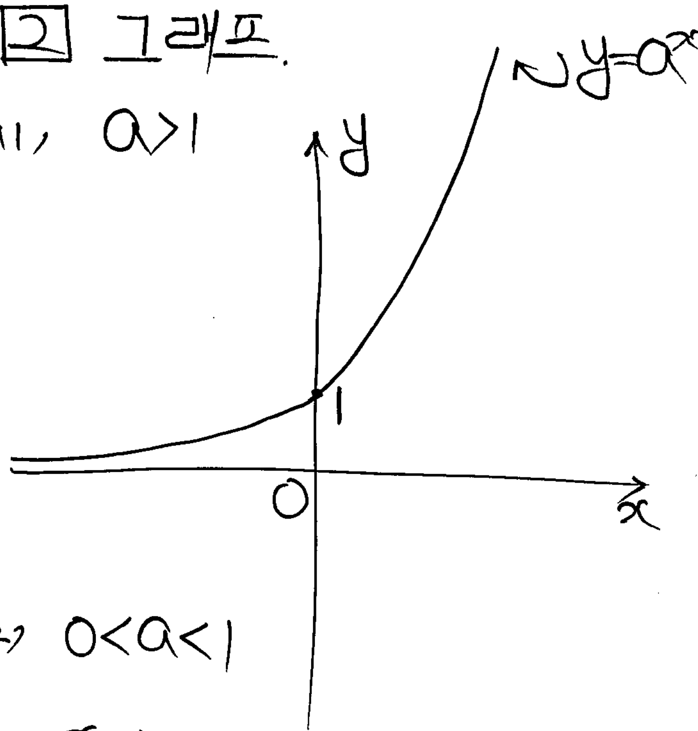
3. 지수 함수

1 정의

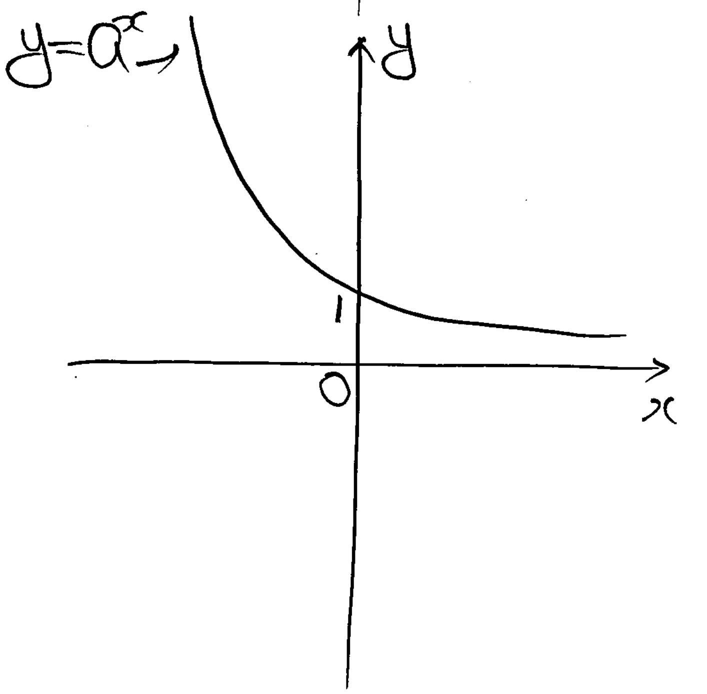
$$y = a^x \quad (a>0,\ a\neq 1)$$

2 그래프.

(1) $a>1$



(2) $0<a<1$



정의역 : $\{x \mid x$는 실수$\}$

치역 : $\{y \mid y>0$ 인 실수$\}$

$a>1$; 증가함수

$0<a<1$; 감소함수

정점 $(0, 1)$

점근선 : $y=0$

3 평행이동 / 대칭이동

$$y=a^{x} \xrightarrow[y\text{축}:\,n]{x\text{축}:\,m} y-n=a^{x-m}$$

$$y=a^{x-m}+n$$

점근선 : $y=n$

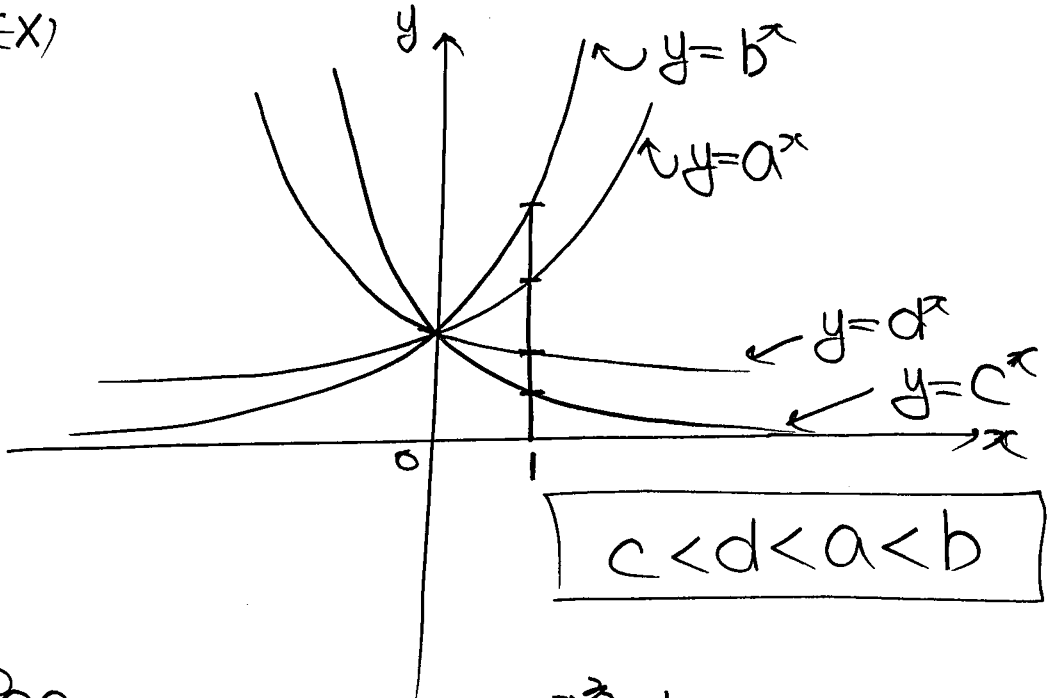
x축 대칭 : $-y=a^{x}$ $\therefore y=-a^{x}$

y축 대칭 : $y=a^{-x}$

원점대칭 : $-y=a^{-x}$ $y=-a^{-x}$

P96

EX)



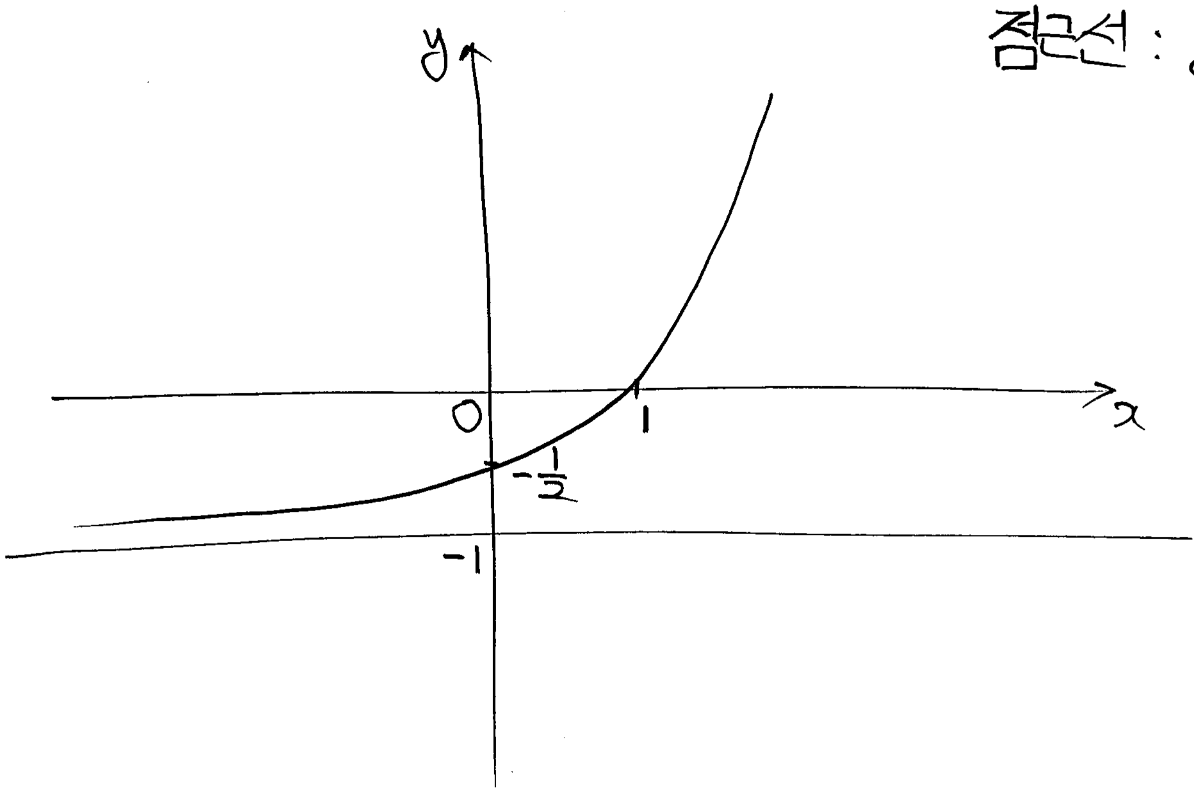
$\boxed{c<d<a<b}$

P99

EX) $y=2^x \xrightarrow[y축:-1]{x축:1} y+1=2^{x-1}$

$y=2^{x-1}-1$

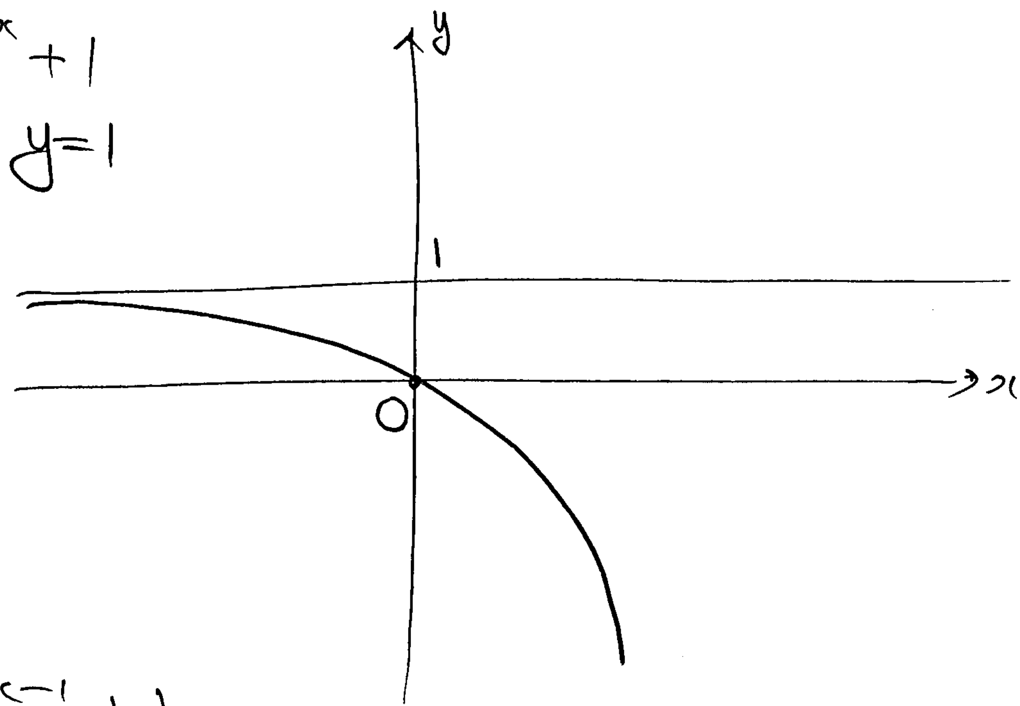
점근선 : $y=-1$



P100

EX) $y=-3^x+1$

점근선 : $y=1$



P101

EX)

(1) $y=2^{x-1}+1$

점근선 : $y=1$

(2) $y=-4\times 2^x-1$ 점근선 : $y=-1$

P103

EX) $\sqrt{8}=\sqrt{2^3}=2^{\frac{3}{2}}$, $\sqrt[3]{16}=\sqrt[3]{2^4}=2^{\frac{4}{3}}$

$\sqrt[5]{64}=\sqrt[5]{2^6}=2^{\frac{6}{5}}$

$$\frac{6}{5}<\frac{4}{3}<\frac{3}{2}$$

$$2^{\frac{6}{5}}<2^{\frac{4}{3}}<2^{\frac{3}{2}}$$

$$\therefore \sqrt[5]{64}<\sqrt[3]{16}<\sqrt{8}$$

P104

EX) (1) $y=f(x)=3^{-x+2}$ $(0\leqq x\leqq 2)$

$M=f(0)=3^2=9$

$m=f(2)=3^0=1$

(2) $y=f(x)=\left(\frac{1}{3}\right)^{-x+2}$ $(0\leqq x\leqq 2)$

$M=f(2)=\left(\frac{1}{3}\right)^0=1$

$m=f(0)=\left(\frac{1}{3}\right)^2=\frac{1}{9}$

P105

EX) (1) $y=4^x-2^{x+2}+1$ $\quad 2^x=t$ $(t>0)$

$f(t)=t^2-4t+1$

$=(t^2-4t+4)-3$

$=(t-2)^2-3$

$\boxed{m=f(2)=-3}$

(2) $y=-\left(\frac{1}{4}\right)^x+\left(\frac{1}{2}\right)^{x-2}+1$

$f(t)=-t^2+4t+1$

$=-(t^2-4t+4)+5$

$=-(t-2)^2+5$

$\boxed{M=f(2)=5}$

$\left(\frac{1}{2}\right)^x=t$ $(t>0)$

$\left(\frac{1}{4}\right)^x=t^2$

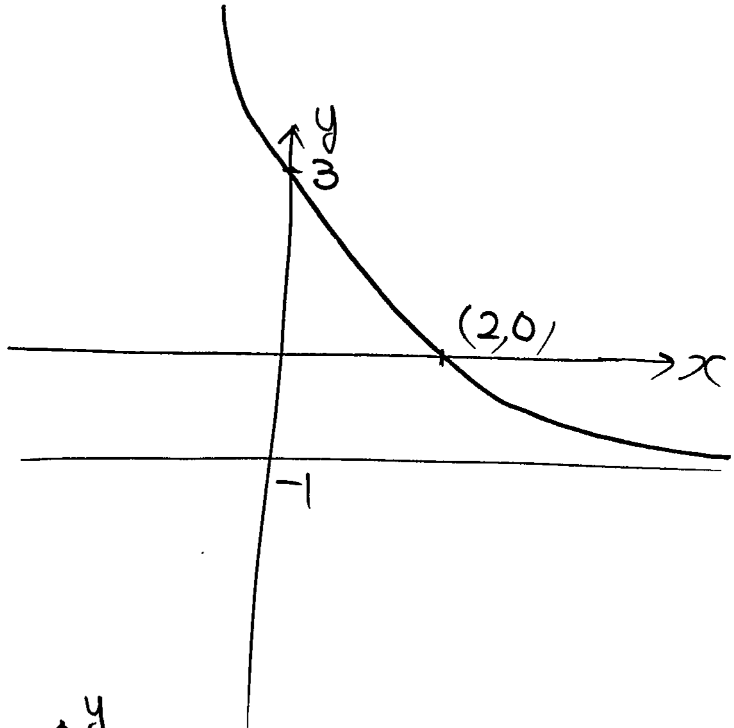
$\left(\frac{1}{2}\right)^{x-2}=\left(\frac{1}{2}\right)^x\cdot\left(\frac{1}{2}\right)^{-2}$

