

2. 로그

II 정의

$$a^x = b \iff x = \log_a b$$

↙ 진수
↘ 밑수

$$a > 0, a \neq 1, b > 0$$

II 성질

$$a > 0, a \neq 1, M > 0, N > 0$$

$$\log_a 1 = 0 \iff a^0 = 1$$

$$\log_a a = 1 \iff a^1 = a$$

$$(2) \log_a MN = \log_a M + \log_a N$$

$$\log_a M = x, \log_a N = y$$

$$M = a^x, N = a^y$$

$$a^{x+y} = MN \quad x+y = \log_a MN$$

$$\log_a MN = \log_a M + \log_a N$$

$$(3) \log_a \frac{M}{N} = \log_a M - \log_a N$$

$$\log_a M = x, \log_a N = y$$

$$M = a^x, N = a^y$$

$$a^{x-y} = \frac{M}{N} \quad x-y = \log_a \frac{M}{N}$$

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

$$(4) \log_a M^k = k \log_a M$$

$$\log_a M^k = x$$

$$a^x = M^k$$

$$a^{\frac{x}{k}} = M$$

$$\frac{x}{k} = \log_a M$$

$$x = k \log_a M$$

$$\boxed{\log_a M^k = k \log_a M}$$

$$(5) \log_{a^n} M^m = \frac{m}{n} \log_a M$$

$$\log_{a^n} M^m = x \iff (a^n)^x = M^m$$

$$a^{\frac{nx}{m}} = M$$

$$\frac{nx}{m} = \log_a M$$

$$x = \frac{m}{n} \log_a M$$

$$\boxed{\log_{a^n} M^m = \frac{m}{n} \log_a M}$$

$$(6) \log_a b = \frac{\log_c b}{\log_c a}$$

$$(c > 0, c \neq 1)$$

$$\log_a b = x \iff a^x = b$$

$$\log_c a^x = \log_c b$$

$$x \log_c a = \log_c b$$

$$x = \frac{\log_c b}{\log_c a}$$

$$\boxed{\log_a b = \frac{\log_c b}{\log_c a}}$$

$$(6-1) \quad \log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$$

$$(7) \quad a^{\log_b c} = c^{\log_b a}$$

$$a^{\log_b c} = x$$

$$\log_b a^{\log_b c} = \log_b x$$

$$\log_b x = \log_b c \cdot \log_b a = \log_b a \cdot \log_b c = \log_b c^{\log_b a}$$

$$x = c^{\log_b a}$$

$$\boxed{a^{\log_b c} = c^{\log_b a}}$$

$$(7-1) \quad a^{\log_a b} = b^{\log_a a} = b$$

(8) 로그 : 밑수가 같아야 연산 가능

$$\log_{10} A = \log A \quad (\text{상용로그})$$

P48

EX1)

$$(1) 2^4 = 16 \iff 4 = \log_2 16$$

$$(2) 3^{-4} = \frac{1}{81} \iff -4 = \log_3 \frac{1}{81}$$

$$(3) 3 = \log_3 27 \iff 3^3 = 27$$

$$(4) 2 = \log_{10} 100 \iff 10^2 = 100$$

EX2)

$$(1) \log_4 32 = \log_{2^2} 2^5 = \frac{5}{2} \log_2 2 = \frac{5}{2}$$

$$(2) \log_{\frac{1}{5}} 25 = \log_{5^{-1}} 5^2 = \frac{2}{-1} \log_5 5 = -2$$

EX3)

$$(1) \log_7 N = 2 \quad N = 7^2 = 49$$

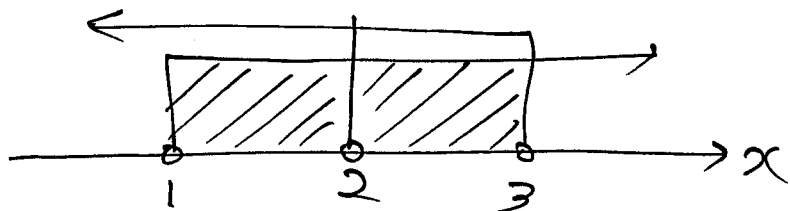
$$(2) \log_{\frac{1}{3}} N = 3 \quad N = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

P50

$$\text{EX) } \log_{x-1} (3-x)$$

$$x-1 > 0, \quad x-1 \neq 1, \quad 3-x > 0$$

$$x > 1, \quad x \neq 2, \quad x < 3$$



$$1 < x < 2 \text{ and } 2 < x < 3$$

P51

EX1)

$$(1) \log_3 1 = 0, \quad \log_3 3 = 1$$

$$(2) \log_2 6 = \log_2 2 + \log_2 3 = 1 + \log_2 3$$

$$(3) \log_2 \frac{3}{2} = \log_2 3 - \log_2 2 = \log_2 3 - 1$$

$$(4) \log_2 8 = \log_2 2^3 = 3 \log_2 2 = 3$$

EX2)

$$(1) \log_{10} \sqrt{2} = \log_{10} 2^{\frac{1}{2}} = \frac{1}{2} \log_{10} 2$$

$$\begin{aligned} (\text{준식}) &= 2 \log_{10} 5 + 2 \log_{10} 2 = 2(\log_{10} 5 + \log_{10} 2) \\ &= 2 \log_{10} 10 = 2 \end{aligned}$$

$$(2) \log_3 \sqrt{5} = \log_3 5^{\frac{1}{2}} = \frac{1}{2} \log_3 5$$

$$\begin{aligned} (\text{준식}) &= \frac{1}{2} \log_3 45 - \frac{1}{2} \log_3 5 = \frac{1}{2} (\log_3 45 - \log_3 5) \\ &= \frac{1}{2} \log_3 9 = \frac{1}{2} \log_3 3^2 = \log_3 3 = 1 \end{aligned}$$

P52

EX)

$$(1) \log_4 8 = \log_{2^2} 2^3 = \frac{3}{2} \log_2 2 = \frac{3}{2}$$

$$(2) \log_3 10 = \frac{\log_{10} 10}{\log_{10} 3} = \frac{1}{\log_{10} 3}$$

$$(3) \log_2 5 \times \log_5 2 = 1$$

$$(4) (\text{준식}) = \frac{\log 6}{\log 4} \times \frac{\log 4}{\log 7} \times \frac{\log 7}{\log 6} = 1$$

