{\frac{1}{2}} \times 4^{\frac{1}{4}} \times 2^{-2}$

$= 2^{\frac{3}{2}} \times 2^{\frac{1}{2}} \times 2^{-2} = 2^0 = 1$

P27

EX) (1) $5^{\sqrt{3}} \times 5^{2\sqrt{3}} = 5^{3\sqrt{3}}$

(2) $(5^{\sqrt{3}})^{\sqrt{3}} = 5^3 = 125$

P28

EX) $\sqrt[3]{3} > \sqrt[4]{4}$

$3^{\frac{1}{3}} > 4^{\frac{1}{4}}$

$(3^{\frac{1}{3}})^{12} > (4^{\frac{1}{4}})^{12}$

$3^4 > 4^3$

$81 > 64$

P29

11

(1) $(-\frac{1}{2})^0 = 1$ (2) $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(3) $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

(4) $(\frac{1}{5})^{-3} = (5^{-1})^{-3} = 5^3 = 125$

21

(1) $3^3 \times (3^2)^{-2} = 3^3 \times 3^{-4} = 3^{-1} = \frac{1}{3}$

(2) $9^{-4} \div (6^{-2})^3 = (3^2)^{-4} \div 6^{-6}$

$= 3^{-8} \times 6^6 = 3^{-8} \times 2^6 \times 3^6 = 2^6 \times 3^{-2} = \frac{2^6}{3^2} = \frac{64}{9}$

(3) $(2^{-2})^3 \times 4^2 = 2^{-6} \times 2^4 = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

(4) $8^{-3} \div (6^2)^{-2} = (2^3)^{-3} \div 6^{-4} = 2^{-9} \times 6^4$

$= 2^{-9} \times 2^4 \times 3^4 = 2^{-5} \times 3^4 = \frac{3^4}{2^5} = \frac{81}{32}$

31

(1) $3^{\frac{1}{3}+\frac{1}{6}} = 3^{\frac{1}{2}} = \sqrt{3}$

(2) $\sqrt[4]{2} \div 2^{-\frac{3}{4}} = 2^{\frac{1}{4}} \div 2^{-\frac{3}{4}} = 2^1 = 2$

(3) $(\frac{4}{9})^{-\frac{1}{2}} = ((\frac{2}{3})^2)^{-\frac{1}{2}} = (\frac{2}{3})^{-1} = \frac{3}{2}$

(4) $(2^{\frac{1}{2}} \times \sqrt[4]{3^3})^4 = (2^{\frac{1}{2}} \times 3^{\frac{3}{4}})^4 = 2^2 \times 3^3 = 108$

41

(1) $2^{\sqrt{2}} \times 2^{\sqrt{8}} = 2^{\sqrt{2}} \times 2^{2\sqrt{2}} = 2^{3\sqrt{2}}$

(2) $5^{\sqrt{243}} \div 5^{\sqrt{27}} = 5^{9\sqrt{3}} \div 5^{3\sqrt{3}} = 5^{6\sqrt{3}}$

(3) $(2^{\sqrt{2}})^{3\sqrt{2}} = 2^6 = 64$

(4) $(3^{\sqrt{8}} \times 3^{\sqrt{2}})^{\frac{1}{\sqrt{2}}} = (3^{3\sqrt{2}})^{\frac{1}{\sqrt{2}}} = 3^3 = 27$

P31

1-1

(1) (준식) $= \left(\left(\frac{2}{3}\right)^2\right)^{-\frac{1}{2}} = \left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

(2) (준식) $= \left(\left(\frac{5}{4}\right)^3\right)^{\frac{1}{4}} \times \left(\left(\frac{5}{4}\right)^{-3}\right)^{-\frac{1}{12}} = \left(\frac{5}{4}\right)^{\frac{3}{4}} \times \left(\frac{5}{4}\right)^{\frac{1}{4}}$

$= \left(\frac{5}{4}\right)^1 = \frac{5}{4}$

(3) (준식) $= \frac{\sqrt[6]{a}}{\sqrt[3]{a}} \times \frac{\sqrt[12]{a}}{\sqrt[6]{a}} \times \frac{\sqrt[8]{a}}{\sqrt[12]{a}} = 1$