$=4t$

P107

11

(1) $y=\left(\frac{1}{2}\right)^{x-2}-1$



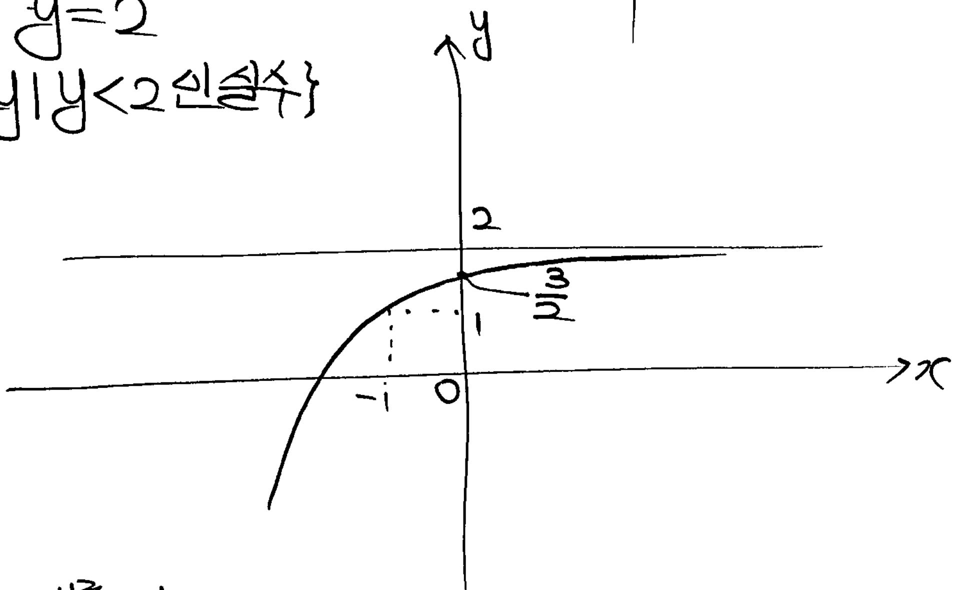
점근선 : $y=-1$

치역 : $\{y \mid y>-1$인 실수$\}$

(2) $y=-\left(\frac{1}{2}\right)^{x+1}+2$

점근선 : $y=2$

치역 : $\{y \mid y<2$인 실수$\}$



12

ㄱ $y=2^x \xrightarrow{y축:\,1} y-1=2^x \quad \therefore y=2^x+1$

ㄴ $y=2^x \xrightarrow{x축대칭} -y=2^x$

$y=-2^x \xrightarrow{x축:\,-\log_2 3} y=-2^{x+\log_2 3}$

$=-3\cdot 2^x$

$\boxed{3=2^{\log_2 3}}$

ㄷ $y=2^x \xrightarrow{y축대칭} y=2^{-x}$

$y=\left(\frac{1}{2}\right)^x \xrightarrow{y축:\,-3} y+3=\left(\frac{1}{2}\right)^x$