P53

$$\text{EX/ (1)} \log_5 2 = \frac{\log_3 2}{\log_3 5} = \frac{b}{a}$$

$$(2) \log_2 3 = \frac{\log_3 3}{\log_3 2} = \frac{1}{b}$$

P54

$$\text{EX1/ (1)} \log_4 27 = \log_{2^2} 3^3 = \frac{3}{2} \log_2 3$$

$$(2) \log_2 3 \times \log_3 2 = 1$$

$$\begin{aligned} (3) (\text{준식}) &= \frac{\log 3}{\log 2} \times \frac{\log 125}{\log 9} \times \frac{\log 16}{\log 5} \\ &= \frac{\log 3}{\log 2} \times \frac{3 \log 5}{2 \log 3} \times \frac{4 \log 2}{\log 5} \\ &= 6 \end{aligned}$$

$$(4) 2^{\log_2 15} = 15$$

$$(5) 4^{\log_2 5} = 5^{\log_2 4} = 5^{\log 2} = 5^2 = 25$$

$$\text{EX2/ } \log 2 = a, \log 3 = b$$

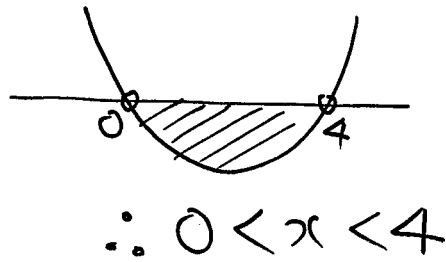
$$\begin{aligned} (1) \log \frac{9}{8} &= \log 9 - \log 8 \\ &= 2 \log 3 - 3 \log 2 = 2b - 3a \end{aligned}$$

$$\begin{aligned} (2) \log \sqrt{24} &= \frac{1}{2} \log 24 = \frac{1}{2} (\log 8 + \log 3) \\ &= \frac{1}{2} (3 \log 2 + \log 3) \\ &= \frac{1}{2} (3a + b) \end{aligned}$$

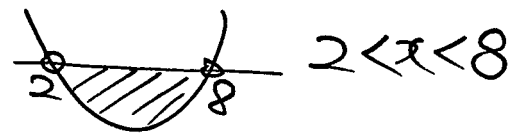
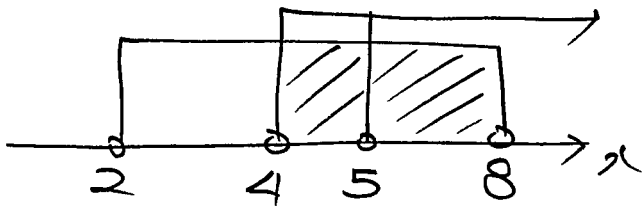
P55

11

$$\begin{aligned} (1) \quad & -x^2 + 4x > 0 \\ & x^2 - 4x < 0 \\ & x(x-4) < 0 \end{aligned}$$



$$\begin{aligned} (2) \quad & x - 4 > 0, \quad x - 4 \neq 1, \quad -x^2 + 10x - 16 > 0 \\ & x > 4, \quad x \neq 5, \quad x^2 - 10x + 16 < 0 \\ & (x-2)(x-8) < 0 \end{aligned}$$



$$4 < x < 5 \quad \text{또는} \quad 5 < x < 8$$

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$$(1) \quad \log_{1000} x = \frac{2}{3} \quad (x > 0)$$

$$x = 1000^{\frac{2}{3}} = (10^3)^{\frac{2}{3}} = 10^2 = 100$$

$$(2) \quad \log_x 4 = -2 \quad (x > 0, x \neq 1)$$

$$x^{-2} = 4 \quad x = 4^{-\frac{1}{2}} = 2^{-1} = \frac{1}{2}$$

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$$(1) \quad \log_{\frac{1}{5}} \sqrt{20} = \frac{1}{2} \log_5 20$$

$$\begin{aligned} (\text{준식}) &= \log_5 \frac{5}{4} + \log_5 20 = \log_5 25 \\ &= 2 \log_5 5 = 2 \end{aligned}$$

$$(2) \log_2 \sqrt{2} = \frac{1}{2} \log_2 2 = \frac{1}{2}, \log_2 9 = 2 \log_2 3$$

$$\begin{aligned} (\text{준식}) &= \frac{1}{2} + \frac{1}{2} \log_2 6 - \frac{1}{2} \log_2 3 \\ &= \frac{1}{2} + \frac{1}{2} (\log_2 6 - \log_2 3) = \frac{1}{2} + \frac{1}{2} \log_2 2 \\ &= \frac{1}{2} + \frac{1}{2} = 1 \end{aligned}$$

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$$(1) x = \underbrace{3 \log_3 2} \times \underbrace{\log_2 3} = 3$$

$$(2) \frac{x}{3} \log_3 2 = 2 \log_3 2 \quad x = 6$$

$$(3) 10^{2 \log_{10} x} = 10^{\log_{10} x^2} = x^2 = 25 \quad (x > 0)$$

$$\boxed{x = 5}$$

$$(4) 3^{\log_{10} 2} = 2^{\log_{10} 3}$$

$$\boxed{\therefore x = 3}$$

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1-11

$$(1) \log_5 75 = 2 \log_5 5 + \log_5 3 = 2 + \log_5 3$$

$$(\text{준식}) = 3 \log_5 3 - 4 - 2 \log_5 3 = -4 + \log_5 3$$

$$\begin{aligned} (2) \log_2 9 &= 2 \log_2 3, \log_2 \left(\frac{8}{3}\right)^3 = 3 \log_2 \left(\frac{8}{3}\right) \\ &= 3(3 \log_2 2 - \log_2 3) = 9 - 3 \log_2 3 \end{aligned}$$

$$(\text{준식}) = 9 - \log_2 3$$

$$(3) (\text{준식}) = \frac{\log 5}{\log 3} \times \frac{\log 7}{\log 5} \times \frac{2 \log 3}{\log 7} = 2$$

$$4) \log_4 9 = \frac{2}{2} \log_2 3 = \log_2 3$$

$$\log_3 4 = 2 \log_3 2, \quad \log_9 2 = \frac{1}{2} \log_3 2$$

$$(\text{준식}) = (2 \log_2 3) \left(\frac{5}{2} \log_3 2 \right) = 5$$

1-2

$$(1) (\text{자수}) = \log_2 (1 \times 2 \times 3 \times 4 \times 5) = \log_2 120$$

$$(\text{준식}) = 2^{\log_2 120} = 120$$

$$(2) (\text{준식}) = \frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \dots \times \frac{\log 1024}{\log 1023}$$

$$= \frac{10 \log 2}{\log 2} = 10$$

1-3

$$9 < 12 < 27$$

$$\log_3 9 < \log_3 12 < \log_3 27$$

$$2 < \log_3 12 < 3$$

$$a=2, \quad b=\log_3 12 - 2$$

$$= \log_3 12 - \log_3 9$$

$$= \log_3 \frac{4}{3}$$

$$3^b = \frac{4}{3}$$

$$(\text{준식}) = \frac{9 + \frac{4}{3}}{\frac{1}{9} + \frac{3}{4}}$$

$$= \frac{\left(\frac{31}{3} \right)}{\frac{31}{36}} = \frac{31 \cdot 12}{31} = 12$$