(4) (준식) $= \sqrt[3]{\frac{4 \cdot 4 \cdot 18 \cdot 18}{24}} = \sqrt[3]{2^3 \cdot 3^3} = \sqrt[3]{6^3} = 6$

1-2

(1) $\sqrt{2\sqrt[3]{4\sqrt[4]{8}}} = 2^{\left(\left(\frac{3}{4}+2\right)\cdot\frac{1}{3}+1\right)\cdot\frac{1}{2}}$

$p = \left(\frac{11}{12}+1\right)\cdot\frac{1}{2} = \frac{23}{24}$

(2) $3^{\frac{1}{16}} \times 3^{\frac{1}{64}} = 3^{\frac{5}{64}} = 3^p$ $\quad p = \frac{5}{64}$

1-3 ②

ㄱ $(\sqrt{2})^{2\sqrt{2}} = (2^{\frac{1}{2}})^{2\sqrt{2}} = 2^{\sqrt{2}}$

$(2\sqrt{2})^{\sqrt{2}} = (2\cdot 2^{\frac{1}{2}})^{\sqrt{2}} = (2^{\frac{3}{2}})^{\sqrt{2}} = 2^{\frac{3\sqrt{2}}{2}}$

ㄴ $(\sqrt{3})^{3\sqrt{3}} = (3^{\frac{1}{2}})^{3\sqrt{3}} = 3^{\frac{3\sqrt{3}}{2}}$

$(3\sqrt{3})^{\sqrt{3}} = (3\cdot 3^{\frac{1}{2}})^{\sqrt{3}} = (3^{\frac{3}{2}})^{\sqrt{3}} = 3^{\frac{3\sqrt{3}}{2}}$

~~ㄷ~~ $(\sqrt{5})^{5\sqrt{5}} = (5^{\frac{1}{2}})^{5\sqrt{5}} = 5^{\frac{5\sqrt{5}}{2}}$

$(5\sqrt{5})^{\sqrt{5}} = (5\cdot 5^{\frac{1}{2}})^{\sqrt{5}} = (5^{\frac{3}{2}})^{\sqrt{5}} = 5^{\frac{3\sqrt{5}}{2}}$

2-11

(1) $(a^{\frac{1}{2}}+b^{\frac{1}{2}})(a^{\frac{1}{2}}-b^{\frac{1}{2}}) = a-b$

(2) $(a^{\frac{1}{3}}-b^{\frac{1}{3}})(a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}})$

$= (A-B)(A^2+AB+B^2)$

$= A^3-B^3 = a-b$

$a^{\frac{1}{3}} = A,\ b^{\frac{1}{3}} = B$

(3) $a^{\frac{1}{3}} = A,\ b^{-\frac{1}{3}} = B$

(준식) $= \dfrac{A^3+B^3}{A+B} = A^2-AB+B^2$

$= a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{-\frac{1}{3}} + b^{-\frac{2}{3}}$

(4) $a^{\frac{1}{2}} = A,\ b^{\frac{1}{2}} = B$

(준식) $= \dfrac{A^3-A^2B+AB^2-B^3}{A-B} = \dfrac{(A-B)(A^2+B^2)}{A-B} = a+b$

2-21

$a^{\frac{1}{8}} = A$

(준식) $= \frac{2}{1-A} + \frac{2}{1+A} + \frac{4}{1+A^2} + \frac{8}{1+A^4} + \frac{16}{1+A^8}$

$= \frac{4}{1-A^2} + \frac{4}{1+A^2} + \frac{8}{1+A^4} + \frac{16}{1+A^8}$

$= \frac{8}{1-A^4} + \frac{8}{1+A^4} + \frac{16}{1+A^8}$

$= \frac{16}{1-A^8} + \frac{16}{1+A^8}$

$= \frac{32}{1-A^{16}} = \frac{32}{1-a^2}$

2-31

(1) $x^{\frac{1}{2}} = a$, $x^{-\frac{1}{2}} = b$, $ab = 1$

$a+b=4$

$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$= 64 - 12 = 52$

(2) $x^{\frac{1}{2}} = a$, $x^{-\frac{1}{2}} = b$ $ab = 1$

$a^2 + b^2 = 7$

$(a+b)^2 = a^2 + 2ab + b^2 = 9$

$a+b = 3$

3-1)