$y=\left(\frac{1}{2}\right)^x-3$

$\cancel{ㄹ}$ $y=2^{2x}=4^x$

31

(1) $y=f(x)=4^{x-4}+1 \quad (2\le x\le 4)$

$M=f(4)=4^{0}+1=1+1=2$

$m=f(2)=4^{-2}+1=\frac{1}{16}+1=\frac{17}{16}$

(2) $y=f(x)=3^{-x+1} \quad (-1\le x\le 1)$

$M=f(-1)=3^{2}=9$

$m=f(1)=3^{0}=1$

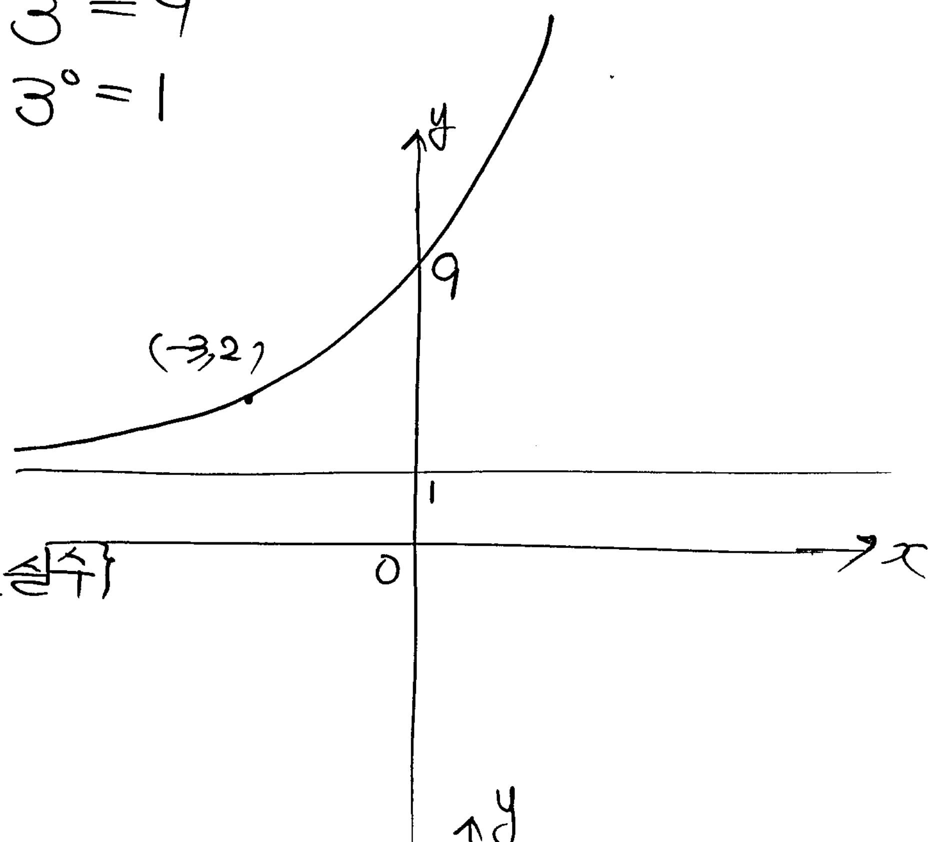
P109

1-11

(1) $y=2^{x+3}+1$

점근선 : $y=1$

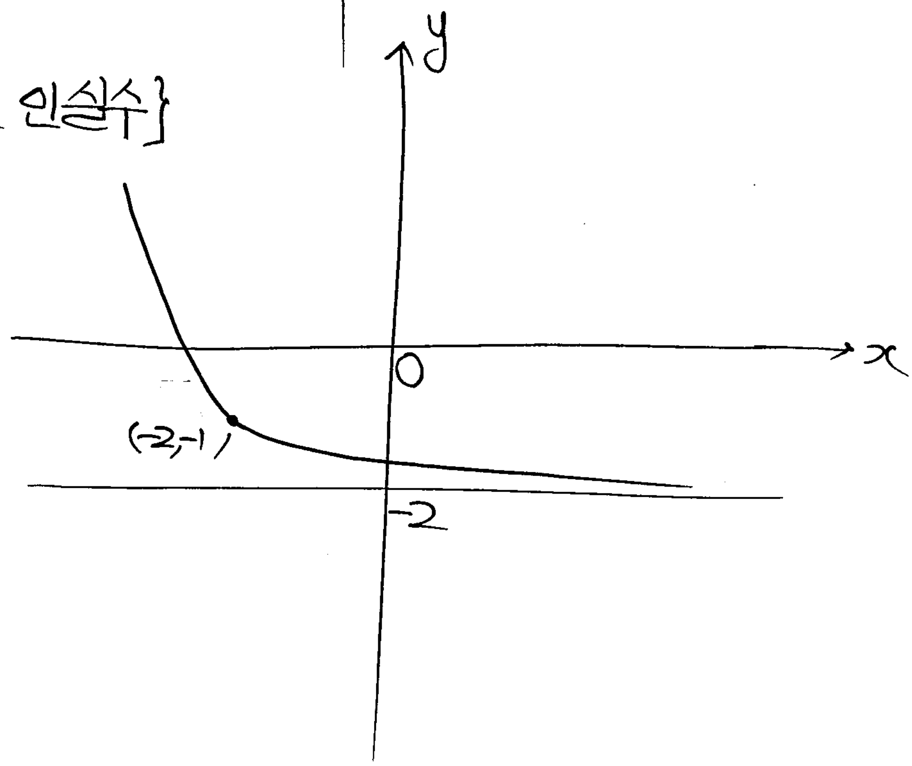
치역 : $\{y \mid y>1$ 인 실수$\}$



(2) $y=4^{x-2}-2$

점근선 : $y=-2$

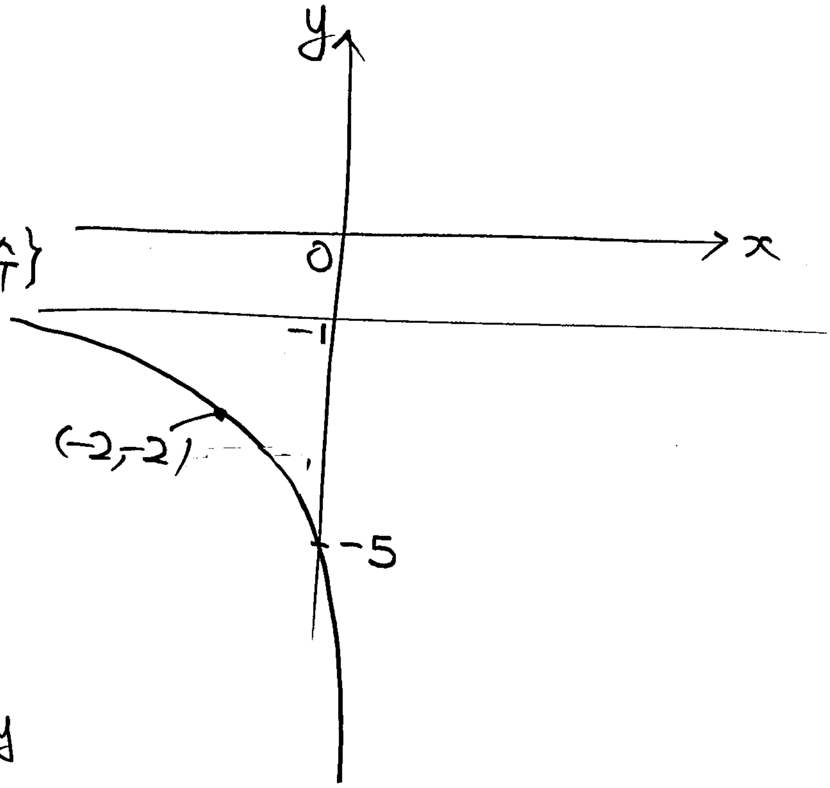
치역 : $\{y \mid y>-2$ 인 실수$\}$



(3) $y=-2^{x+2}-1$

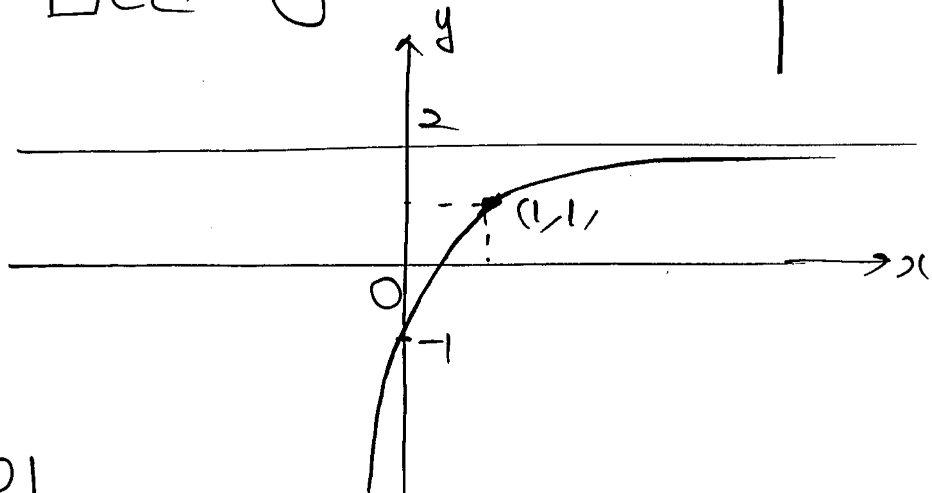
점근선 : $y=-1$

치역 : $\{y \mid y<-1 \text{ 인 실수}\}$



(4) $y=-\left(\frac{1}{3}\right)^{x-1}+2$

점근선 : $y=2$



1-21

(1) $y=3^{2x} \xrightarrow[y\text{축}:\, n]{x\text{축}:\, m} y-n=3^{2(x-m)}$

$y=27\times 3^{2x}-12 \longrightarrow y+12=3^{2x+3}$

$-2m=3\ , \quad -n=12$

$m=-\frac{3}{2}$

$\therefore\ mn=18$

(2) $y=2^{x} \xrightarrow{x축:m,\ y축:n} y-n=2^{x-m}$

$y=2^{x-m}+n$

$(-1,1)$; $1=2^{-1-m}+n$

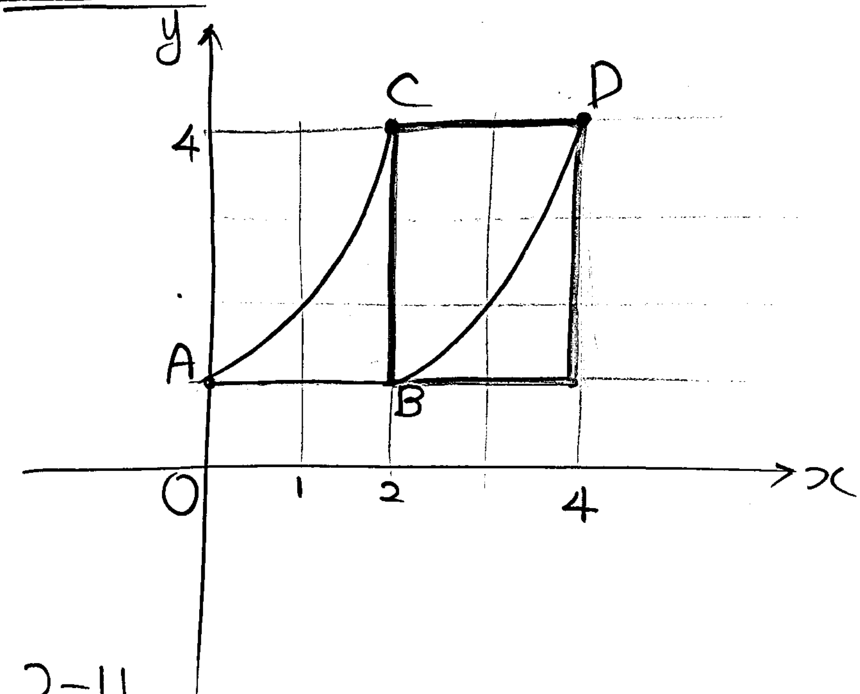
$(0,5)$; $-\underline{|\,5=2^{-m}+n}$

$-4=2^{-m}(2^{-1}-1)$

$2^{-m}=8 \qquad m=-3,\ n=-3$

$m^2+n^2=18$

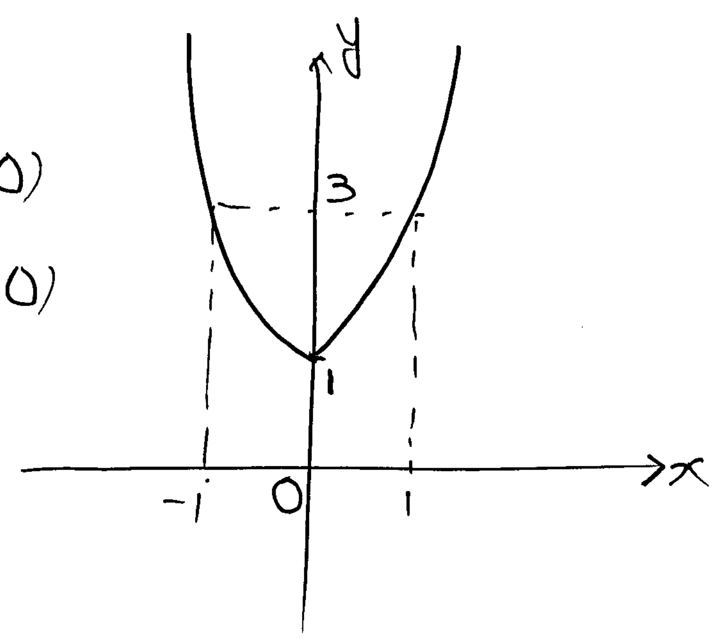
1-3| 6



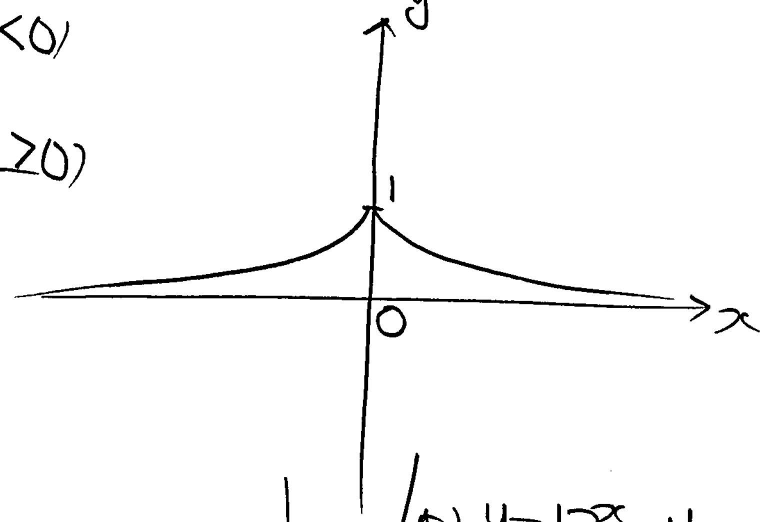
$S=6$

2-1|

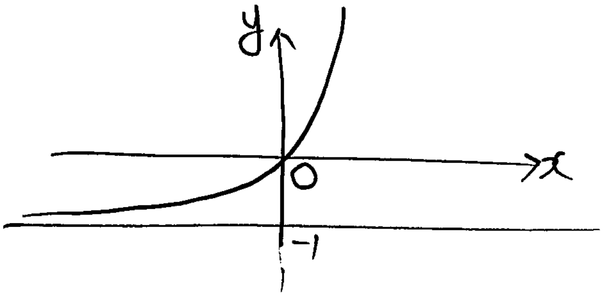
(1) $y=3^{|x|}=\begin{cases} 3^{-x} & (x<0) \\ 3^{x} & (x\geq 0) \end{cases}$



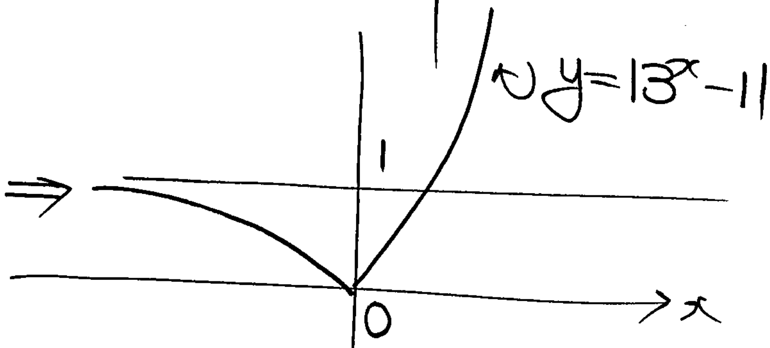
(2) $y=3^{-|x|}=\begin{cases}3^{x} & (x<0)\\ 3^{-x} & (x\geq 0)\end{cases}$



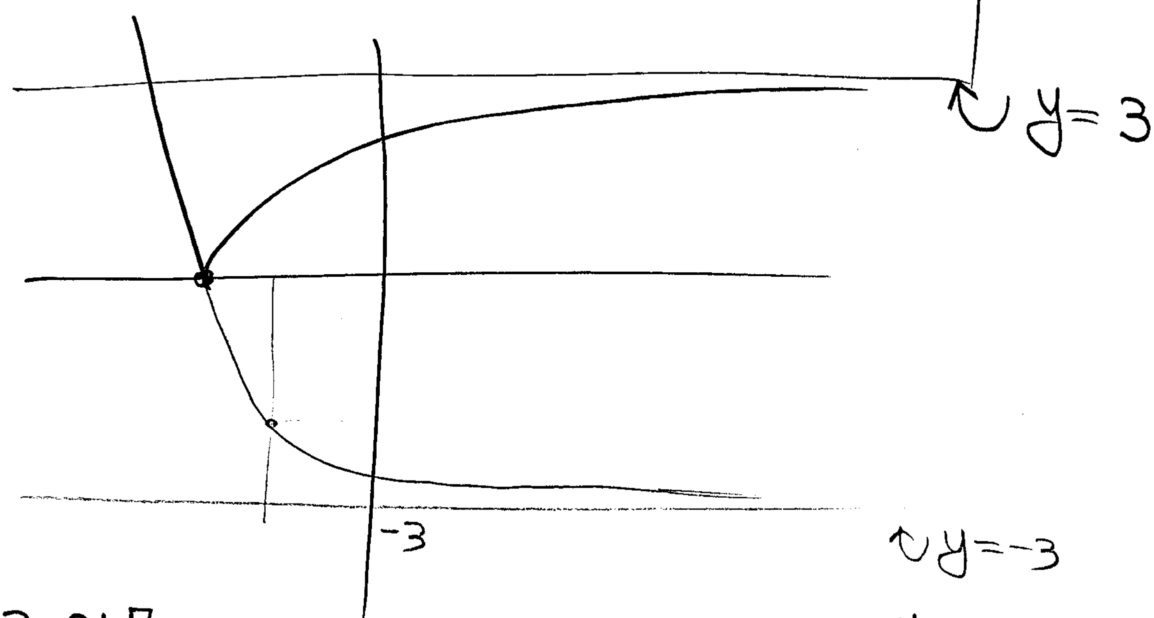
(3) $y=3^{x}-1$



$\Rightarrow$

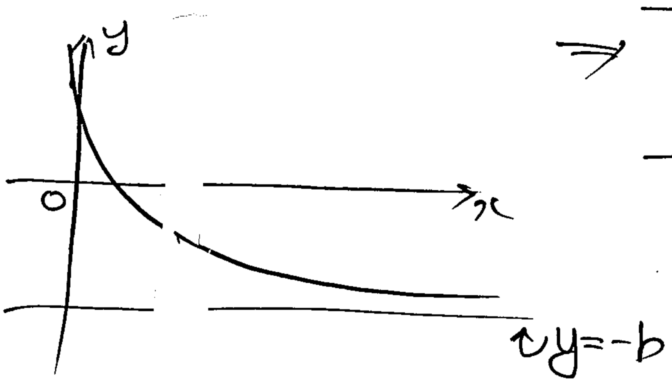


(4) $y=\left|\left(\frac{1}{3}\right)^{x+1}-3\right|$

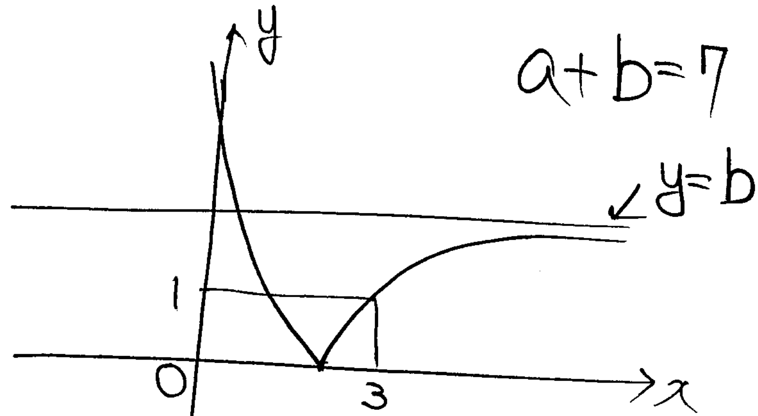


2-217

$y=\left(\frac{1}{2}\right)^{x-a}-b$



$\Rightarrow$

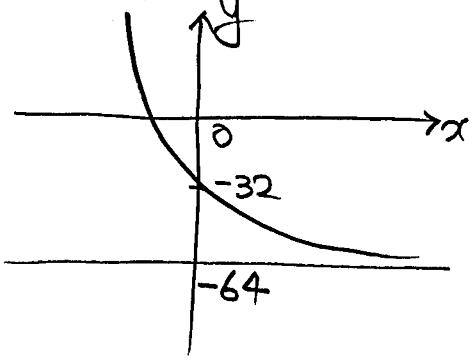


$a+b=7$

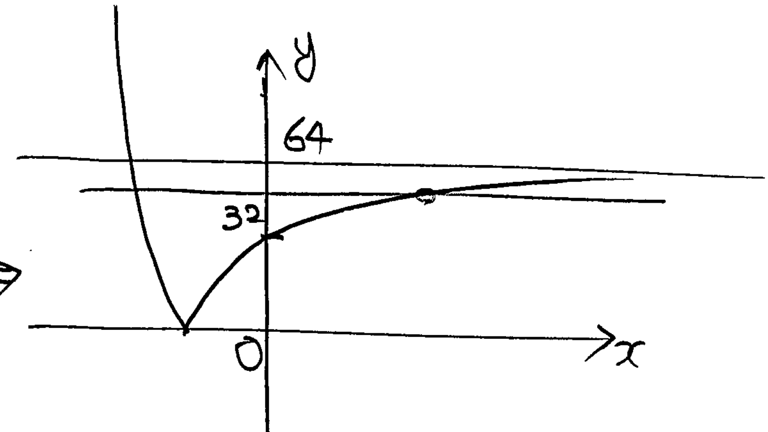
$b=3$ $\quad -\left(\frac{1}{2}\right)^{3-a}+3=1$

$a=4$ $\quad \left(\frac{1}{2}\right)^{3-a}=2$

2-3|31



$\Rightarrow$



$32 < K < 64$

31개

3-11

(1) $f(p) = \frac{1}{2}(3^p - 3^{-p}) = 2$ $\quad \boxed{3^p - 3^{-p} = 4}$

$f(3p) = \frac{1}{2}(3^{3p} - 3^{-3p})$

$= \frac{1}{2}(3^p - 3^{-p})(3^{2p} + 1 + 3^{-2p}) = 38$

$\because \left((3^p - 3^{-p})^2 = 3^{2p} - 2 + 3^{-2p} = 16 \quad 3^{2p} + 3^{-2p} = 18 \right)$

(2) $g(x) = 3^{-x}$

㉠ $3^{-2a} \cdot 3^{-a} \cdot 3^{-2b} = 3^{-3a-2b} = 3^3$ $\quad -3a - 2b = 3$

㉡ $g(a-b) = 3^{-(a-b)} = 3^1$ $\quad -a + b = 1$

$-5a = 5$

$a = -1,\ b = 0$

$3^{2a} + 3^{2b} = 3^{-2} + 3^0$

$= \frac{1}{9} + 1 = \frac{10}{9}$

3-21

㋐ $f(-x) = a^{-x} = \dfrac{1}{a^x} = \dfrac{1}{f(x)}$

㋑ $f(x) = a^x$

$\sqrt{f(2x)} = (f(2x))^{\frac{1}{2}} = (a^{2x})^{\frac{1}{2}} = a^x$

$\cancel{㋒}$ $f(x^3) = a^{x^3}$

$\{f(x)\}^3 = (a^x)^3 = a^{3x}$

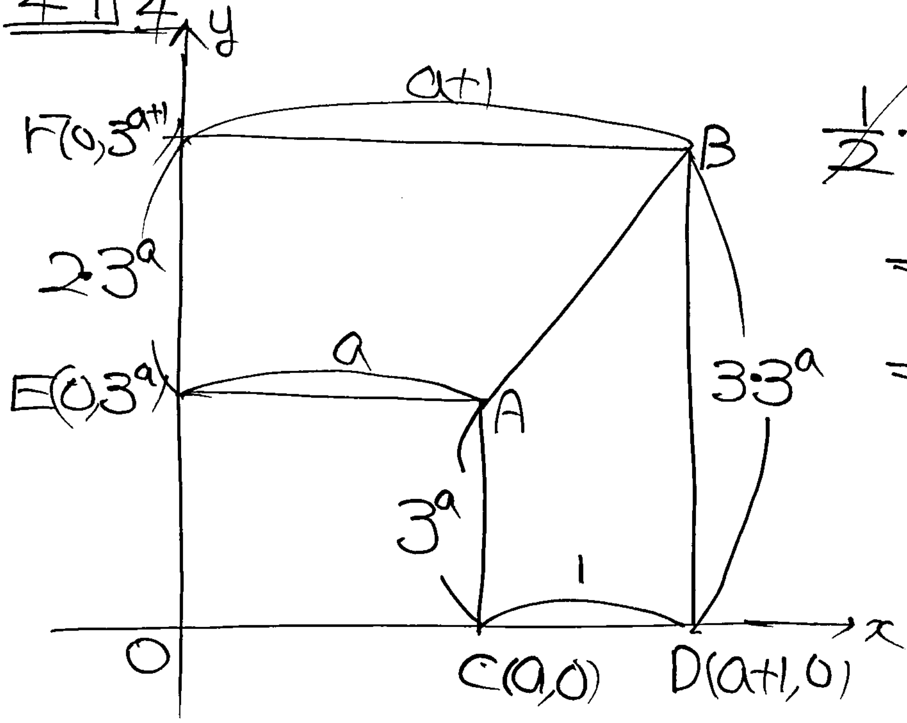
3-31 8

$f(x)f(y) = (2^x+2^{-x})(2^y+2^{-y}) = 2^{x+y}+2^{x-y}+2^{-x+y}+2^{-x-y} = 6$

$g(x)\cdot g(y) = (2^x-2^{-x})(2^y-2^{-y}) = 2^{x+y}-2^{x-y}-2^{-x+y}+2^{-x-y} = 10$

$f(x+y) = 2^{x+y}+2^{-x-y} = 8$

4-11 $\frac{3}{4}$



$\cancel{\frac{1}{2}}\cdot 4\cdot 3^a\cdot 1 : \cancel{\frac{1}{2}}\cdot(2a+1)\cdot 2\cdot 3^a$

$= 4 : 2(2a+1)$

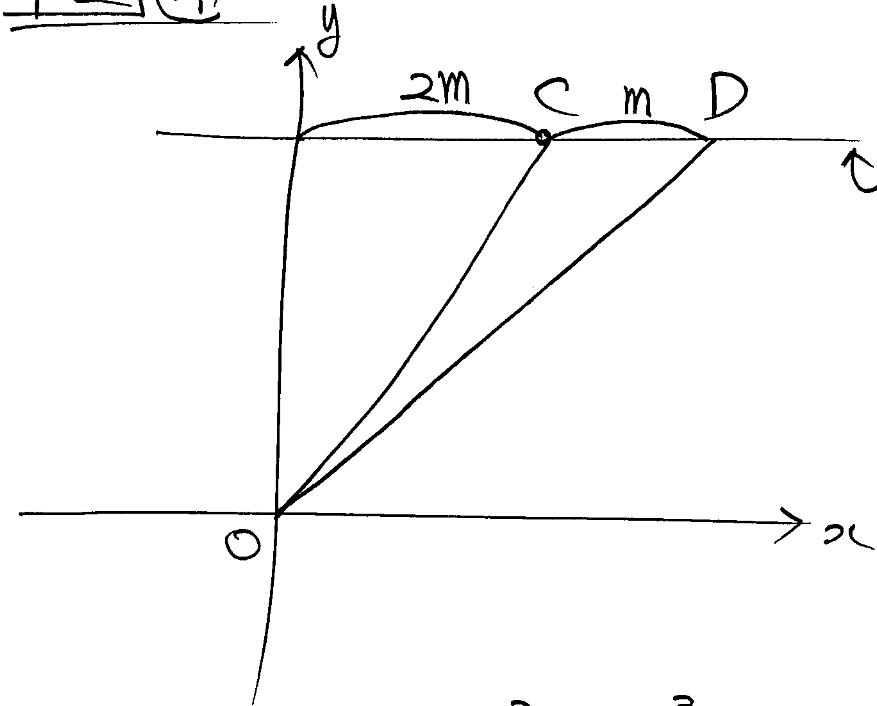
$= 4 : 5$

$2(2a+1) = 5$

$2a+1 = \dfrac{5}{2}$

$a = \dfrac{3}{4}$

4-21 ④



C(2m, k)