2-11

$$\begin{aligned}(1) \log_{10} \frac{7}{2} &= \log_{10} 7 - \log_{10} 2 \\ &= b - (1-a) \\ &= a + b - 1\end{aligned}$$

$$\log_{10} 5 + \log_{10} 2 = 1$$

$$\begin{aligned}(2) \log_{10} \sqrt{14} &= \frac{1}{2} \log_{10} 14 = \frac{1}{2} (\log_{10} 2 + \log_{10} 7) \\ &= \frac{1}{2} (1 - a + b)\end{aligned}$$

2-21④

$$3^a = 2 \Leftrightarrow a = \log_3 2$$

$$5^b = 3 \Leftrightarrow b = \log_5 3 \Rightarrow \log_3 5 = \frac{1}{b}$$

$$\begin{aligned}\log_{120} 150 &= \frac{\log_3 150}{\log_3 120} = \frac{\log_3 (2 \times 3 \times 5^2)}{\log_3 (2^3 \times 3 \times 5)} \\ &= \frac{\log_3 2 + \log_3 3 + 2\log_3 5}{3\log_3 2 + \log_3 3 + \log_3 5} = \frac{a + 1 + \frac{2}{b}}{3a + 1 + \frac{1}{b}} \\ &= \frac{ab + b + 2}{3ab + b + 1}\end{aligned}$$

2-31③

$$\log_2 45 = 2\log_2 3 + \log_2 5 = a$$

$$\log_2 75 = \log_2 3 + 2\log_2 5 = b$$

$$\log_2 \frac{5}{3} = \log_2 5 - \log_2 3 = b - a$$

3-1136

$$x^2 + ax + 6 = 0 \quad \text{등 } \alpha, \beta \quad (\alpha < \beta)$$

$$\alpha + \beta = -a, \quad \alpha\beta = 6$$

$$\frac{1}{3} \log_b \alpha + \frac{1}{3} \log_b \beta = \frac{2}{3}$$

$$\log_b \alpha + \log_b \beta = 2$$

$$\log_b \alpha\beta = 2$$

$$\boxed{\alpha\beta = b^2}$$

$$\underline{b^2 = 6}$$

$$(\beta - \alpha)^2 = 6$$

$$6 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$a^2 = 6 + 4\alpha\beta = 6 + 24 = 30$$

$$a^2 + b^2 = 36$$

3-21

$$x^2 - 5x + 3 = 0 \quad \text{등 } \alpha, \beta$$

$$\alpha + \beta = 5, \quad \alpha\beta = 3$$

$$(1) \quad 3^\alpha \times 3^\beta = 3^{\alpha+\beta} = 3^5 = 243$$

$$\log_3 \alpha + \log_3 \beta = \log_3 \alpha\beta = \log_3 3 = 1$$

$$\therefore 243 + 1 = 244$$

$$(2) \quad (\log_3 3) = \log_3 3\alpha + \log_3 3\beta$$

$$= \log_3 9\alpha\beta$$

$$= \log_3 27 = 3 \log_3 3$$

$$= 3$$

3-31

$$2x^2 - mx + 8 = 0 \quad \text{두근 } \alpha, \beta$$

$$\alpha + \beta = \frac{m}{2}, \quad \alpha\beta = 4 \quad \Rightarrow \quad \log_2 \alpha\beta = \log_2 4$$

$$= 2 \log_2 2 = 2$$

$$\frac{1}{\log_2 \alpha} + \frac{1}{\log_2 \beta} = -\frac{2}{3}$$

$$\log_2 \beta + \log_2 \alpha = -\frac{2}{3} \cdot \log_2 \alpha \cdot \log_2 \beta$$

$$\log_2 \alpha + \log_2 \beta = 2$$

$$\log_2 \alpha \cdot \log_2 \beta = -3$$

$$\log_2 \alpha = 3$$

$$\log_2 \beta = -1$$

$$\alpha = 8, \quad \beta = \frac{1}{2}$$

$$\frac{m}{2} = \frac{17}{2}$$

$$\log_2 \alpha = 1$$

$$\log_2 \beta = 3$$

$$\alpha = \frac{1}{2}, \quad \beta = 8$$

$$m = 17$$

P63. 상용로그

$$\log_{10} A = \log A$$

$$\text{EX 1), } \log 10 = 1 \quad (2, \log 10^3 = 3)$$

$$(3, \log 10^{-2} = -2)$$

$$(4, \log 10^{\frac{1}{2}} = \frac{1}{2})$$

* 상용 로그

III 정의

$$\log_{10} A = \log A = \underbrace{n}_{\text{정수부분}} + \underbrace{\alpha}_{\text{소수부분}} \quad (n \text{은 정수} \rightarrow 0 \leq \alpha < 1)$$

$$\text{ex, } \log A = 3.6 = \underline{3} + \underline{0.6}$$

$$\log B = -2.2 = \underline{-3} + \underline{0.8}$$

II 상용로그의 정수부분의 성질

$$\textcircled{1} \text{ 1자리수 : } 1 \leq A < 10$$

$$\text{2자리수 : } 10 \leq A < 10^2$$

⋮

$$n\text{자리수 : } 10^{n-1} \leq A < 10^n$$

$$\Leftrightarrow n-1 \leq \log A < n$$

A의 정수부분이 n자리수 이면

$\log A$ 의 정수부분은 $n-1$ 이다 (n은 자연수)

$$\textcircled{2} \text{ 1번째 : } 10^{-1} \leq B < 1$$

$$\text{2번째 : } 10^{-2} \leq B < 10^{-1}$$

⋮

$$n\text{번째 : } 10^{-n} \leq B < 10^{-n+1} \Leftrightarrow -n \leq \log B < -n+1$$

B가 소수점 아래 n번째 자리에서 처음으로 0이 아닌 숫자

이면 $\log B$ 의 정수부분은 $-n$ 이다.

$$\log 1 = 0, \log 10 = 1$$

상용로그표 : 소수부분

$$\log 1.00 \sim \log 9.99$$

P63

EX1) (1) $\log 4.69 = 0.6712$

(2) $\log 9.62 = 0.9832$

EX2)

(1) $\log 50.1 = \log 10 + \log 5.01$
 $= 1 + 0.6998 = 1.6998$

(2) $\log 0.0172 = \log 10^{-2} + \log 1.72$
 $= -2 + 0.2355$
 $= -1.7645$

(3) $\log 73500 = \log 10^4 + \log 7.35$
 $= 4 + 0.8663 = 4.8663$

(4) $\log \sqrt{6.52} = \frac{1}{2} \log 6.52$
 $= \frac{1}{2} \times 0.8142$
 $= 0.4071$

P64

$$\text{Ex, (1), } \log x = 1.0492$$

$$= 1 + 0.0492$$

$$= \log 10 + \log 1.12 = \log 11.2$$

$$\boxed{x = 11.2}$$

$$(2) \log x = 3.5051 = 3 + 0.5051$$

$$= \log 10^3 + \log(3.20)$$

$$= \log 3200$$

$$\boxed{x = 3200}$$

P65

EX1)

$$(1) \log \underline{321} = \log 100 + \log 3.21 = \underline{2} + \underline{0.5065}$$

$$(2) \log \underline{32100} = \log 10^4 + \log 3.21 = \underline{4} + \underline{0.5065}$$