$2^x = 3$

(1) (준식) $= \dfrac{2^x(2^{3x}+2^{-3x})}{2^x(2^x+2^{-x})} = \dfrac{2^{4x}+2^{-2x}}{2^{2x}+1}$

$= \dfrac{9+\frac{1}{3}}{3+1} = \dfrac{28}{12} = \dfrac{7}{3}$

(2) (준식) $= \dfrac{2^x(2^{3x}-2^{-3x})}{2^x(2^x-2^{-x})} = \dfrac{2^{4x}-2^{-2x}}{2^{2x}-1}$

$= \dfrac{9-\frac{1}{3}}{3-1} = \dfrac{27-1}{6} = \dfrac{26}{6} = \dfrac{13}{3}$

3-2) ④

$(a^x + a^{-x})^2 = a^{2x} + 2 + a^{-2x} = 8$

$a^x + a^{-x} = 2\sqrt{2}$

$a^{3x} + a^{-3x} = (a^x + a^{-x})^3 - 3(a^x + a^{-x})$

$= 16\sqrt{2} - 6\sqrt{2}$

$= 10\sqrt{2}$

$A^3 + B^3$

$= (A+B)^3 - 3AB(A+B)$

3-31

$f(p)=\frac{1}{2}(a^p-a^{-p})=2$ $\quad a^p-a^{-p}=4$

$f(2p)=\frac{1}{2}(a^{2p}-a^{-2p})$

$=\frac{1}{2}(a^p-a^{-p})(a^p+a^{-p})$

$=2(a^p+a^{-p})$

$A^2-B^2=(A-B)(A+B)$

$(a+b)^2=(a-b)^2+4ab$

$(a^p+a^{-p})^2=(a^p-a^{-p})^2+4=20$

$a^p+a^{-p}=2\sqrt{5}$

$f(2p)=4\sqrt{5}$

$$a^m=b^n$$

$$(a^m)^{\frac{1}{m}}=(b^n)^{\frac{1}{m}} \Rightarrow a=b^{\frac{n}{m}}$$

4-11

(1) $a^x=3^4 \Rightarrow a=3^{\frac{4}{x}}$

$b^y=3^4 \Rightarrow b=3^{\frac{4}{y}}$

$c^z=3^4 \Rightarrow c=3^{\frac{4}{z}}$

$3^2=3^{4(\frac{1}{x}+\frac{1}{y}+\frac{1}{z})}$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{2}$$

(2) $\frac{1}{x}+\frac{1}{y}=2$

$81^x = k \qquad 81 = k^{\frac{1}{x}}$

$9^y = k \times \quad 9 = k^{\frac{1}{y}}$

$729 = k^2$

$k = (3^6)^{\frac{1}{2}} = 3^3 = 27$ $\boxed{k=27}$

4-21

(1) $a^x = 2^y = 3^z = k$, $\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=0$

$a = k^{\frac{1}{x}}$

$2 = k^{\frac{1}{y}} \Rightarrow 2^2 = k^{\frac{2}{y}}$

$3 = k^{\frac{1}{z}} \Rightarrow 3^3 = k^{\frac{3}{z}}$

$a \cdot 4 \cdot 27 = 1$

$\boxed{\frac{1}{a} = 108}$

(2) $2^{4a} = 3^{3b} = x^c = k$, $\frac{1}{4a}+\frac{1}{3b}=\frac{1}{2c}$

$2 = k^{\frac{1}{4a}}$

$3 = k^{\frac{1}{3b}}$

$x = k^{\frac{1}{c}}$

$\Rightarrow x^{\frac{1}{2}} = k^{\frac{1}{2c}}$

$6 = x^{\frac{1}{2}}$

$\boxed{36 = x}$

4-31

(1) $a^x = b^y = k$, $a^2b = 1$

$a = k^{\frac{1}{x}} \Rightarrow a^2 = k^{\frac{2}{x}}$

$b = k^{\frac{1}{y}}$

$a^2b = k^{\frac{2}{x}+\frac{1}{y}} = 1$

$$\boxed{\frac{2}{x}+\frac{1}{y}=0}$$

(2) $3^{3a} = x^b = 6^c = k$ $\quad \frac{1}{3a}+\frac{1}{2b}=\frac{1}{c}$

$3 = k^{\frac{1}{3a}}$

$x = k^{\frac{1}{b}} \Rightarrow x^{\frac{1}{2}} = k^{\frac{1}{2b}}$

$6 = k^{\frac{1}{c}}$

$3 \cdot x^{\frac{1}{2}} = 6$ $\quad x^{\frac{1}{2}} = 2$ $\quad \boxed{x=4}$

5-11 ⑤

① $\sqrt[4]{4\sqrt[3]{5}} = \sqrt[4]{\sqrt[3]{4^3 \cdot 5}} = \sqrt[12]{4^3 \cdot 5}$