D(3m, k)

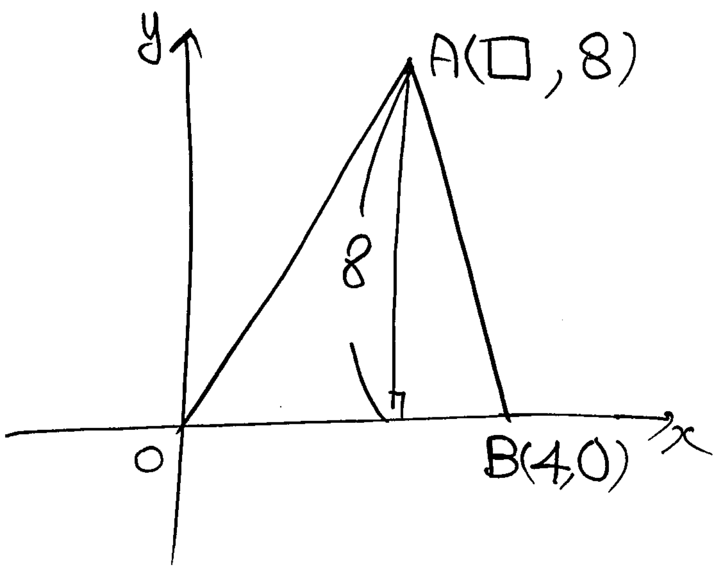
$2^{2m}=k$

$a^{3m}=k$

$2^2=a^3$

$a=2^{\frac{2}{3}}=\sqrt[3]{2^2}=\sqrt[3]{4}$

4-3 71



㋐ $2^x-1=8$

$2^x=9$

㋑ $2^{-x}+\frac{a}{9}=8$

$\frac{1}{9}+\frac{a}{9}=8$

$1+a=72$

$a=71$

5-11

(1) $2^{0.5}=2^{\frac{1}{2}}$, $\sqrt[5]{4}=\sqrt[5]{2^2}=2^{\frac{2}{5}}$, $0.5^{-\frac{3}{4}}=(2^{-1})^{-\frac{3}{4}}=2^{\frac{3}{4}}$

$\frac{2}{5}<\frac{1}{2}<\frac{3}{4}$ $\therefore 2^{\frac{2}{5}}<2^{\frac{1}{2}}<2^{\frac{3}{4}}$

$\therefore \sqrt[5]{4}<2^{0.5}<0.5^{-\frac{3}{4}}$

(2) $\left(\frac{1}{2}\right)^{\frac{1}{3}}$ $\left(\frac{1}{3}\right)^{\frac{1}{4}}$ $\left(\frac{1}{5}\right)^{\frac{1}{6}}$

$\left(\frac{1}{2}\right)^{4}$ $\left(\frac{1}{3}\right)^{3}$ $\left(\frac{1}{5}\right)^{2}$ $\left(\frac{1}{3}\right)^{\frac{1}{4}}<\left(\frac{1}{5}\right)^{\frac{1}{6}}<\left(\frac{1}{2}\right)^{\frac{1}{3}}$

$\frac{1}{16}$ $\frac{1}{27}$ $\frac{1}{25}$

① ③ ②

(3) $2^{444}=(2^4)^{111}$, $3^{333}=(3^3)^{111}$, $5^{222}=(5^2)^{111}$

$2^4<5^2<3^3$

$\therefore 2^{444}<5^{222}<3^{333}$

(4) $\sqrt[3]{0.2}=\sqrt[3]{\frac{1}{5}}=\left(\frac{1}{5}\right)^{\frac{1}{3}}$

$\sqrt[4]{0.04}=\sqrt[4]{\left(\frac{1}{5}\right)^2}=\left(\frac{1}{5}\right)^{\frac{2}{4}}=\left(\frac{1}{5}\right)^{\frac{1}{2}}$

$\sqrt[15]{0.008}=\sqrt[15]{\left(\frac{1}{5}\right)^3}=\left(\frac{1}{5}\right)^{\frac{1}{5}}$

$\frac{1}{5}<\frac{1}{3}<\frac{1}{2}$

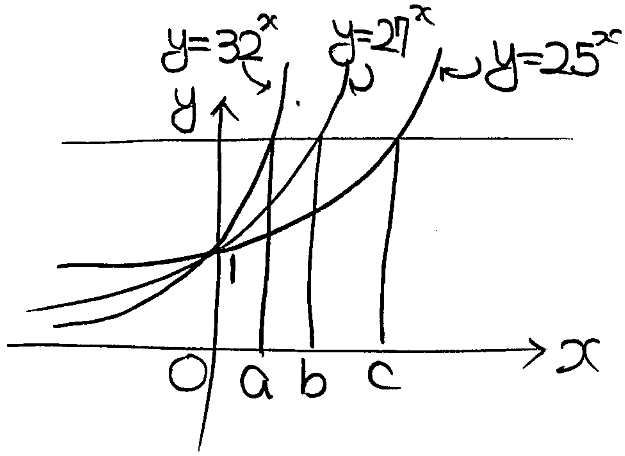
$\sqrt[4]{0.04}<\sqrt[3]{0.2}<\sqrt[15]{0.008}$

5-21 ①

$$a^m < \underset{\sim\sim\sim}{a^n < b^n} < b^m$$

$$a < b \Rightarrow \quad 0 < a < 1 < b \ , \ m > n$$

5-31



(1) $(2^5)^a = (3^3)^b = (5^2)^c$

$$a < b < c$$

(2) ㋐ $2^x = 3^y$

$$2^{6x} = 3^{6y}$$

$$(2^3)^{2x} = (3^2)^{3y} \qquad 3y < 2x$$

㋑ $2^x = 5^z$

$$2^{10x} = 5^{10z} \qquad 2x < 5z \qquad \therefore 3y < 2x < 5z$$

$$(2^5)^{2x} = (5^2)^{5z}$$

6-11

(1) $f(x) = 3^{3-2x}$

$(0 \leq x \leq 2)$

$$M = f(0) = 3^3 = 27$$

$$m = f(2) = 3^{-1} = \frac{1}{3}$$

(2) $f(x) = \left(\frac{1}{2}\right)^{-(x-1)^2+2}$ $(-1 \leq x \leq 2)$

$M = f(-1) = \left(\frac{1}{2}\right)^{-2} = 2^2 = 4$

$m = f(1) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

<u>6-2| 63</u>

$g(x) = 2^{x^2-6x-1} = 2^{(x-3)^2-10}$ $(-1 \leq x \leq 4)$

$M = g(-1) = 2^6 = 64$

$a = -1,\ b = 64$ $a+b = 63$

<u>6-3| ③</u>

$f(g(x)) = a^{x^2+2x+3} = a^{(x+1)^2+2}$

$f(g(-1)) = a^2 = 4$ $a = 2$ $(\because a > 1)$

$(g \circ f)(1) = g(f(1))$

$= g(2)$

$= 4+4+3 = 11$

$f(1) = a^1 = a = 2$

<u>7-11</u>

(1) $y = f(t)$

$f(t) = t^2 + 2t$ $(1 \leq t \leq 4)$

$= (t+1)^2 - 1$

$M = f(4) = 24$

$m = 3$

$4^{-x} = \left(\frac{1}{4}\right)^x = \left(\left(\frac{1}{2}\right)^x\right)^2$

$\left(\frac{1}{2}\right)^{x-1} = 2\left(\frac{1}{2}\right)^x$

$\left(\frac{1}{2}\right)^x = t$

(2) $2^x = t \quad (\frac{1}{2} \leq t \leq 4)$

$y = f(t) = 4t^2 - 8t + 1 = 4(t-1)^2 - 3$

$$\boxed{\begin{aligned} M &= f(4) = 33 \\ m &= f(1) = -3 \end{aligned}}$$

7-21

(1) $y = 4^x + 4^{-x} - 4(2^x + 2^{-x}) + 2$

$2^x + 2^{-x} = t \quad (t \geq 2)$

$2^x + 2^{-x} \geq 2\sqrt{2^x \cdot 2^{-x}}$

$= 2$

$4^x + 2 + 4^{-x} = t^2$

$y = f(t)$

$f(t) = t^2 - 4t \quad (t \geq 2)$

$= (t^2 - 4t + 4) - 4$

$= (t-2)^2 - 4$

$m = f(2) = -4$

(2) $3^x + 3^{-x} = t \quad (t \geq 2)$

$3^x + 3^{-x} \geq 2\sqrt{3^x \cdot 3^{-x}} = 2$

$9^x + 2 + 9^{-x} = t^2 \qquad 9^x + 9^{-x} = t^2 - 2$

$y = f(t)$

$f(t) = 6t - (t^2 - 2) + 2 \quad (t \geq 2)$

$= -t^2 + 6t + 4$

$= -(t-3)^2 + 13$

$M = f(3) = 13$

7-31 ⑤

$f(x) = 3^a(\underbrace{3^x + 3^{-x}}) + 2 \geq 3^a \cdot 2 + 2 = 20$

$\because \left(3^x + 3^{-x} \geq 2\sqrt{3^x \cdot 3^{-x}} = 2\right)$ $\quad 3^a = 9 \quad \boxed{a=2}$

P123

EX1) $2^{3x+3} = 2^{2x-2}$ $\quad 3x+3 = 2x-2 \quad \boxed{x=-5}$

EX2) $3^x = t \quad (t>0)$

$t^2 - 2t - 3 = 0$

$(t-3)\underbrace{(t+1)}_{>0} = 0$

$t = 3$

$3^x = 3$

$\boxed{x=1}$

P125

EX1) (1) $2^x > 2^3$ $\quad \boxed{x>3}$

(2) $\left(\frac{1}{2}\right)^{2x} < \left(\frac{1}{2}\right)^{3x-6}$ $\quad 2x > 3x-6$

$-x > -6 \quad \therefore x < 6$

EX2) $2^x = t \quad (t>0)$

$t^2 + 2t - 8 \leq 0$

$\underbrace{(t+4)}_{>0}(t-2) \leq 0$

$t \leq 2$

$2^x \leq 2^1$

$\boxed{\therefore x \leq 1}$

P126

EX) $x^{2x+6} = x^{x^2+x}$ $(x>0)$

i) $x=1$; $1^8 = 1^2$ (O) $\boxed{x=1}$

ii) $x \neq 1$; $2x+6 = x^2+x$

$x^2-x-6=0$ $\boxed{x=3}$

$(x-3)(x+2)=0$

P127

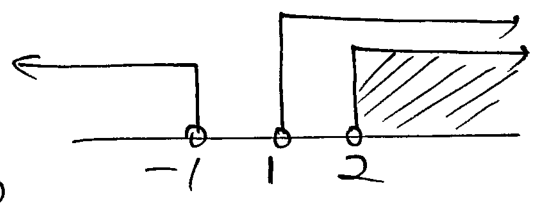
EX) $x^{2x+3} < x^{2x^2-1}$ $(x>0)$

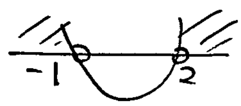
㉠ $x=1$; $1^5 < 1^1$ (X)

㉡ $x>1$; $2x+3 < 2x^2-1$

$x^2-x-2>0$

$(x-2)(x+1)>0$

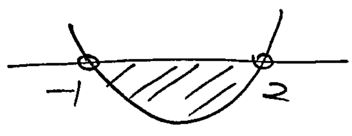


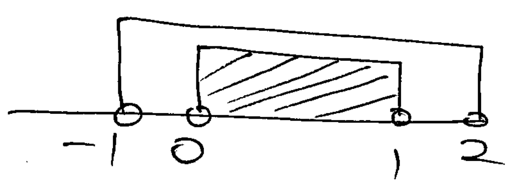


$x<-1$ 또는 $x>2$ $\boxed{x>2}$

㉢ $0<x<1$; $2x+3 > 2x^2-1$

$(x-2)(x+1)<0$





$-1<x<2$

㉠, ㉡, ㉢ 에서

$\boxed{0<x<1}$ $\boxed{0<x<1 \text{ 또는 } x>2}$

P127

<개념>

1|

(1) $3^{2x+1}=3^4$ $2x+1=4$ $x=\frac{3}{2}$

(2) $5^{2x+1}=5^{3x}$ $2x+1=3x$ $x=1$

2|

(1) $2^x=t\ (t>0)$ $t=2$

$t^2-t-2=0$ $2^x=2$

$(t-2)\underbrace{(t+1)}_{>0}=0$ $\boxed{x=1}$

(2) $3^x=t\ (t>0)$ $t=1$

$3^x=1$

$t+\frac{1}{t}=2$ $\boxed{x=0}$

$t^2-2t+1=0$

$(t-1)^2=0$

3|

(1) $5^x>5^{-3}$ $\boxed{x>-3}$

(2) $\left(\frac{3}{4}\right)^{3x}>\left(\frac{3}{4}\right)^1$ $3x<1$ $\boxed{x<\frac{1}{3}}$

(3) $3^{-x-1}<3^{2x}$ $-x-1<2x$

$-3x<1$ $\boxed{x>-\frac{1}{3}}$

4|

$3^x=t\ (t>0)$ $t<9$

$3^x<3^2$

$t^2-8t-9<0 \Rightarrow (t-9)\underbrace{(t+1)}_{>0}<0$ $\boxed{x<2}$

8-11

(1) $3^{-x^2+4}=3^{3x}$

$-x^2+4=3x$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$\boxed{x=-4 \text{ 또는 } x=1}$

(2) $2\sqrt{2}=2\cdot 2^{\frac{1}{2}}=2^{1+\frac{1}{2}}=2^{\frac{3}{2}}$

$2^{\frac{3}{2}x^2}=2^{2x+2}$

$\frac{3}{2}x^2=2x+2$

$3x^2-4x-4=0$

$(3x+2)(x-2)=0$

$\boxed{x=-\frac{2}{3} \text{ 또는 } x=2}$

(3) $x^2-1=0$, $x+1=0$ $\boxed{x=-1}$

(4) ㉠ $x-1=2$ $\boxed{x=3}$

㉡ $x-4=0$ $\boxed{x=4}$

8-21 ③

$2^{\alpha}\cdot 8=49$

$2^{\alpha}=\frac{49}{8}=6.xx$

$\boxed{2<\alpha<3}$

8-31

(1) $x^{2x-1} = x^{x+2}$ $(x>0)$

㋐ $x=1$; $1^1 = 1^3$ $\boxed{\therefore x=1}$

㋑ $x \neq 1$; $2x-1 = x+2$ $\boxed{x=3}$

(2) $(x+1)^{x^2} = (x+1)^{2x}$ $(x>-1)$

㋐ $x+1=1$ $1^0 = 1^0 = 1$ $\boxed{x=0}$

$x=0$

㋑ $x+1 \neq 1$; $x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$ $\boxed{x=2}$