EX2)

$$(1) \log N = -3 - 0.0672 \\ = \underline{-4} + \underline{0.9328}$$

P66

$$(2) \log \underline{0.321} = \log 10^{-1} + \log 3.21 \\ = \underline{-1} + \underline{0.5065}$$

$$(3) \log \underline{0.0321} = \log 10^{-2} + \log 3.21 \\ = \underline{-2} + \underline{0.5065}$$

P66

EX) $\log 5.67 = 0.7536$

$$(1) \log 56700 = \log 10^4 + \log 5.67 \\ = \underline{4} + \underline{0.7536}$$

$$(2) \log 0.00567 = \log 10^{-3} + \log 5.67 \\ = \underline{-3} + \underline{0.7536}$$

$$(3) \log \sqrt{567} = \frac{1}{2} \log 567 = \frac{1}{2} (\log 10^2 + \log 5.67) \\ = \frac{1}{2} (2 + 0.7536) = \underline{1} + \underline{0.3768}$$

$$(4) \log \frac{1}{5.67} = -\log 5.67 = -0.7536 \\ = -\underline{1} + \underline{0.2464}$$

P69

EX1)

$$(1) \log 2850 = \log 10^3 + \log 2.85 \\ = \underline{3} + \underline{0.4548}$$

$$(2) \log 0.00285 = \log 10^{-3} + \log 2.85 \\ = -\underline{3} + \underline{0.4548}$$

P71

11

$$(1) \log 4 = 2 \log 2 = 0.6020$$

$$(2) \log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - 0.3010 \\ = 0.6990$$

$$(3) \log 6 = \log 2 + \log 3 = 0.7781$$

$$(4) \log 8 = 3 \log 2 = 0.9030$$

$$(5) \log 9 = 2 \log 3 = 0.9542$$

21

$$\log 6.12 = 0.7868$$

$$(1) \log 612 = \log 100 + \log 6.12 = 2 + 0.7868 = 2.7868$$

$$(2) \log 0.0612 = \log 10^{-2} + \log 6.12 = -2 + 0.7868 \\ = -1.2132$$

3)

$$\log 31 = \log 10 + \log 3.1 = 1.4914$$

$$\therefore \log 3.1 = 0.4914$$

$$(1) \log x = 3 + 0.4914 = \log 10^3 + \log 3.1 = \log 3100$$

$$x = 3100$$

$$(2) \log x = -2 + 0.4914 = \log 10^{-2} + \log 3.1 = \log 0.031$$

$$x = 0.031$$

P73

4-116

$$\textcircled{7} \quad x \leq \log A < x+1$$

$$\log 10^x \leq \log A < \log 10^{x+1}$$

$$10^x \leq A < 10^{x+1}$$

$$f(x) = 10^{x+1} - 10^x$$

$$= 10^x \cdot (10 - 1)$$

$$= 9 \cdot 10^x$$

$$\textcircled{8} \quad y \leq \log \frac{1}{B} < y+1$$

$$10^y \leq \frac{1}{B} < 10^{y+1}$$

$$10^{y+1} < B \leq 10^{y+2}$$

$$g(y) = 10^{y+2} - 10^{y+1}$$

$$= 10^{y+1} \cdot \left(10 - \frac{1}{10}\right)$$

$$= 10^{y+1} \cdot \frac{9}{10}$$

$$\log f(10) - \log f(5)$$

$$= \log \left(\frac{9 \times 10^{10}}{9 \times 10^4} \right) = \log 10^6 = 6$$

4-2) ④

$$\textcircled{7} \quad 2 \leq \log A < 3$$

$$\log 100 \leq A < \log 1000$$

$$100 \leq A < 1000$$

$$a = 999$$

$$\textcircled{8} \quad -2 \leq \log B < -1$$

$$\log 10^{-2} \leq \log B < \log 10^{-1}$$

$$\frac{1}{100} \leq B < \frac{1}{10}$$

$$b = \frac{1}{100}$$

$$\boxed{ab = 9.99}$$

4-3) ③

$$\textcircled{7} \quad m \leq \log A < m+1$$

$$\log 10^m \leq \log A < \log 10^{m+1}$$

$$10^m \leq A < 10^{m+1}$$

$$\begin{aligned} x &= 10^{m+1} - 10^m \\ &= 10^m \cdot (10 - 1) \\ &= 9 \cdot 10^m \end{aligned}$$

$$\textcircled{8} \quad -n \leq \log \frac{1}{B} < -n+1$$

$$-n \leq -\log B < -n+1$$

$$n-1 < \log B \leq n$$

$$\log 10^{n-1} < \log B \leq \log 10^n$$

$$10^{n-1} < B \leq 10^n$$

$$\begin{aligned} y &= 10^n - 10^{n-1} \\ &= 10^{n-1} \cdot (10 - 1) \\ &= 9 \cdot 10^{n-1} \end{aligned}$$

$$\begin{aligned} \log x - \log y &= \log \frac{x}{y} = \log \left(\frac{9 \cdot 10^m}{9 \cdot 10^{n-1}} \right) \\ &= \log 10^{m-n+1} = m - n + 1 \end{aligned}$$

5-1-1

$$\textcircled{7} \log 40 = \log 10 + \log 4 = \underline{1} + \underline{\log 4} \quad f(40) = \log 4$$

$$\textcircled{8} \log 50^2 = \log 2500 = \log 10^3 + \log 2.5 \\ = \underline{3} + \underline{\log 2.5} \quad f(50^2) = \log 2.5$$

$$\textcircled{9} \log 100^3 = \log 10^6 = \underline{6} \quad f(100^3) = 0$$

$$(\text{준식}) = \log 4 + \log 2.5 + 0 = \log 10 = 1$$

5-2-1 ④

$$f(1) = f(10) = f(100)$$

$$f(2) = f(20)$$

$$f(3) = f(30)$$

$$f(4) = f(40)$$

$$f(5) = f(50)$$

$$f(6) = f(60)$$

5-3-1 ③

$$f(7) = f(70)$$

$$f(8) = f(80)$$

$$f(9) = f(90)$$

$$f(11) = f(110)$$

$$f(12) = f(120)$$

$$f(13) = f(130)$$

$$f(14) = f(140)$$

$$f(15) = f(150)$$

$$150 - 15$$

$$= 135$$

6-1141

$$\textcircled{A} \log 7^{40} = 40 \log 7 = 33.8040 \\ = \underline{33} + \underline{0.8040}$$

$$\begin{array}{r} 0.8451 \\ 40 \\ \hline 33.8040 \end{array}$$

$$\boxed{n=34}$$

$$\textcircled{B} 7^{40} = 10^{33 + 0.8040} = 10^{0.8040} \times 10^{33} = \underline{6.44} \times 10^{33}$$