② $\sqrt[4]{5\sqrt[3]{4}} = \sqrt[4]{\sqrt[3]{5^3 \cdot 4}} = \sqrt[12]{5^3 \cdot 4}$

③ $\sqrt[4]{\sqrt[3]{4 \cdot 5}} = \sqrt[12]{4 \cdot 5}$

④ $\sqrt[3]{4\sqrt[4]{5}} = \sqrt[3]{\sqrt[4]{4^4 \cdot 5}} = \sqrt[12]{4^4 \cdot 5}$

⑤ $\sqrt[3]{5\sqrt[4]{4}} = \sqrt[3]{\sqrt[4]{5^4 \cdot 4}} = \sqrt[12]{5^4 \cdot 4}$

5-21 ⑤

3^{55} 4^{44} 5^{33}

$(3^5)^{11}$ $(4^4)^{11}$ $(5^3)^{11}$

3^5 4^4 5^3

243 256 125

② ① ③

$5^{33} < 3^{55} < 4^{44}$

5-31 ③

$\sqrt{2}$ $\sqrt[3]{4}$ $\sqrt[4]{6}$ $\sqrt[6]{12}$

$2^{\frac{1}{2}}$ $4^{\frac{1}{3}}$ $6^{\frac{1}{4}}$ $12^{\frac{1}{6}}$

$(2^{\frac{1}{2}})^{12}$ $(4^{\frac{1}{3}})^{12}$ $(6^{\frac{1}{4}})^{12}$ $(12^{\frac{1}{6}})^{12}$

2^6 4^4 6^3 12^2

64 256 216 144

④ ① ② ③

↑

$\sqrt{2}$, $\sqrt[3]{4}$

P40

1-1

(1) 64의 세제곱근 $\Leftrightarrow x^3=64$ (거짓)

(2) -8의 세제곱근 $\Leftrightarrow x^3=-8$ $x=\sqrt[3]{-8}=\sqrt[3]{(-2)^3}$
$=-2$ (거짓)

(3) $\pm\sqrt[4]{8}$ (거짓)

(4) $\sqrt[4]{(-3)^6}>0$ (거짓)

(5) $\pm\sqrt[6]{64}=\pm\sqrt[6]{2^6}=\pm 2$, (거짓)
$\sqrt[3]{8}=\sqrt[3]{2^3}=2$

1-2 96

$x=\sqrt[4]{2}=2^{\frac{1}{4}}$ $x^n=(2^{\frac{1}{4}})^n=2^{\frac{n}{4}}$ = 세자리자연수

$\frac{n}{4}=7, 8, 9$

$n=28, 32, 36$ $28+32+36=96$

1-3 ⑤

$a^{9a}=(9a)^9$

$(a^9)^a=(9a)^9$

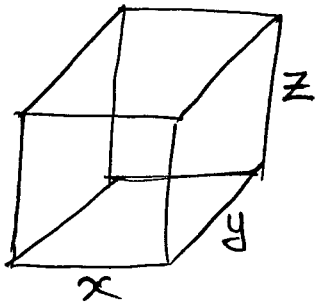
$a^9=9a$

$a^8=9$

$a^8=3^2$

$a=3^{\frac{1}{4}}$
$=\sqrt[4]{3}$

1-4 ③



$xy\times xy\times yz\times yz\times zx\times zx$
$=(xyz)^4=a$
$V=xyz=a^{\frac{1}{4}}=\sqrt[4]{a}$

1-5|30

$a^6=3$ $a=3^{\frac{1}{6}}$ $(abc)^n$

$b^5=7$ $b=7^{\frac{1}{5}}$ $=(3^{\frac{1}{6}}\cdot 7^{\frac{1}{5}}11^{\frac{1}{2}})^n$