P131

9-11

(1) $3^x = t \quad (t>0)$

$t^2 - 30t + 81 = 0$

$(t-3)(t-27) = 0$

$t = 3$ 또는 $t = 27$

$3^x = 3$ 또는 $3^x = 27$

$\boxed{x = 1 \text{ 또는 } x = 3}$

(2) $2^x = t \quad (t>0)$

$t^2 - 4t - 32 = 0$

$(t-8)(t+4) = 0$

$t = 8$

$2^x = 8$

$\boxed{x = 3}$

(3) $2^{\frac{x}{2}} = t \quad (t>0)$

$t(t-2) = 8$

$t^2 - 2t - 8 = 0$

$(t-4)(t+2) = 0$

$t = 4$

$2^{\frac{x}{2}} = 2^2$

$\boxed{x = 4}$

(4) $2^x = t \quad (t>0)$

$t^3 - 12t^2 + 32t = 0$

$t(t-4)(t-8) = 0$

$t = 4$ 또는 $t = 8$

$2^x = 4$ 또는 $2^x = 8$

$\boxed{x = 2 \text{ 또는 } x = 3}$

9-21

(1) $2^x + 2^{-x} = t$

$2^x + 2^{-x} \geq 2\sqrt{2^x \cdot 2^{-x}} = 2 \qquad (t \geq 2)$

$4^x + 2 + 4^{-x} = t^2 \qquad 4^x + 4^{-x} = t^2 - 2$

$2(t^2-2) - 3t - 1 = 0$

$2t^2 - 3t - 5 = 0$

$(2t-5)(t+1)=0$

$t=\frac{5}{2}$

$2^x+2^{-x}=\frac{5}{2}$

$2^x=a \quad (a>0)$

$a+\frac{1}{a}=\frac{5}{2}$

$2a^2-5a+2=0$

$(2a-1)(a-2)=0$

$a=\frac{1}{2}$ 또는 $a=2$

$2^x=2^{-1}$ 또는 $2^x=2^1$

$\boxed{x=-1 \text{ 또는 } x=1}$

(2) $(3+2\sqrt{2})^x=t \quad (t>0)$

$(3-2\sqrt{2})^x=\frac{1}{t}$

$t+\frac{1}{t}=6$

$t^2-6t+1=0$

$t=3\pm2\sqrt{2}$

$(3+2\sqrt{2})^x=3+2\sqrt{2}$ $\boxed{x=1}$

$(3+2\sqrt{2})^x=3-2\sqrt{2}$ $\boxed{x=-1}$

9-31

(1) $2^x=t \quad (t>0)$

$t^2-7t+12=0$

$(t-3)(t-4)=0$

$t=3$ 또는 $t=4$

$2^x=3$ 또는 $2^x=4$

$2^\alpha=3$, $2^\beta=4$

$2^{2\alpha}+2^{2\beta}=9+16$

$=25$

(2) $4^x = t \ (t>0)$

$t^2 - 12t + 9 = 0$ 두근 $4^\alpha, 4^\beta$

$4^\alpha + 4^\beta = 12$

$4^\alpha \cdot 4^\beta = 9 \Rightarrow 2^{2\alpha+2\beta} = 9$ $\boxed{2^{\alpha+\beta} = 3}$

$(2^\alpha + 2^\beta)^2 = 4^\alpha + 2 \cdot 2^{\alpha+\beta} + 4^\beta$ $\therefore 2^\alpha + 2^\beta = 3\sqrt{2}$

$= 12 + 6 = 18$

P133

10-11

(1) $2^{-2x+4} < 2^5$ $\quad -2x + 4 < 5$ $\quad \boxed{\therefore x > -\frac{1}{2}}$

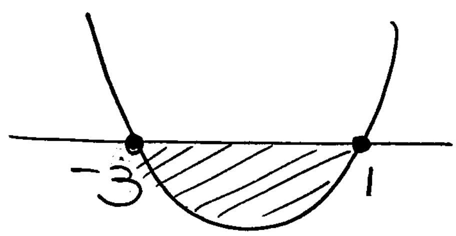
$-2x < 1$

(2) $5^{-2} < 5^x < 5^3$ $\quad \boxed{\therefore -2 < x < 3}$

(3) $\left(\frac{3}{2}\right)^{x^2} \le \left(\frac{3}{2}\right)^{-2x+3}$

$x^2 \le -2x + 3$

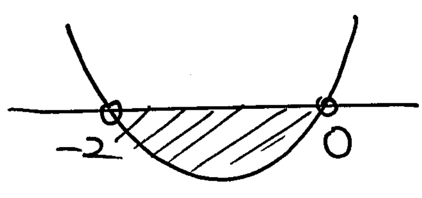
$x^2 + 2x - 3 \le 0$



$(x+3)(x-1) \le 0$

$\boxed{-3 \le x \le 1}$

(4) $2^{2x^2} < 2^{-4x}$



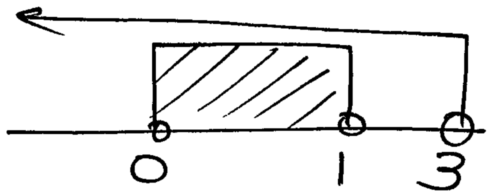
$2x^2 < -4x$

$2x^2 + 4x < 0$ $\quad \boxed{-2 < x < 0}$

$2x(x+2) < 0$

10-21

(1) $x^{3x-2} > x^{x+4}$ $(x>0)$



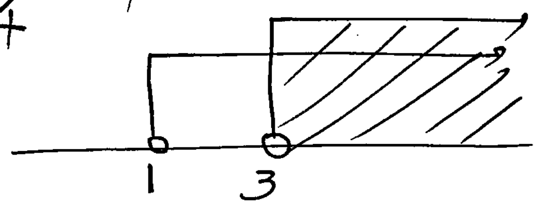
㉠ $0<x<1$; $3x-2<x+4$

$2x<6$

$x<3$

$\therefore 0<x<1$

㉡ $x>1$; $3x-2>x+4$



$2x>6$

$x>3$

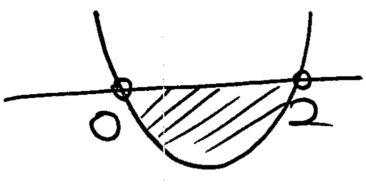
$\therefore x>3$

(2) $(x^2-2x+1)^{x-1} < (x^2-2x+1)^0$ $(x \neq 1)$

㉠ $0<x^2-2x+1<1$

$x^2-2x<0$

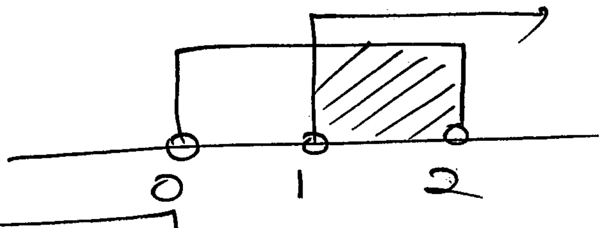
$x(x-2)<0$



$0<x<2$

$x-1>0$

$x>1$

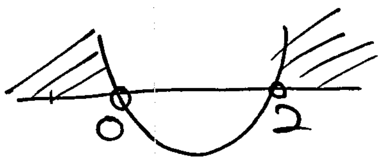


$1<x<2$

㉡ $x^2-2x+1>1$

$x^2-2x>0$

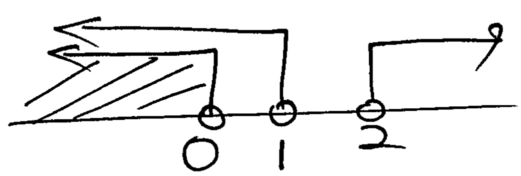
$x(x-2)>0$



$x<0$ 또는 $x>2$

$x-1<0$

$x<1$



$x<0$

10-31 7

$0<a<b<1$

㉠ $a^6 \le a^{6-x} \cdot b^x$ $\qquad \therefore 0 \le x$

$1 \le \left(\frac{b}{a}\right)^x$

$\left(\frac{b}{a}\right)^0 \le \left(\frac{b}{a}\right)^x$

㉡ $a^{6-x} b^x \le b^6$ $\qquad \therefore x-6 \le 0$

$a^{6-x} \cdot b^{x-6} \le 1$ $\qquad x \le 6$

$\left(\frac{b}{a}\right)^{x-6} \le \left(\frac{b}{a}\right)^0$

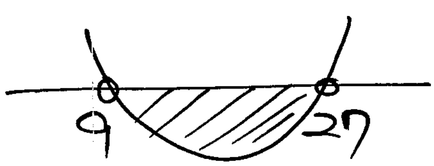
㉠, ㉡에서 $0 \le x \le 6$ $\quad \boxed{7개}$

11-11

(1) $3^x = t \quad (t>0)$ $\qquad 9<t<27$

$t^2 - 36t + 243 < 0$ $\qquad 3^2 < 3^x < 3^3$

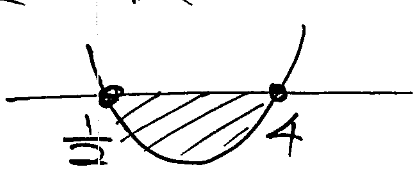
$(t-9)(t-27)<0$ $\qquad \boxed{\therefore 2<x<3}$



(2) $2^x = t \quad (t>0)$ $\qquad \frac{1}{2} \le t \le 4$

$2t^2 - 9t + 4 \le 0$ $\qquad 2^{-1} \le 2^x \le 2^2$

$(2t-1)(t-4) \le 0$ $\qquad \boxed{\therefore -1 \le x \le 2}$



(3) $3^x = t \quad (t>0)$

$3t^2 - 26t - 9 \geq 0$

$(\underbrace{3t+1}_{>0})(t-9) \geq 0$

$t \geq 9$

$3^x \geq 3^2$

$\boxed{\therefore x \geq 2}$

(4) $\left(\frac{1}{2}\right)^x = t \quad (t>0)$

$t^2 - \frac{1}{2}t - 3 > 0$

$2t^2 - t - 6 > 0$

$(\underbrace{2t+3}_{>0})(t-2) > 0$

$t > 2$

$\left(\frac{1}{2}\right)^x > \left(\frac{1}{2}\right)^{-1}$

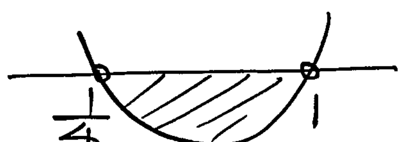
$\boxed{x < -1}$

11-21

(1) $a^x = t \quad (t>0)$

$4t^2 - 5t + 1 < 0$

$(4t-1)(t-1) < 0$



$a^2 = \frac{1}{4}$

$\boxed{a = \frac{1}{2}}$

$\frac{1}{4} < t < 1$

$\frac{1}{4} < a^x < 1$

$\Leftrightarrow \quad 0 < x < 2$

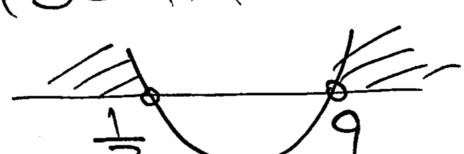
$a^0 > a^x > a^2$

$a^2 < a^x < 1$

(2) $a^x = t \quad (t>0)$

$3t^2 - 28t + 9 > 0$

$(3t-1)(t-9) > 0$



$t < \frac{1}{3}$ 또는 $t > 9$

$a^x < \frac{1}{3}$ 또는 $a^x > 9$

$\Leftrightarrow \quad x < -1$ 또는 $x > 2$

$a^x < a^{-1}$ 또는 $a^x > a^2$

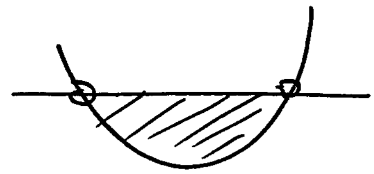
$\boxed{a = 3}$

11-31

(1) $a^x = t \quad (t>0)$

$t^2 - a^2 t - a^{-2} t + 1 < 0$

$(t - a^2)(t - a^{-2}) < 0$



㉠ $a>1$; $a^{-2} < t < a^2$

$a^{-2} < a^x < a^2$ $\qquad \therefore -2 < x < 2$

㉡ $0<a<1$; $a^2 < t < a^{-2}$

$a^2 < a^x < a^{-2}$ $\qquad \therefore -2 < x < 2$

$2 > x > -2$

㉠, ㉡에서 $\boxed{-2<x<2}$

(2) $a^x = t \quad (t>0)$

$\dfrac{t^2}{a^2} - 1 < at - \dfrac{t}{a^3}$ $\qquad t < a^3$

$a^x < a^3$

$at^2 - (a^4 - 1)t - a^3 < 0$

$\underbrace{(at+1)}_{>0}(t - a^3) < 0$

$\boxed{\begin{array}{ll} 0<a<1 \text{ 일때} & x>3 \\ a>1 \text{ 일때} & x<3 \end{array}}$

12-1

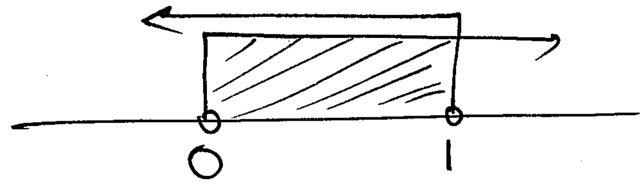
(1) $3^x = t \quad (t>0)$ 두근 α, β

$t^2 - 2t + a = 0$ 두근 $2^\alpha, 2^\beta$

㉠ $D/4 = 1 - a > 0 \quad \therefore a < 1$

㉡ $2^\alpha + 2^\beta = 2 > 0$

$2^\alpha \cdot 2^\beta = a > 0$



$\boxed{0 < a < 1}$

(2) $2^x = t \quad (t>0)$ 두근 α, β

$t^2 - 2^a \cdot t + 2^{a+1} = 0$ 두근 $2^\alpha, 2^\beta$

㉠ $D = 2^{2a} - 4 \cdot 2^{a+1} = 2^{2a} - 2^{a+3} \geq 0$

$2^{2a} \geq 2^{a+3}$ $\quad 2a \geq a+3$

$a \geq 3$

㉡ $2^\alpha + 2^\beta = 2^a > 0$

$2^\alpha \cdot 2^\beta = 2^{a+1} > 0$

㉠, ㉡에서 $\boxed{a \geq 3}$

12-2

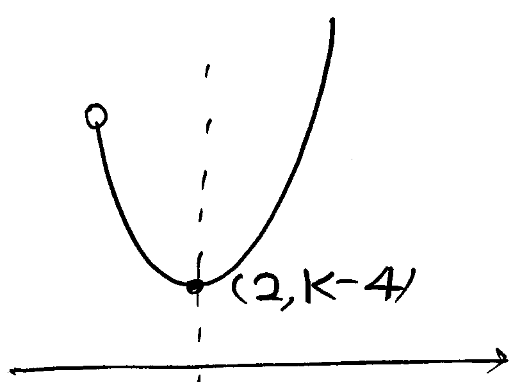
(1) $2^x = t \quad (t>0)$

$t^2 - 4t + K \geq 0$

$f(t) = t^2 - 4t + K$

$= (t^2 - 4t + 4) + K - 4$

$= (t-2)^2 + K - 4$



$K - 4 \geq 0 \quad \boxed{\therefore K \geq 4}$

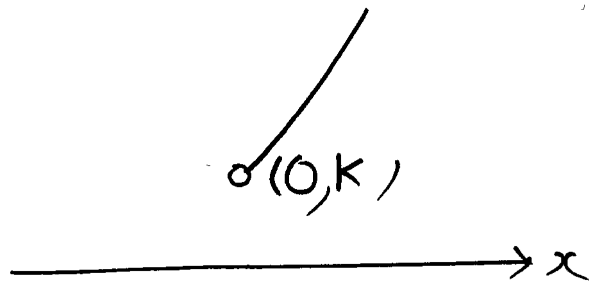
(2) $\left(\frac{1}{2}\right)^x = t \quad (t>0)$