$$\log 6 = \log 2 + \log 3 = 0.7781$$

$$\boxed{\log 6 < 0.8040 < \log 7}$$

$$\boxed{a=6}$$

$$\textcircled{C} 6 < 10^{0.8040} < 7$$

$$\textcircled{D} 7^n \text{의各位차라수: } \underline{7, 9, 3, 1}, \underline{7, 9, 3, 1}, \dots$$

$$\boxed{b=1}$$

$$n + a + b = 41$$

6-212

$$10^9 \leq 3^n < 10^{10}$$

$$9 \leq n \log 3 < 10$$

$$\frac{9}{0.4771} \leq n < \frac{10}{0.4771}$$

$$18.xx \leq n < 20.xx$$

$$n=19, 20$$

$$19 + 20 = 39$$

$$\begin{array}{r} 18.xx \\ 4771 \overline{) 90000} \\ \underline{4771} \\ 42290 \\ \underline{38168} \\ 4122 \end{array}$$

$$\begin{array}{r} 20.xx \\ 4771 \overline{) 100000} \\ \underline{9542} \\ 4580 \end{array}$$

6-31 4자리

$$\textcircled{7} \quad 11 \leq \log x^6 < 12$$
$$11 \leq 6 \log x < 12$$

$$\frac{11}{6} \leq \log x < 2$$

$$\textcircled{8} \quad 13 \leq \log y^8 < 14$$
$$13 \leq 8 \log y < 14$$

$$\frac{13}{8} \leq \log y < \frac{14}{8}$$

$$\textcircled{7} + \textcircled{8} \quad \frac{44+39}{24} \leq \log xy < \frac{8+7}{4}$$
$$\frac{83}{24} \leq \log xy < \frac{15}{4}$$

$\log xy$ 의 정수부분: 3

xy : 4자리수

7-1135

$$\textcircled{7} \quad \log \left(\frac{1}{7}\right)^{40} = -40 \log 7$$
$$= -33.8040$$
$$= -34 + 0.1960$$

$$\begin{array}{r} 0.8451 \\ 40 \\ \hline 33.8040 \end{array}$$

$$n = 34$$

$$\textcircled{8} \quad \left(\frac{1}{7}\right)^{40} = 10^{-34+0.1960} = 10^{0.1960} \times 10^{-34} = 1.44 \times 10^{-34}$$

$$\log 1 < 0.1960 < \log 2$$

$$a = 1$$

$$1 < 10^{0.1960} < 2$$

$$n+a=35$$

7-2 ④

$$\textcircled{7} \log\left(\left(\frac{1}{3}\right)^{10} \times \left(\frac{2}{3}\right)^{20}\right) = \log\left(\frac{2^{20}}{3^{30}}\right)$$

$$= 20\log 2 - 30\log 3.$$

$$= 6.02 - 14.313$$

$$= -8.293 = -\underline{9} + \underline{0.707}$$

$$\boxed{n=9}$$

$$\textcircled{8} \left(\frac{1}{3}\right)^{10} \times \left(\frac{2}{3}\right)^{20} = 10^{-9+0.707} = 10^{0.707} \times 10^{-9} = 5.4 \times 10^{-9}$$

$$\log 5 = \log \frac{10}{2} = 1 - \log 2 = 0.699$$

$$\log 6 = \log 2 + \log 3 = 0.7781$$

$$\log 5 < 0.707 < \log 6$$

$$\textcircled{5} < 10^{0.707} < 6$$

$$\boxed{a=5}$$

$$n+a=14$$

7-3 14

$$136 \leq \log 23^{100} < 137$$

$$136 \leq 100 \log 23 < 137$$

$$1.36 \leq \log 23 < 1.37$$

$$-13.7 < -10 \log 23 \leq -13.6$$

$$-13.7 < \log \left(\frac{1}{23}\right)^{10} < -13.6$$

$$\log \left(\frac{1}{23}\right)^{10} \text{의 정수부분} : -14$$

$$a=14$$

$$\begin{array}{r} 0.4771 \\ \times \quad 3 \\ \hline 14.313 \end{array}$$

8-1 10^7

$$10 \leq x < 100 \quad \therefore 1 \leq \log x < 2$$

$$\log x^4 - \log \frac{1}{x} = 4 \log x + \log x \\ = 5 \log x = \text{정수}$$

$$5 \leq 5 \log x < 10$$

$$5 \log x = 5, 6, 7, 8, 9$$

$$\log x = 1, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}$$

$$x = 10^1, 10^{\frac{6}{5}}, 10^{\frac{7}{5}}, 10^{\frac{8}{5}}, 10^{\frac{9}{5}}$$

$$10^{1 + \frac{6}{5} + \frac{7}{5} + \frac{8}{5} + \frac{9}{5}} \\ = 10^7$$

8-2

$$(1) \textcircled{1} 2 \leq \log x < 3$$

$$\textcircled{6} \log x^4 - \log x^2 = \log x^2 = 2 \log x = \text{정수}$$

$$4 \leq 2 \log x < 6$$

$$2 \log x = 4, 5$$

$$\log x = 2, \frac{5}{2}$$

$$x = 10^2, 10^{\frac{5}{2}}$$

$$\therefore 10^{2 + \frac{5}{2}} = 10^{\frac{9}{2}}$$

$$(2) 1 \leq \log x < 2$$

$$\begin{aligned} \text{④ } \log x^3 - \log x &= 3\log x - \frac{1}{2}\log x \\ &= \frac{5}{2}\log x = \text{정수} \end{aligned}$$

$$\text{④, ⑤ } \frac{5}{2} \leq \frac{5}{2}\log x < 5$$

$$\frac{5}{2}\log x = 3, 4 \quad \log x = \frac{6}{5}, \frac{8}{5}$$

$$x = 10^{\frac{6}{5}}, 10^{\frac{8}{5}}$$

$$10^{\frac{6}{5} + \frac{8}{5}} = 10^{\frac{14}{5}}$$

$$\underline{8-3) 16}$$

$$(7) \quad 5 \leq \log x < 6$$

$$\begin{aligned} (14) \quad \log x^2 - \log x &= 2\log x + \log x \\ &= 3\log x = \text{정수} \end{aligned}$$

$$(7), (14) \quad 15 \leq 3\log x < 18$$

$$3\log x = 15, 16, 17$$

$$\log x = 5, \frac{16}{3}, \frac{17}{3}$$

$$x = 10^5, 10^{\frac{16}{3}}, 10^{\frac{17}{3}}$$

$$M = 10^{5 + \frac{16}{3} + \frac{17}{3}} = 10^{16}$$

$$\log M = 16$$

9-11 1000

$$10 < x < 100$$

$$1 < \log x < 2$$

$$3 < 3\log x < 6$$

$$3\log x = 4, 5$$

$$x = 10^{\frac{4}{3}}, 10^{\frac{5}{3}}$$

$$\log x + \log x^2$$

$$= \log x + 2\log x = 3\log x = \text{정수}$$

$$\log x = \frac{4}{3}, \frac{5}{3}$$

$$10^{\frac{4}{3} + \frac{5}{3}} = 10^3 = 100$$

9-21

$$(1) \log x = \underline{3} + \underline{\alpha} \quad (0 \leq \alpha < 1)$$

$$\log \sqrt{x} = \frac{1}{2} \log x = \frac{3}{2} + \frac{\alpha}{2} = \underline{1} + \frac{1+\alpha}{2}$$