$c^2=11$ $c=11^{\frac{1}{2}}$ $=3^{\frac{n}{6}}7^{\frac{n}{5}}11^{\frac{n}{2}}$ = 자연수

$\therefore n=30$

1-6|

(1) $2^n+2^n+2^n+2^n=4\cdot 2^n=2^2\cdot 2^n=2^{n+2}$

$8^4=(2^3)^4=2^{12}$ $n+2=12$ $\boxed{n=10}$

(2) $\frac{3^{20}}{3^{10}-3^8}=\frac{3^{20}}{3^8(3^2-1)}=\frac{3^{12}}{2^3}=2^{-3}\times 3^{12}$

$m=-3,\ n=12$ $\boxed{m+n=9}$

1-7|

(1) $\sqrt[n]{2^{n-1}}=2^{\frac{n-1}{n}}$ $2^{\frac{1}{2}}\times 2^{\frac{3}{4}}\times 2^{\frac{5}{6}}\times 2^{\frac{11}{12}}$

$=2^{\frac{6+9+10+11}{12}}=2^3=8$

(2) $3^6-1=(3^2-1)(3^4+3^2+1)$

$\therefore a^3-b^3=(a-b)(a^2+ab+b^2)$

$x=\sqrt[2n]{\frac{2^{11}}{8}}=\sqrt[2n]{2^8}=2^{\frac{4}{n}}$

$16+4+2=22$

1-8| 4

$$\frac{1}{a^{-1}+1}=\frac{a}{a(a^{-1}+1)}=\frac{a}{1+a}$$

(준식) $=4$

1-9| ④

① $\sqrt{(\sqrt{2^{\sqrt{2}}})^{\sqrt{2}}}=\left((2^{\frac{\sqrt{2}}{2}})^{\sqrt{2}}\right)^{\frac{1}{2}}=2^{\frac{1}{2}}$

② $\left\{\sqrt{(\sqrt{2})^{\sqrt{2}}}\right\}^{\sqrt{2}}=\left(\left((2^{\frac{1}{2}})^{\sqrt{2}}\right)^{\frac{1}{2}}\right)^{\sqrt{2}}=2^{\frac{1}{2}}$

③ $\sqrt{\{(\sqrt{2})^{\sqrt{2}}\}^{\sqrt{2}}}=\{(2^{\frac{1}{2}})^2\}^{\frac{1}{2}}=2^{\frac{1}{2}}$

④ $(\sqrt{2})^{\sqrt{(\sqrt{2})^{\sqrt{2}}}}=(2^{\frac{1}{2}})^{((2^{\frac{1}{2}})^{\sqrt{2}})^{\frac{1}{2}}}=(2^{\frac{1}{2}})^{2^{\frac{\sqrt{2}}{4}}}=2^{\frac{1}{2}\times 2^{\frac{\sqrt{2}}{4}}}$

⑤ $(\sqrt{\sqrt{2^{\sqrt{2}}}})^{\sqrt{2}}=(\sqrt[4]{2^{\sqrt{2}}})^{\sqrt{2}}=(2^{\frac{\sqrt{2}}{4}})^{\sqrt{2}}=2^{\frac{1}{2}}$