$t^2+4t+k>0$

$f(t)=t^2+4t+k$

$=(t+2)^2+k-4$

$f(0)=k$



$\boxed{k \geq 0}$

12-31

$4^x+4^{-x}-2(2^x+2^{-x})+a=0$

$2^x+2^{-x} \geq 2\sqrt{2^x \cdot 2^{-x}}=2$

$2^x+2^{-x}=t \quad (t \geq 2)$

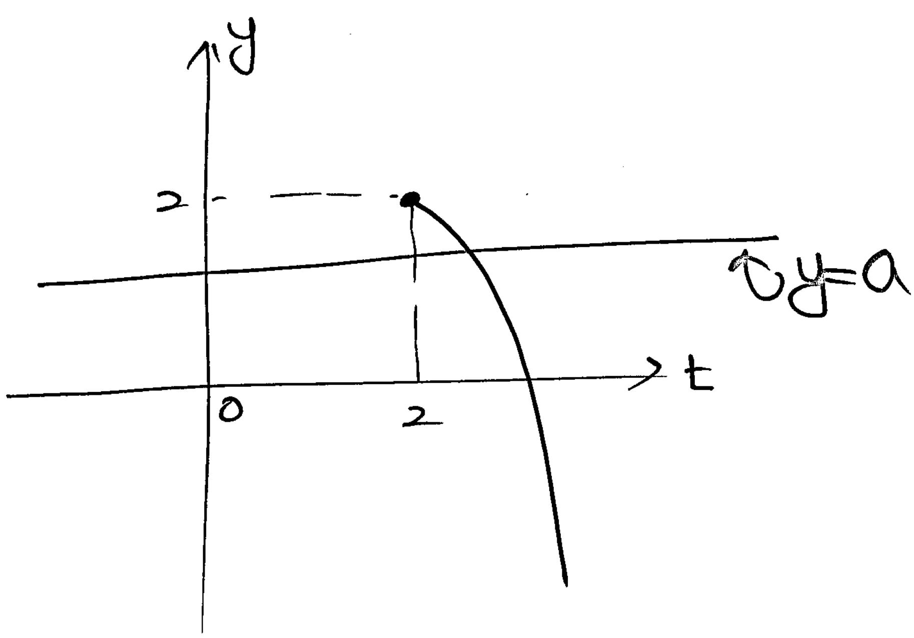
$4^x+2+4^{-x}=t^2 \qquad 4^x+4^{-x}=t^2-2$

$t^2-2-2t+a=0 \qquad (t \geq 2)$

$a=-t^2+2t+2$

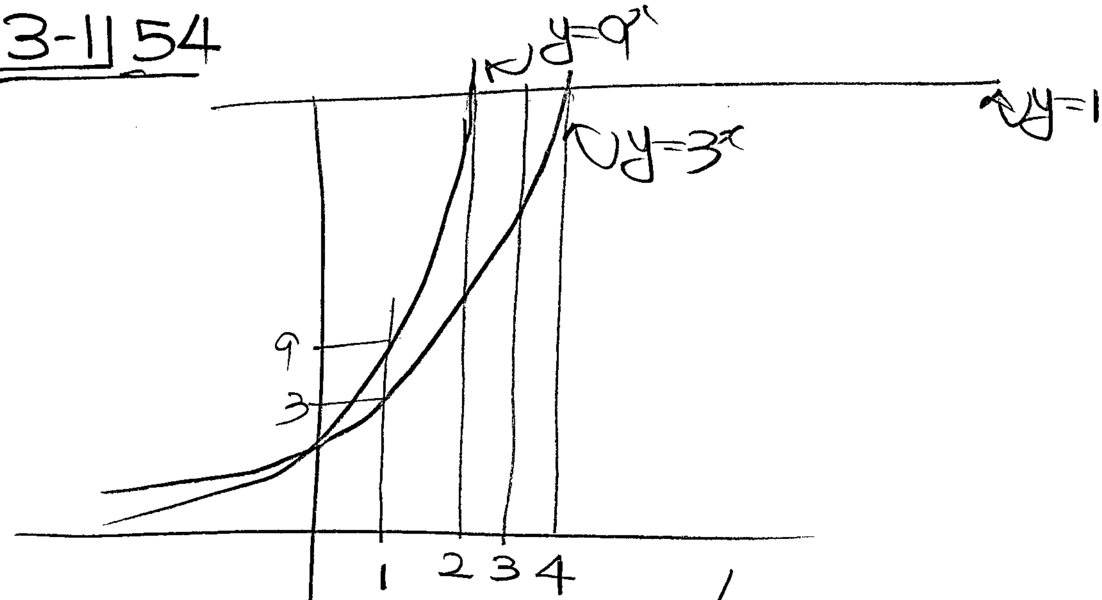
$\begin{cases} y=a \\ y=f(t)=-t^2+2t+2=-(t-1)^2+3 \end{cases}$

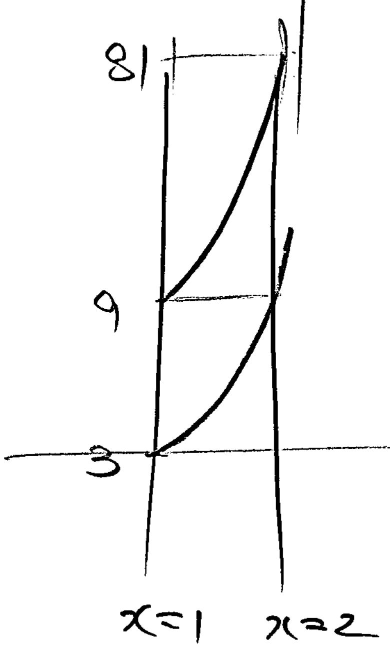
$f(2)=2$

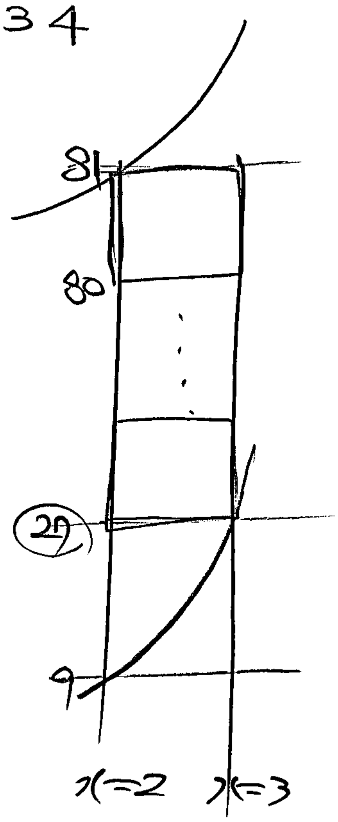


$\boxed{a \leq 2}$

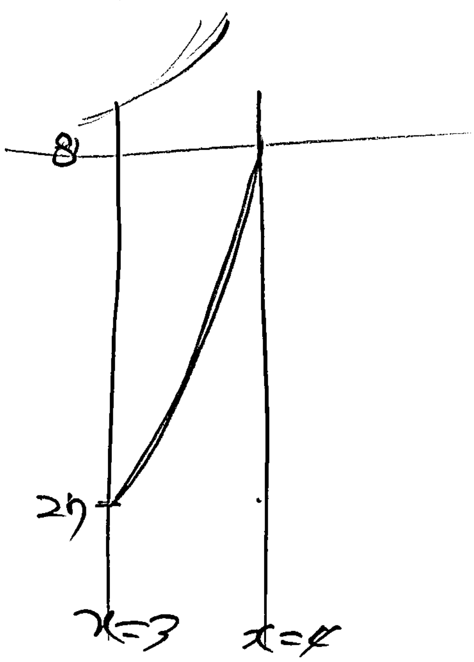
13-11 54





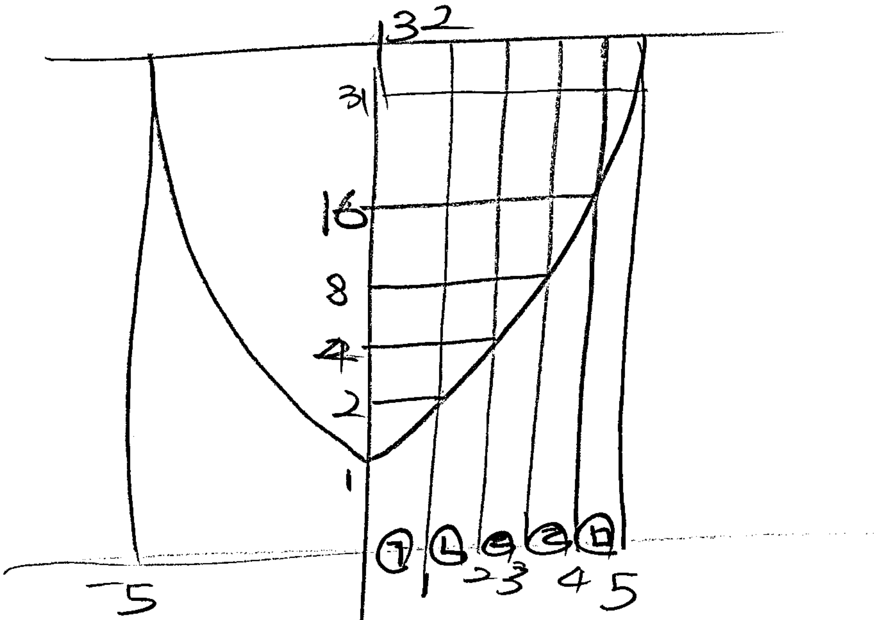


$27 \sim 80 : \boxed{54}$



13-21 196

$$y = 2^{|x|} = \begin{cases} 2^{x} & (x \geq 0) \\ 2^{-x} & (x < 0) \end{cases}$$



ㄱ 2~31 : 30) 58
ㄴ 4~31 : 28) 98
ㄷ 8~31 : 24) 40
ㄹ 16~31 ~ 16
ㅁ 0

98×2 = 196

3-1| 4

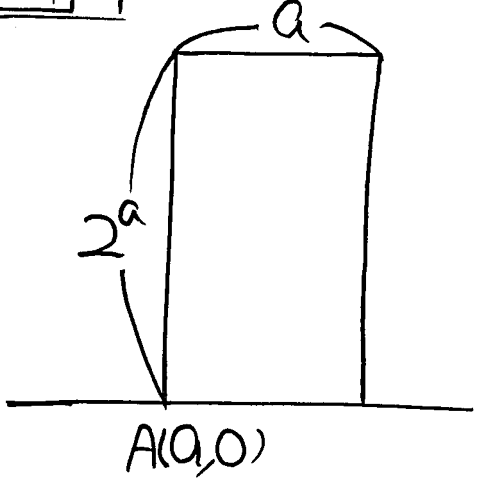
(2, 2) $\alpha=2$, $\beta=2$ $\alpha+\beta=4$

3-2| ③

$y=\dfrac{a^x}{b^x}=\left(\dfrac{a}{b}\right)^x$ | $\dfrac{a}{b}>1$

3-3| 9

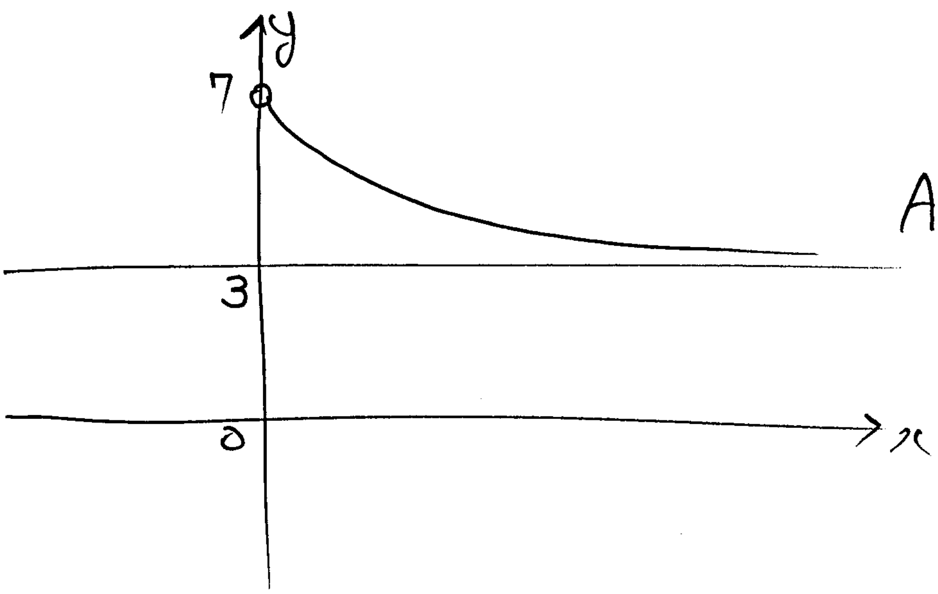
3-4| 4



$a\cdot 2^a=64$

$a=4$

3-5| 18



$A=\{3, 4, 5, 6\}$

$\therefore$ 18

3-6) 3

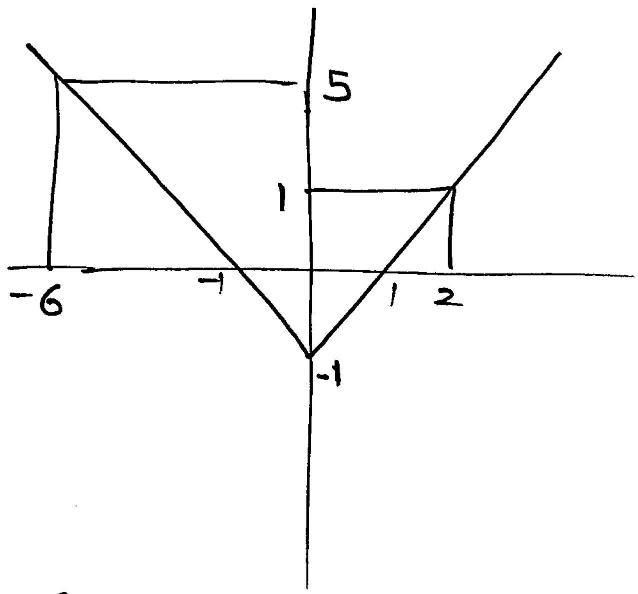
$y = 2^{2x+a} + 2 \qquad b = 2$

$\xrightarrow{\text{y축 대칭}} f(x) = 2^{-2x+a} + 2$

$f(-1) = 2^{2+a} + 2 = 10 \qquad a = 1 \qquad \boxed{a+b=3}$

3-7) 16

$y = |x| - 1$



$M = 2^5$

$\times\; m = 2^{-1}$

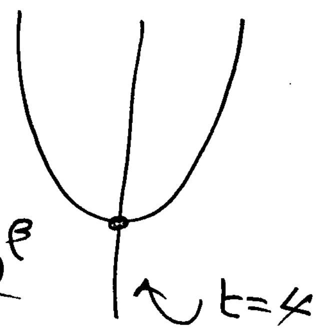
$Mm = 2^4 = 16$

3-8) 40

$2^x = t \quad (t > 0) \qquad x = 2 \rightarrow t = 4$

$f(t) = t^2 - at + 12$

$a = 8$



$t^2 - 8t + 12 = 0$ 두근 $2^\alpha, 2^\beta$

$(t-2)(t-6) = 0$

$2^\alpha = 2,\ 2^\beta = 6 \qquad 2^{2\alpha} + 2^{2\beta} = 4 + 36 = 40$

3-9 ⑤

$2^x = X,\ 3^y = Y \qquad (X>0, Y>0)$

$4X - \frac{1}{3}Y = 55$

$\frac{1}{2}X + 3Y = 89$

$\times 9$

$36X - 3Y = 495$

$\frac{73X}{2} = 584$

$X = 16$

$Y = 27$

$2^x = 2^4,\quad 3^y = 3^3$

$\alpha = 4,\ \beta = 3 \qquad \alpha + \beta = 7$

$$\begin{array}{r} 55 \\ \times\ 9 \\ \hline 45 \\ 45\ \ \\ \hline 495 \end{array}$$

$$\begin{array}{r} 8 \\ 73\overline{)584} \\ 584 \\ \hline 0 \end{array}$$

3-10 1

$$f(x) = \frac{13}{3^x(1+3+3^2)} = 3^{-x}$$

$6 \cdot 3^{-x} + 3^x = 5$ 두근 α, β

$3^x = t \ (t>0)$

$\frac{6}{t} + t = 5$

$t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$ 두근 $3^\alpha, 3^\beta$

$3^\alpha = 2,\ 3^\beta = 3$

$9^\alpha + 9^\beta = 4 + 9 = 13$

3-11 ④

$f(1)=64$

$a=2,\ b=\frac{1}{2}$ $\quad ab=1$ $\quad f(1)=\left(f\left(\frac{1}{2}\right)\right)^2=64$

$\boxed{f\left(\frac{1}{2}\right)=8}$

$a=3,\ b=\frac{1}{3}$ $\quad ab=1$ $\quad f(1)=\left(f\left(\frac{1}{3}\right)\right)^3=64=4^3$

$\boxed{f\left(\frac{1}{3}\right)=4}$

$a=\frac{1}{2},\ b=\frac{1}{3}$; $\boxed{f\left(\frac{1}{6}\right)=\left\{f\left(\frac{1}{3}\right)\right\}^{\frac{1}{2}}=4^{\frac{1}{2}}=2}$