$$\alpha + \frac{1+\alpha}{2} = \frac{3}{4}$$

$$4\alpha + 2 + 2\alpha = 3$$

$$\alpha = \frac{1}{6}$$

$$\log \sqrt{x} \text{의 소수부분} : \frac{7}{12}$$

$$(2) \log x = \underline{3} + \underline{\alpha} \quad (0 \leq \alpha < 1)$$

$$\log \sqrt[3]{x} = \frac{1}{3} \log x = \underline{1} + \underline{\frac{1}{3}\alpha}$$

$$\alpha + \frac{1}{3}\alpha = 1$$

$$\alpha = \frac{3}{4}$$

$$\log \sqrt[3]{x} \text{의 소수부분} : \frac{1}{4}$$

9-315

(13) $2 \leq \log x < 3$

(14) $\log x^2 - [\log x^2] + \log x - [\log x] = 1$

$$\log x^2 + \log x = 3 \log x = \text{정수}$$

$$\log x \neq \text{정수}$$

$$\log x^2 \neq \text{정수}$$

$$6 \leq 3 \log x < 9$$

$$3 \log x = 6, 7, 8$$

$$\log x = \cancel{2}, \frac{7}{3}, \frac{8}{3}$$

$$x = 10^{\frac{7}{3}}, 10^{\frac{8}{3}}$$

$$M = 10^{\frac{7}{3} + \frac{8}{3}} = 10^5$$

$$\log M = 5$$

P84

2-11②

$$\frac{\log 3}{\log x} = \frac{\log 27}{\log y} \Rightarrow \frac{\log 3}{\frac{1}{2} \log x} = \frac{3 \log 3}{\log y}$$

$$\boxed{\log y = \frac{3}{2} \log x}$$

$$\textcircled{7} \log_x \sqrt{y} = \frac{\log \sqrt{y}}{\log x} = \frac{\frac{1}{2} \log y}{\log x} = \frac{3}{4}$$

$$\textcircled{8} \log_{xy} (xy)^{\frac{2}{3}} = \frac{2}{3}$$

$$\frac{3}{4} + \frac{2}{3} = \frac{17}{12}$$

2-2) ⑤

$$\log_w x = \frac{1}{24}, \log_w y = \frac{1}{40}$$

$$\log_w xyz = \log_w x + \log_w y + \log_w z = \frac{1}{12}$$

$$\log_w z = \frac{1}{12} - \frac{1}{24} - \frac{1}{40} = \frac{10-5-3}{120} = \frac{1}{60}$$

$$\boxed{\log_z w = 60}$$

2-3) ⑤

$$365^a = 10 \quad 365 = 10^{\frac{1}{a}}$$

$$(0.365)^b = 10 \div 10.365 = 10^{\frac{1}{b}}$$

$$10^3 = 10^{\frac{1}{a} - \frac{1}{b}}$$

$$\boxed{\frac{1}{a} - \frac{1}{b} = 3}$$

2-4)

$$(1) \quad 5^{\log_n 4} = 4^{\log_n 5} = 2^{2 \log_n 5} = 2^{\log_n 25}$$

$$n = 25$$

$$(2) \quad 2^x = \sqrt{1+\sqrt{2}} \quad 4^x = 1+\sqrt{2}$$

$$\begin{aligned} (\text{준식}) &= 4^x - 4^{-x} \\ &= 2 \end{aligned}$$

$$4^{-x} = \frac{1}{\sqrt{2}+1} = \sqrt{2}-1$$

2-51

$$(1) A = 10^{\log 3 \times \log 6}, B = 10^{\log 6 \times \log 30}$$

$$\frac{B}{A} = 10^{\log 6 \times \log 30 - \log 6 \times \log 3} \\ = 10^{\log 6 \cdot (\log 30 - \log 3)} = 10^{\log 6} = 6$$

$$(2) A = 4a, B = 5a, C = 2a$$

$$(\text{지수}) = \log_3 A^2 + \log_3 B - \log_3 C^3$$

$$= \log_3 \left(\frac{16a^2 \cdot 5a}{8a^3} \right) = \log_3 10$$

$$(\text{준식}) = 3^{\log_3 10} = 10$$

2-6156

$$\log_2 a = 3, \log_2 b = 4, \log_2 c = 5$$

$$a = 8, b = 16, c = 32 \quad a + b + c = 56$$

2-7125

$$\log A = n + \alpha \quad (n \text{ 정수}, 0 \leq \alpha < 1)$$

$$x^2 - (\log_2 5) \cdot x + k = 0 \quad \exists n, \alpha$$

$$n + \alpha = \log_2 5$$

$$n \cdot \alpha = k$$

$$k = 2 \log_2 \frac{5}{4} = \log_2 \frac{25}{16}$$

$$2^k = \frac{25}{16}$$

$$\boxed{2^{k+4} = 25}$$

$$4 < 5 < 8$$

$$2 < \log_2 5 < 3$$

$$n = 2, \alpha = \log_2 5 - 2 \\ = \log_2 \frac{5}{4}$$

$$2^\alpha = \frac{5}{4}$$

$$\underline{2-8} \mid 43$$

$$\log 243 = \log 100 + \log 2.43 = 2 + \log 2.43 = 2.3856$$

$$\log 2.43 = 0.3856$$

$$\log 0.0541 = \log 10^{-2} + \log 5.41 = -2 + \log 5.41 = -1.2668$$

$$\log 5.41 = 0.7332$$

$$\log \frac{24300^{10}}{54.1}$$

$$= 10 \log 24300 - \log 54.1$$

$$= 43.856 - 1.7332$$

$$= 42.1228$$

$$= \underline{\underline{42}} + 0.1228$$

$$n = 43$$

$$\log 24300$$

$$= \log 10^4 + \log 2.43$$

$$= 4 + 0.3856$$

$$\log 54.1$$

$$= \log 10 + \log 5.41$$

$$= 1.7332$$

$$\underline{2-9} \mid 640$$

$$\Rightarrow \log n = 4 + \alpha \quad (0 \leq \alpha < 1)$$

$$\log \frac{\sqrt[3]{n}}{4} = \frac{1}{3} \log n - \log 4$$

$$= \frac{4}{3} + \frac{\alpha}{3} - \log 4$$

$$= 1 + \frac{1+\alpha}{3} - \log 4$$

$$= 0$$

$$1 + \alpha = 3 \log 4 \Rightarrow \log n = 3 + 3 \log 4$$

$$\log n = \log 1000 + \log 64 = \log 64000$$

$$n = 64000 \quad \frac{n}{100} = 640$$

$$\underline{2-10|37}$$

$$\textcircled{7} \quad 1 < a < 10$$

$$0 < \log a < 1 \quad 0 < 3 \log a < 3$$

$$0 < \log a^3 < 3$$

$$3 \log a = 1, 2 \quad \log a = \frac{1}{3} \text{ 또는 } \frac{2}{3}$$

$$a = 10^{\frac{1}{3}} \text{ 또는 } \textcircled{10^{\frac{2}{3}}}$$

$$\textcircled{4} \quad 1 < b < 10$$

$$0 < \log b < 1 \quad 0 < 5 \log b < 5$$

$$0 < \log b^5 < 5$$

$$5 \log b = 1, 2, 3, 4$$

$$\log b = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$$

$$b = 10^{\frac{1}{5}}, 10^{\frac{2}{5}}, 10^{\frac{3}{5}}, \textcircled{10^{\frac{4}{5}}}$$