1-10| $\frac{9}{2}$

㉠ $2=K\cdot 2^x$

㉡ $3=K\cdot 4^x$

㉢ $T=K\cdot 8^x$

$=\frac{4}{3}\cdot\frac{27}{8}$

$=\frac{9}{2}$

$4^x=(2^2)^x=2^{2x}$

㉠ ÷ ㉡ $\frac{2}{3}=\frac{1}{2^x}$

$2^x=\frac{3}{2}$

$2=K\cdot\frac{3}{2}$ $K=\frac{4}{3}$

$8^x=(2^3)^x=2^{3x}=(2^x)^3$

1-11 ③

$2 * \sqrt{2} = (\sqrt{2})^2 = 2$

$(2 * \sqrt{2}) * 2\sqrt{2} = 2 * 2\sqrt{2} = 2^{2\sqrt{2}}$

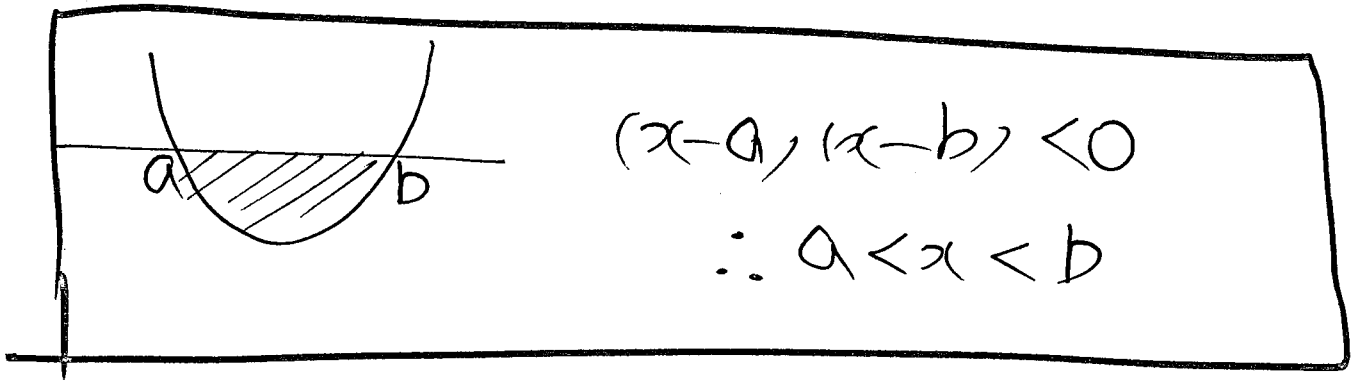
1-12 ⑤

$$y = \frac{2^x(2^x - 2^{-x})}{2^x(2^x + 2^{-x})} = \frac{4^x - 1}{4^x + 1}$$

$4^x \cdot y + y = 4^x - 1$

$(1-y) \cdot 4^x = 1 + y$ $\quad 4^x = \frac{1+y}{1-y}$ $\quad 4^{-x} = \frac{1-y}{1+y}$

$$\frac{1+y}{1-y} + \frac{1-y}{1+y} = \frac{(1+y)^2 + (1-y)^2}{(1-y)(1+y)} = \frac{2(1+y^2)}{1-y^2}$$



$(x-a)(x-b) < 0$

$\therefore a < x < b$

1-13 ④

㉠ $(x - \sqrt{3})(x - \sqrt[3]{4}) < 0$

$\sqrt[3]{4} < x < \sqrt{3}$

㉡ $(x - \sqrt[4]{6})(x - \sqrt[6]{20}) < 0$

$\sqrt[4]{6} < x < \sqrt[6]{20}$

$\sqrt{3} > \sqrt[3]{4}$

$3^{\frac{1}{2}} > 4^{\frac{1}{3}}$

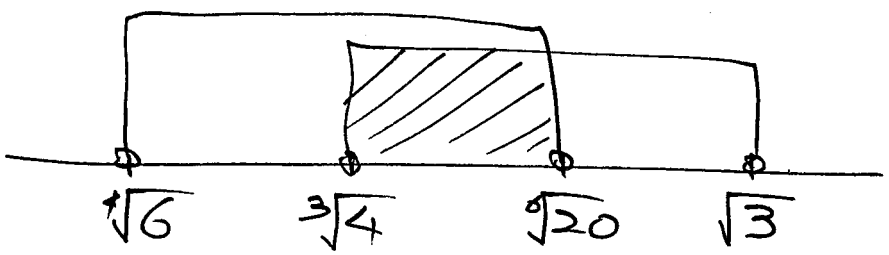
$3^3 > 4^2$

$\sqrt[4]{6} < \sqrt[6]{20}$

$6^{\frac{1}{4}} < 20^{\frac{1}{6}}$

$6^3 < 20^2$

$\sqrt{3}$	$\sqrt[3]{4}$	$\sqrt[4]{6}$	$\sqrt[6]{20}$
$3^{\frac{1}{2}}$	$4^{\frac{1}{3}}$	$6^{\frac{1}{4}}$	$20^{\frac{1}{6}}$
3^6	4^4	6^3	20^2
729	256	216	400
①	③	④	②