$ab=\frac{1}{6}$

$\therefore 8+4+2=14$

3-12 ⑤

$$(1+r_1)^{10}=\left(1+\frac{r_2}{2}\right)^{20}=\left(1+\frac{r_3}{4}\right)^{40}$$

$$\underbrace{1+r_1=\left(1+\frac{r_2}{2}\right)^2}_{㉠}=\left(1+\frac{r_3}{4}\right)^4$$

($\left(1+\frac{r_2}{2}\right)^2=\left(1+\frac{r_3}{4}\right)^4$: ㉡)

㉠ $1+r_1=1+r_2+\frac{r_2^2}{4}$ $\qquad r_1-r_2=\frac{r_2^2}{4}>0$

$r_1>r_2$

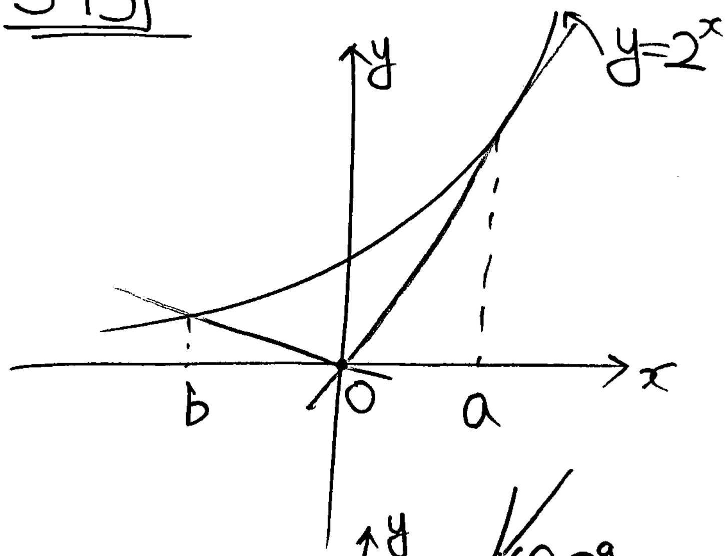
㉡ $\left(1+\frac{r_2}{2}\right)=\left(1+\frac{r_3}{4}\right)^2$

$\cancel{1}+\frac{r_2}{2}=\cancel{1}+\frac{r_3}{2}+\frac{r_3^2}{16}$ $\qquad r_2-r_3=\frac{r_3^2}{8}>0$

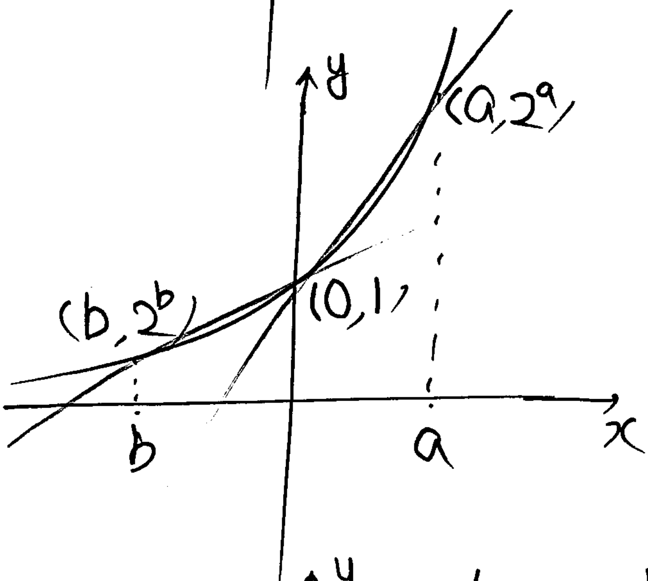
$r_2>r_3$

$\boxed{r_3<r_2<r_1}$

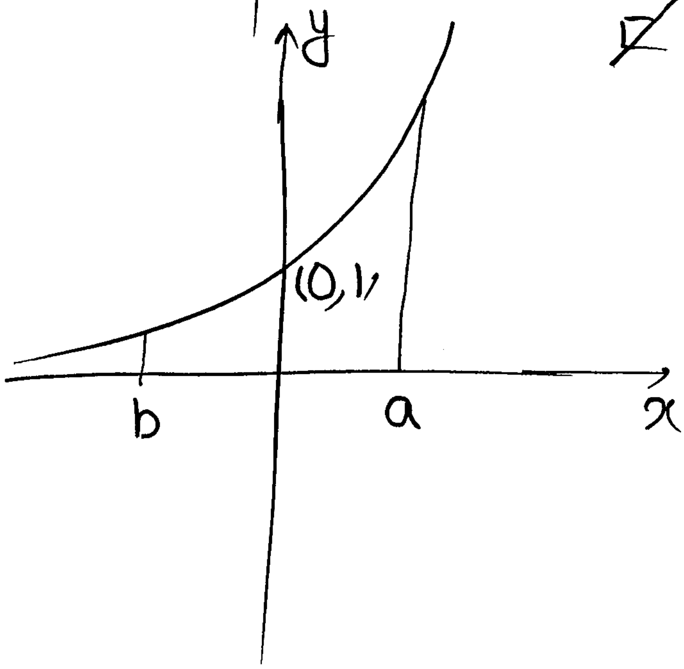
3-131



㋐ $\dfrac{2^b}{b} < \dfrac{2^a}{a}$



㋑ $\dfrac{2^b-1}{b} < \dfrac{2^a-1}{a}$



㋒ $b=-a$

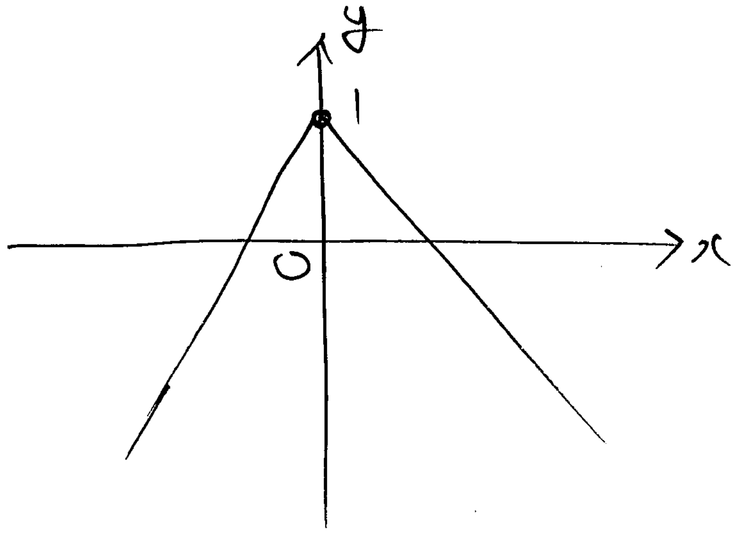
$$\frac{2^a-2^b}{a-b} = \frac{2^a-2^{-a}}{2a} \boxed{?} 1$$

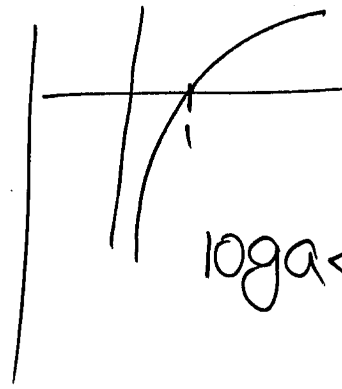
$$a=4 ; \quad \frac{16-\frac{1}{16}}{8} > 1$$

3-14 ①

$$f(x)=\begin{cases}2a^{-x} & (x<0)\\ 2a^{x} & (x\geqq 0)\end{cases}\qquad 0<a<1$$

$$y=\log_2 2a^{|x|}=1+|x|\log a$$





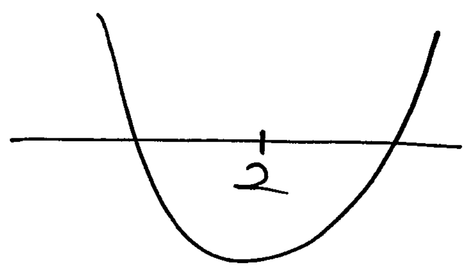
$\log a<0$

3-15 ア

$2^x=t\quad(t>0)$

$x=1\to t=2$

$t^2-4at+a^2=0$

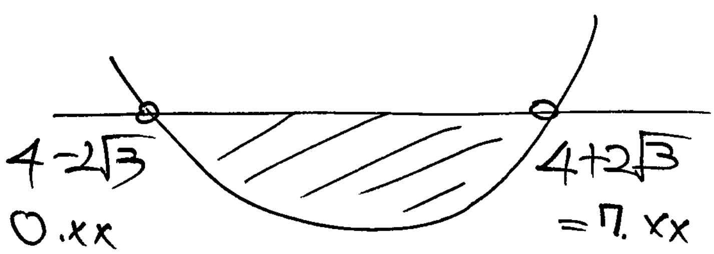


$f(t)=t^2-4at+a^2$

$f(2)=4-8a+a^2<0$

$a^2-8a+4=0$

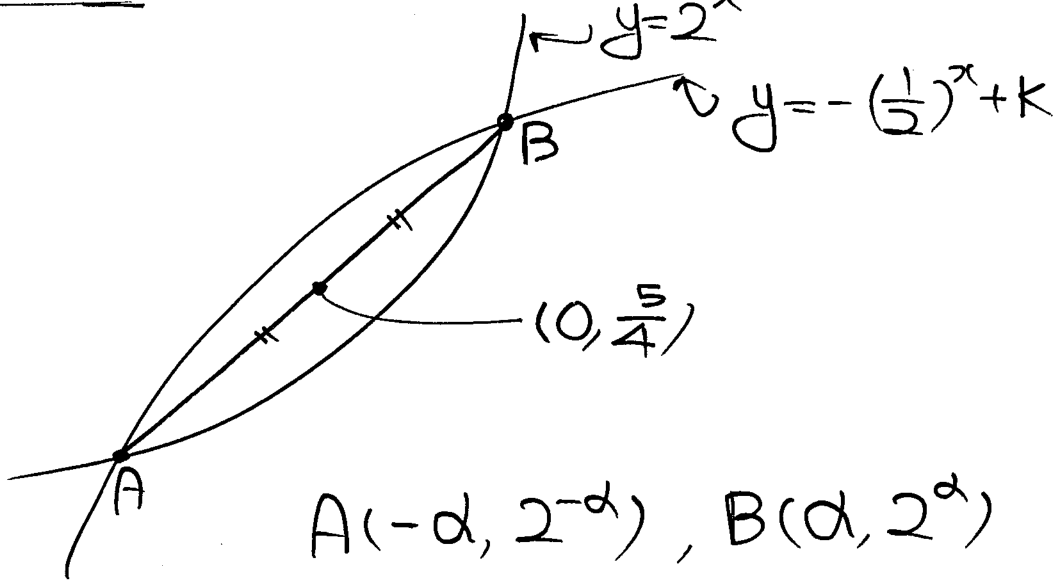
$a=4\pm\sqrt{12}$



$$\boxed{a=1,2,3,4,5,6,7}$$

7コ

3-16 $\frac{5}{2}$



$A(-\alpha, 2^{-\alpha})$, $B(\alpha, 2^{\alpha})$ $(\alpha>0)$

$\frac{2^{-\alpha}+2^{\alpha}}{2}=\frac{5}{4}$ $\quad 2^{\alpha}+2^{-\alpha}=\frac{5}{2}$

$2^{\alpha}=t$ $(t>1)$

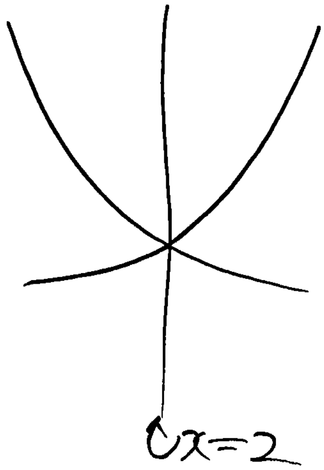
$t+\frac{1}{t}=\frac{5}{2}$ $\quad 2t^2-5t+2=0$

$(t-2)(2t-1)=0$

$t=2$ $\quad 2^{\alpha}=2$ $\quad \alpha=1$

$B(1,2)$ $\quad 2=-\left(\frac{1}{2}\right)^1+k$ $\quad \boxed{k=\frac{5}{2}}$

3-17 1



$f(2)=f(2)$

$a^{2b-1}=a^{1-2b}$ $\quad \boxed{b=\frac{1}{2}}$

$a+a^{-1}=\frac{5}{2}$

$2a^2-5a+2=0$ $\quad \boxed{a=\frac{1}{2}}$

$(a-2)(2a-1)=0$

$\boxed{a+b=1}$

3-18) 3

$3^x = 4$ $\quad x = \log_3 4$ $\quad A(\log_3 4, 4)$

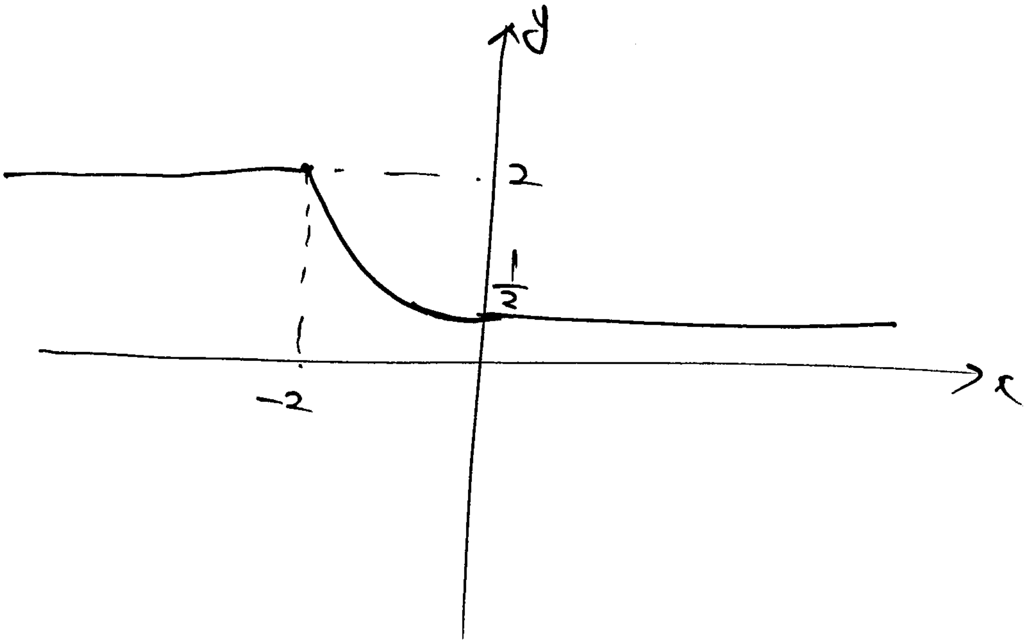
$9^x = 4$ $\quad x = \log_9 4 = \frac{1}{2}\log_3 4$ $\quad B(\frac{1}{2}\log_3 4, 4)$

$27^x = 4$ $\quad x = \log_{27} 4 = \frac{1}{3}\log_3 4$ $\quad C(\frac{1}{3}\log_3 4, 4)$

$\overline{AB} = \frac{1}{2}\log_3 4, \quad \overline{BC} = \frac{1}{6}\log_3 4 \quad \frac{\overline{AB}}{\overline{BC}} = 3$