$$ab \text{의 최댓값} : 10^{\frac{2}{3}} \times 10^{\frac{4}{5}}$$

$$= 10^{\frac{22}{15}}$$

$$p = 15, q = 22$$

$$p + q = 37$$

2-11) ③

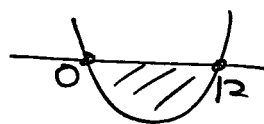
$$a-4>0, \quad a-4 \neq 1,$$

$$a>4, \quad a \neq 5$$

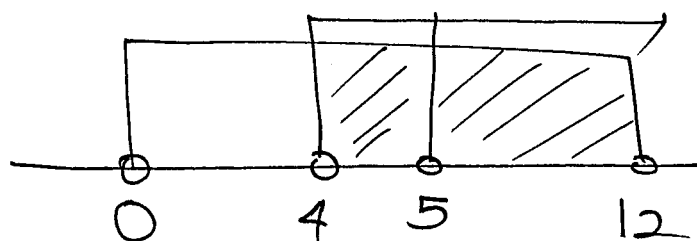
$$x^2+ax+3a>0$$

$$x^2+ax+3a=0$$

$$D=a^2-12a<0$$



$$0 < a < 12$$



$$4 < a < 5 \text{ 또는 } 5 < a < 12$$

$$a = 6, 7, 8, 9, 10, 11 \quad \boxed{6개}$$

2-12) ⑤

$$\log ab = \log(a+b+14)$$

$$ab = a+b+14$$

$$ab - a - b = 14$$

$$(a-1)(b-1) = 15 \quad (a \leq b)$$

$$\begin{array}{cc} 1 & 15 \\ 3 & 5 \end{array}$$

$$(a, b) = (2, 16), (4, 6)$$

$$ab \text{의 최댓값} : 32$$

2-13) ③

$$5 \leq \log x < 6$$

$$\underline{1 \leq \log y < 2 \Rightarrow -2 < -\log y \leq -1}$$

$$3 < \log \frac{x}{y} < 5$$

$$-25 < (\log \frac{x}{y})(\log \frac{y}{x}) < -9$$

15개

2-14) ③

$$10^9 \leq \frac{b}{a} < 10^{10} \quad 10^n \leq \frac{b}{a} < 10^{n+3}$$

$$n = 7, 8, 9$$

$$7 + 8 + 9 = 24$$

2-15) 21

$$\log_{400} 2^a + \log_{400} 5^b = c$$

$$\log_{400} (2^a \times 5^b) = c$$

$$2^a \times 5^b = 400^c = (2^4 \times 5^2)^c = 2^{4c} \times 5^{2c}$$

$$a = 4c, \quad b = 2c$$

$$c = 3, \quad a = 12, \quad b = 6$$

$$a + b + c = 21$$

2-16|10

$$a = \frac{\log 1000 - \log 125}{\log 121 - \log 100} = \frac{\log 8}{\log 1.21} = \log_{1.21} 8$$

$$(1.21)^a = 8$$

$$a = 10.xx$$

$$[a] = 10$$

2-17|28

$$\log 8^{30} = 30 \log 8 = 90 \log 2$$

$$= 27.09$$

$$= 27 + \underline{0.09}$$

$$\begin{array}{r} 0.301 \\ 9 \\ \hline 2709 \end{array}$$

$$\log(a+b) + \log 10^n = n + \log(a+b)$$

$$n = 27$$

$$1 < a+b < 2$$

$$a = 1$$

$$n + a = 28$$

2-18|87

$$5^{20} \div 2^{40} = \frac{5^{20}}{2^{40}}$$

$$\log \frac{5^{20}}{2^{40}} = 20 \log 5 - 40 \log 2$$

$$= 13.980 - 12.04$$

$$= 1.94$$

$$= 1 + 0.94$$

$$\log 5 = \log \frac{10}{2}$$

$$= 1 - \log 2$$

$$= 0.699$$

$$\begin{array}{r} 0.699 \\ 20 \\ \hline 13980 \\ 301 \\ \hline 12.00 \end{array}$$