$A \cap B = \{x \mid \sqrt[3]{4} < x < \sqrt[6]{20}\}$

1-14 | 400

$\sqrt[3]{16} = 16^{\frac{1}{3}} = a$ $\quad a^3 = 16$ $\quad ab = \sqrt[3]{64}$

$\sqrt[3]{4} = 4^{\frac{1}{3}} = b$ $\quad b^3 = 4$ $\quad = \sqrt[3]{4^3} = 4$

$x = a + b$

$x^3 = (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= 20 + 3ab(a+b) = 20 + 12x$

$\therefore (x^3 - 12x)^2 = 20^2$

$= 400$

$x^2=a$ $\quad x=\pm\sqrt{a}$

$(\sqrt{a})^2=a$ $\quad (-\sqrt{a})^2=a$

$$\sqrt{a^2}=|a|=\begin{cases} a & (a\geq 0) \\ -a & (a<0) \end{cases}$$

$(a\neq 0, b\neq 0)$

$\sqrt{a}\sqrt{b}=-\sqrt{ab}$ ($a<0, b<0$ 일때만 성립)

$\dfrac{\sqrt{b}}{\sqrt{a}}=-\sqrt{\dfrac{b}{a}}$ ($a<0, b>0$ 일때만 성립)

1-15

ㄱ (x), ㄴ (x), $\sqrt{(-a)^2}=|-a|$

ⓒ ⓔ ⓓ

1-16 -2

$(x^{\frac{1}{4}}+x^{-\frac{1}{4}})(x^{\frac{1}{4}}-x^{-\frac{1}{4}})(x^{\frac{1}{2}}+x^{-\frac{1}{2}})$

$=(x^{\frac{1}{2}}-x^{-\frac{1}{2}})(x^{\frac{1}{2}}+x^{-\frac{1}{2}})$

$=x-x^{-1}=-2$

(°0°)

$(x-x^{-1})^2=(x+x^{-1})^2-4$

$=8-4$

$=4$

$x-x^{-1}<0$

$(a-b)^2=(a+b)^2-4ab$

1-17 | 75

$(2^x+2^{-x})(2^y+2^{-y}) = 2^{x+y}+2^{x-y}+2^{-x+y}+2^{-x-y} = 20$

$(2^x-2^{-x})(2^y-2^{-y}) = 2^{x+y}-2^{x-y}-2^{-x+y}+2^{-x-y} = 10$

$2^{x+y}+2^{-x-y} = 15$ $\quad$ (준식) $= 75$

$2^{x-y}+2^{-x+y} = 5$

1-18 | 4√5

㉠ $3^m - \frac{3^m}{3} = \left(1-\frac{1}{3}\right)3^m = \frac{2}{3}\cdot 3^m = 30$

$3^m = 45$

㉡ $9\cdot 3^n - 3\cdot 3^n = 6\cdot 3^n = 6\sqrt{5}$ $\quad 3^n = \sqrt{5}$

$9^{\frac{m}{4}} = (3^2)^{\frac{m}{4}} = 3^{\frac{m}{2}} = \sqrt{3^m} = \sqrt{45} = 3\sqrt{5}$

$27^{\frac{n}{3}} = (3^3)^{\frac{n}{3}} = 3^n = \sqrt{5}$

(준식) $= 4\sqrt{5}$

1-19 | 648

㉠ $A:C = B:D$ $\quad AD = BC$

$2^{2a+1}\cdot 3^{2b+1} = 2^{3a-2}\cdot 3^{2b+1}$ $\quad 2a+1 = 3a-2$

$a = 3$

㉡ $A = 8\cdot 3^b$, $B = 12\cdot 3^b$, $C = 32\cdot 3^b$, $D = 48\cdot 3^b$

$100\cdot 3^b = 8100$ $\quad 3^b = 81 = 3^4$ $\quad b = 4$

$A = 2^3\cdot 3^4 = 8\cdot 81 = 648$

1-20110

㉠ $\sqrt[3]{\frac{P}{3}} = \sqrt[3]{2^m \cdot 3^{n-1}} = 2^{\frac{m}{3}} \cdot 3^{\frac{n-1}{3}}$ = 자연수

㉡ $\sqrt[4]{\frac{P}{4}} = \sqrt[4]{2^{m-2} \cdot 3^n} = 2^{\frac{m-2}{4}} \cdot 3^{\frac{n}{4}}$ = 자연수

$m=6$, $n=4$ $\quad m+n=10$