$$\log \frac{5^{20}}{2^{40}} = \log 10 + \log 87.4 = \log 87.4$$

$$\left\lfloor \frac{5^{20}}{2^{40}} \right\rfloor = 87$$

$$\underline{2-19 \mid 83}$$

$$2 < b < 4$$

$$3 < b < 9$$

$$4 < b < 16$$

$$b = 8$$

$$5 < b < 25$$

$$6 < b < 36$$

$$7 < b < 49$$

$$8 < b < 64$$

$$b = 16, 32$$

$$9 < b < 81$$

$$b = 27$$

$$8 + 16 + 32 + 27 = 83$$

$$\underline{2-20 \mid 416}$$

$$20 + 180 \times 3^{-\frac{t}{256}} = 50$$

$$3^{\frac{t}{256}} = 6$$

$$\frac{t}{256} \log 3 = \log 6$$

$$\frac{t}{256} \times \frac{48}{100} = \frac{78}{100}$$

$$t = \frac{78 \times 256 \times 100}{48}$$

$$t = 416$$

$$\boxed{n = 416}$$

$$\log 6 = \log 2 + \log 3 = 0.78$$

$$\begin{array}{r} 26 \\ 16 \\ \hline 156 \\ 26 \\ \hline 416 \end{array}$$

2-21) 6

$$1 \leq a < b < 100$$

$$0 \leq \log a < \log b < 2$$

$$\log a + \log b = \log ab = \text{정수}$$

$$\log a \neq \text{정수}$$

$$\log b \neq \text{정수}$$

$$\textcircled{7} \log ab = 1$$

$$ab = 10$$

$$(a, b) = (2, 5)$$

$$\textcircled{8} \log ab = 2$$

$$ab = 100$$

$$(a, b) = (2, 50), (4, 25),$$

$$(5, 20)$$

$$\textcircled{9} \log ab = 3$$

$$ab = 1000$$

$$(a, b) = (20, 50)$$

$$(25, 40)$$

2-22) 77

$$\log A = n + \alpha \quad (n \text{은 정수}, 0 \leq \alpha < 1)$$

$$\textcircled{7} \underline{n=0}; 0 \leq 2\alpha$$

$$\underline{0 \leq \alpha < 1}$$

$$0 \leq \log A < 1$$

$$1 \leq A < 10$$

9개

$$\textcircled{8} n=1; 1 \leq 2\alpha$$

$$\underline{\frac{1}{2} \leq \alpha < 1}$$

$$\frac{3}{2} \leq \log A < 2$$

$$10^{\frac{3}{2}} = 10 \cdot 10^{\frac{1}{2}} = 10\sqrt{10}$$

$$10^{\frac{3}{2}} \leq A < 10^2$$

$$32 \sim 99 \quad 68개$$

$$31.6 \leq A < 100$$

$$9+68=77$$

2-23] 875

$$\log_2 n - \log_2 1000 = \log_2 \frac{n}{1000} = \text{정수}$$

$$\frac{1}{10} \leq \frac{n}{1000} \leq \frac{5}{10}$$

$$\frac{n}{1000} = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$$

$$n = 500, 250, 125$$

$$\therefore 500 + 250 + 125 = 875$$

2-24] 55

$$\textcircled{1} 1 \leq n < 10$$

$$0 \leq \log n < 1$$

$$\lfloor \log n \rfloor = 0$$

$$\lfloor \log_2 n \rfloor = 1$$

$$1 \leq \log_2 n < 2$$

$$10 \leq 2n < 100$$

$$5 \leq n < 50$$

$$5 \leq n < 10$$

$$\boxed{574}$$

$$\textcircled{2} 10 \leq n < 100$$

$$1 \leq \log n < 2$$

$$\lfloor \log n \rfloor = 1$$

$$\lfloor \log_2 n \rfloor = 2$$

$$2 \leq \log_2 n < 3$$

$$100 \leq 2n < 1000$$

$$50 \leq n < 500$$

$$\therefore 50 \leq n < 100$$

$$\boxed{5074}$$

$$\textcircled{3} n = 100$$

$$\log n = 2$$

$$\lfloor \log n \rfloor = 2$$

$$\lfloor \log_2 n \rfloor = 3$$

$$3 \leq \log_2 n < 4$$

$$1000 \leq 2n < 10000$$

$$5 + 50 = 55$$

(X)

$$\underline{2-25} \mid 218$$

$$100 < 500 < 1000$$

$$2 < \log 500 < 3$$

$$\lfloor \log 500 \rfloor = 2$$

$$\lfloor \log A \rfloor = 2$$

$$2 \leq \log A < 3$$

$$100 \leq A < 1000$$

$$A = 218$$

$$\underline{2-26} \mid 16$$

$$ax^2 + (3a+1)x + a-4 = 0 \quad \exists n, \alpha$$

$$n + \alpha = \frac{-(3a+1)}{a}$$

$$\lceil \log A \rceil = n + \alpha$$

$$n \in \mathbb{Z}, 0 \leq \alpha < 1$$

$$= -3 - \frac{1}{a} = -4 + \underline{\left(1 - \frac{1}{a}\right)}$$

$$n = -4, \quad \alpha = 1 - \frac{1}{a}$$

$$n\alpha = \frac{a-4}{a} = -4 + \frac{4}{a}$$

$$a-4 = -4a+4$$

$$a = \frac{8}{5}$$

$$10a = 16$$

2-27|100

$$\log a = f(a) + g(a),$$

$$- \log b = f(b) + g(b),$$

$$\log \frac{a}{b} = 2 - \log 3 = \log 100 - \log 3 = \log \frac{100}{3}$$

$$\boxed{\frac{a}{b} = \frac{100}{3}}$$

$$3a + \frac{25}{b} \geq 2\sqrt{3a \cdot \frac{25}{b}} = 2 \cdot 50 = 100$$

2-28|10

$$\text{Ans } \log x = 2n + \alpha \quad (n \in \mathbb{Z}, 0 \leq \alpha < 1)$$

$$\log y = n + \beta \quad 0 \leq \beta < 1$$

$$\Rightarrow \log x - \log \frac{1}{y} = \log x + \log y = 3n + \alpha + \beta = \text{정수}$$

$$0 \leq \alpha + \beta < 2 \quad \alpha + \beta = 0 \text{ 또는 } 1$$

$$\textcircled{1} \alpha + \beta = 0 \quad \alpha = 0, \beta = 0$$

$$\log x^2 y^3 = 2 \log x + 3 \log y = 4n + 3n = 7n = 23.4$$

(X)

$$\textcircled{2} \alpha + \beta = 1$$

$$\log x^2 y^3 = 2 \log x + 3 \log y = 4n + 2\alpha + 3n + 3\beta$$

$$= 7n + 2 + \beta = 23.4$$

$$n = 3, \beta = 0.4, \alpha = 0.6$$

$$\log x = 6.6, \log y = 3.4$$

$$\log xy = \log x + \log y = 10$$

2-29(5)

$$\log a = f(a) + g(a), \quad (f(a) : \text{정수})$$

$$0 \leq g(a) < 1$$

$$\textcircled{A} \log 1004 = \underbrace{\log 1000}_{=3} + \log 1.004$$

$$f(1004) = 3$$

$$\textcircled{B} \log 2 = 0 + \log 2 \quad g(2) = \log 2$$

$$\log 6 = 0 + \log 6 \quad g(6) = \log 6$$

$$\log 12 = \log 10 + \log 1.2 \quad g(2) + g(6)$$

$$= 1 + g(12) \quad = 1 + g(12)$$

$$\textcircled{C} \log ab = f(ab) + g(ab)$$

$$\log a = f(a) + g(a)$$

$$\log b = f(b) + g(b)$$

$$\underline{2-30 \mid 474}$$

$$\textcircled{1} \quad A = 2^{50} + 3^{100} \doteq 3^{100}$$

$$\log 3^{100} = 100 \log 3 = 47.71 = 47 + 0.71$$

$$f(50, 100) = 48$$

$$\textcircled{2} \quad A = 2^k + 3^{50} \doteq 2^k$$

$$\log 2^k = k \log 2$$

$$47 \leq k \log 2 < 48$$

$$\frac{47}{0.301} \leq k < \frac{48}{0.301}$$

$$156.xx \leq k < 159.xx$$

$$k = 157, 158, 159$$

$$157 + 158 + 159$$

$$= 474$$

$$\begin{array}{r} 156 \\ 301 \overline{) 47000} \\ \underline{301} \\ 1690 \\ \underline{1505} \\ 1850 \\ \underline{1806} \end{array}$$

$$\begin{array}{r} 159 \\ 301 \overline{) 48000} \\ \underline{301} \\ 1790 \\ \underline{1505} \\ 2850 \\ \underline{2707} \end{array}$$