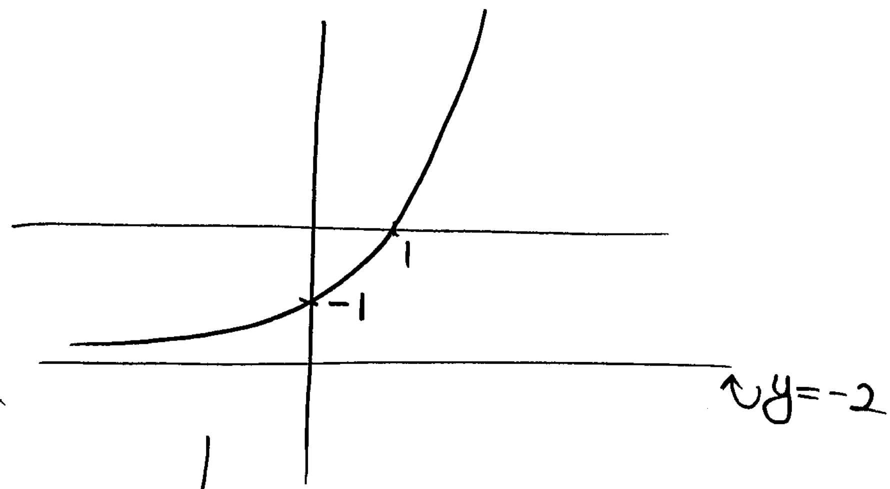
3-19) ④

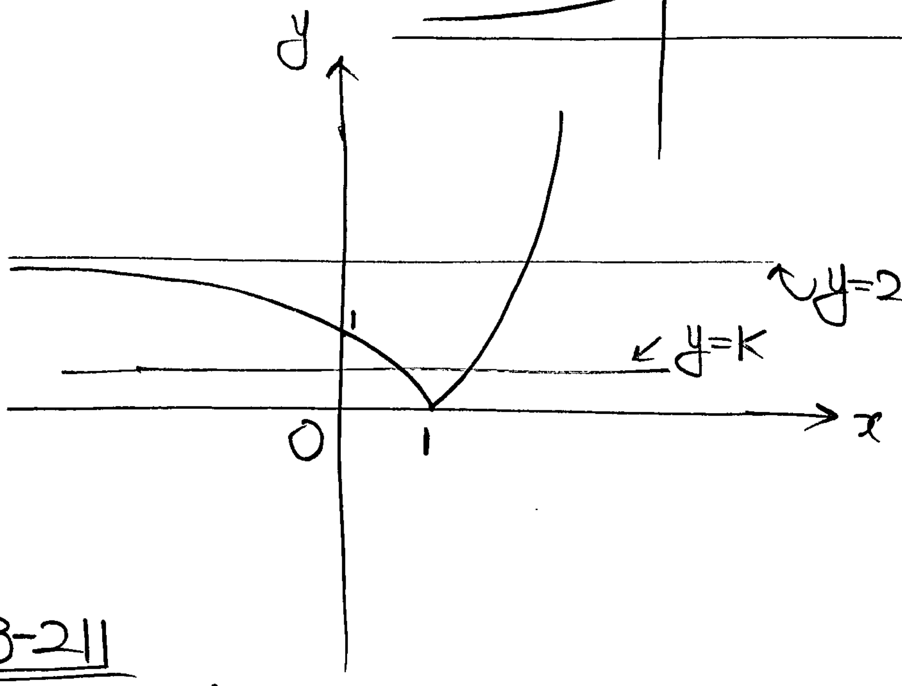
$$f(x) = \begin{cases} -1 & (x<-2) \\ x+1 & (-2 \le x < 0) \\ 1 & (x \ge 0) \end{cases} \Rightarrow y = 2^{-f(x)} = \begin{cases} 2 & (x<-2) \\ 2^{-x-1} & (-2 \le x < 0) \\ 2^{-1} & (x \ge 0) \end{cases}$$



3-20

$y = 2^x - 2$





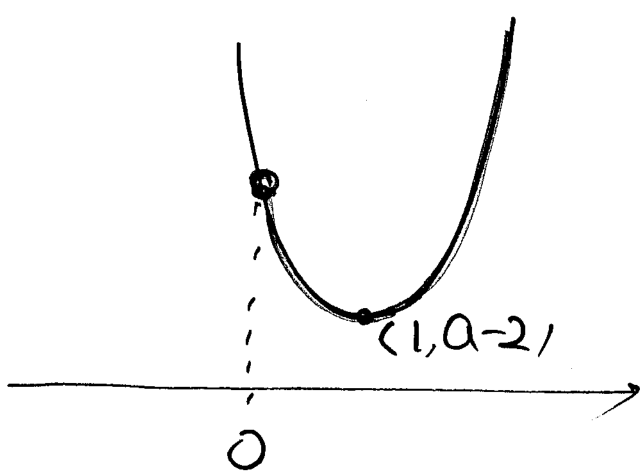
$\boxed{0<K<2}$

3-21

(1) $2^{\frac{x}{2}} = t \ (t>0)$

$2t^2 - 4t + a \geq 0$

$f(t) = 2t^2 - 4t + a = 2(t-1)^2 + a - 2.$



$a - 2 \geq 0$

$\boxed{a \geq 2}$

(2) $5^x = t \ (t>0)$

$t^2 - 2at + 9 \ge 0$

$t^2 + 9 \ge 2at$

$t + \frac{9}{t} \ge 2a$

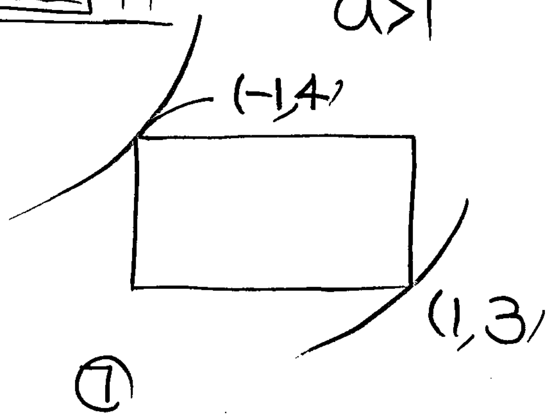
$2a \le 6$

$t + \frac{9}{t} \ge 2\sqrt{t \cdot \frac{9}{t}} = 6$

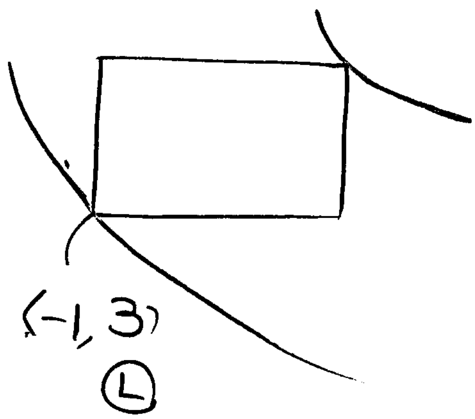
$\boxed{\therefore a \le 3}$

3-22177

$a>1$



$0<a<1$



㉠ $a^2+1=4$

$a^2=3$

$\alpha^2=3$

$\boxed{\alpha^8=3^4=81}$

$a^2+1=3$

$a^2=2$

(x)

$a^4+1=3$

$a^4=2$

$\beta^4=2$

$\boxed{\beta^8=4}$

$\alpha^8-\beta^8=77$

3-23| 20

$P(\alpha, k\cdot 3^{\alpha}) \Leftrightarrow P(\alpha, 3^{-\alpha})$

$Q(2\alpha, k\cdot 3^{2\alpha}) \Leftrightarrow Q(2\alpha, -4\times 3^{2\alpha}+8)$

$k\cdot 3^{\alpha} = 3^{-\alpha}$ $\quad$ $k\cdot 3^{2\alpha}=1$

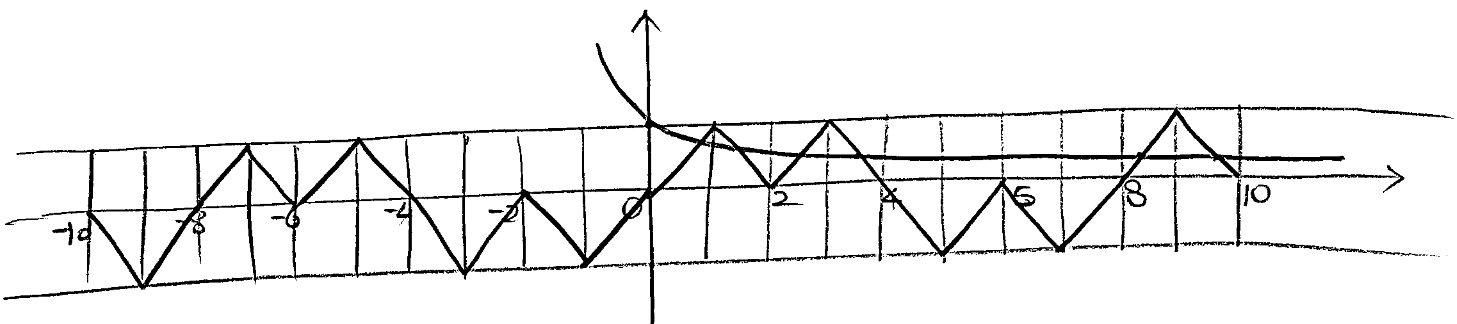
$k\cdot 3^{2\alpha} = -4\times 3^{2\alpha}+8$ $\quad$ $3^{2\alpha}=\frac{7}{4}$ $\quad$ $k=\frac{4}{7}$

$35k=20$

3-24| 6

(나) $f(-x)=-f(x)$ ⇔ 기함수 ⇔ 원점에 대칭인함수

(다) $f(2-x)=f(2+x)$ ⇔ $x=2$ 에 대칭인 함수



6개

3-25|

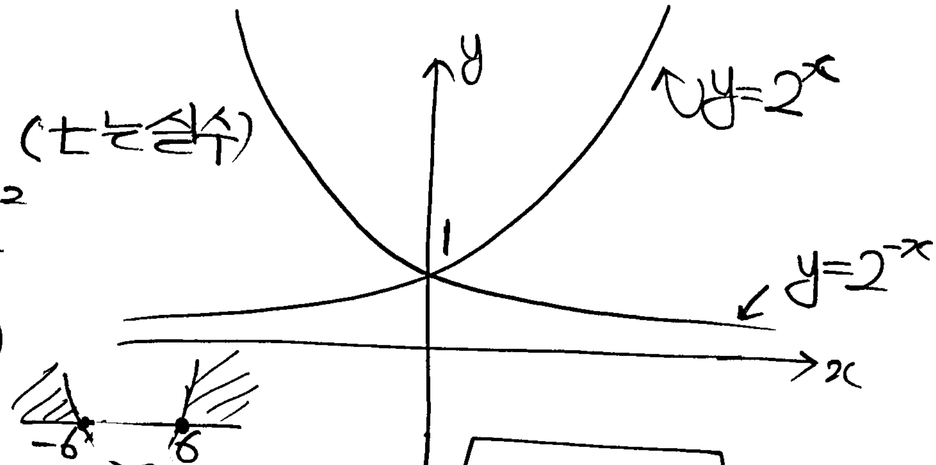
(1) $2^x-2^{-x}=t$ (t는실수)

$4^x-2+4^{-x}=t^2$

$t^2+at+9=0$

$D=a^2-36\ge 0$

$a\le -6$ 또는 $a\ge 6$



$m=6$ $\quad$ $\boxed{m^2=36}$

(2) $2^x+2^{-x} \geq 2\sqrt{2^x \cdot 2^{-x}}=2$

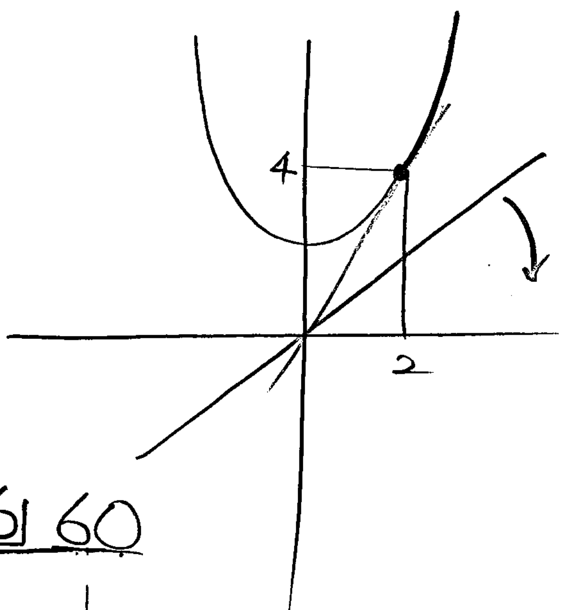
$2^x+2^{-x}=t \quad (t \geq 2)$

$4^x+2+4^{-x}=t^2 \qquad 4^x+4^{-x}=t^2-2$

$(t^2-2)-2kt+6=0$

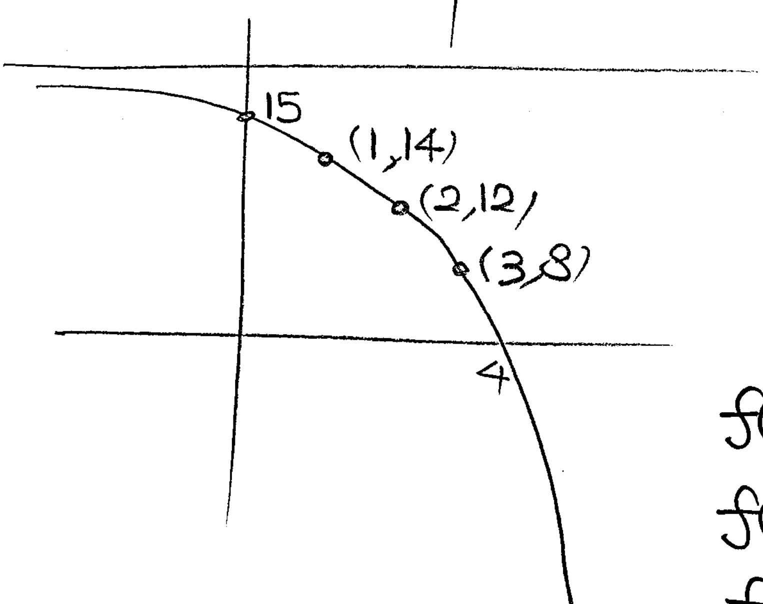
$t^2-2kt+4=0 \qquad D/4=k^2-4=0$

$t^2+4=2kt \qquad k=2;\ t=2$



$\boxed{k<2}$

3-26 60



$y=16 \quad y=a^x \quad (1,a)$

$(2,a^2)$

$(3,a^3)$

$f(2)=13+9+1=23$

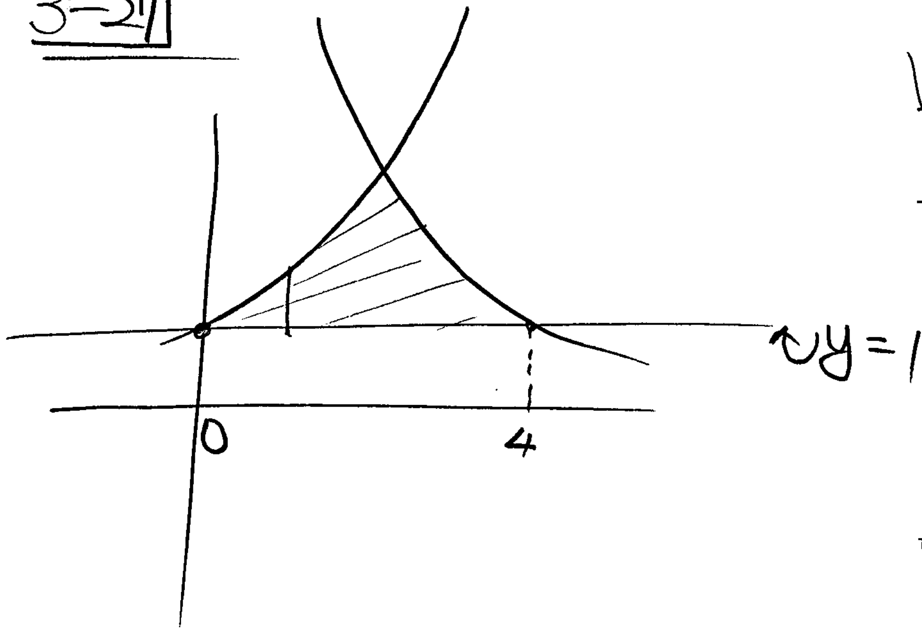
$f(3)=12+4 \quad =16$

$f(4)=11$

$f(5)=10$

$23+16+11+10=60$

3-27 |



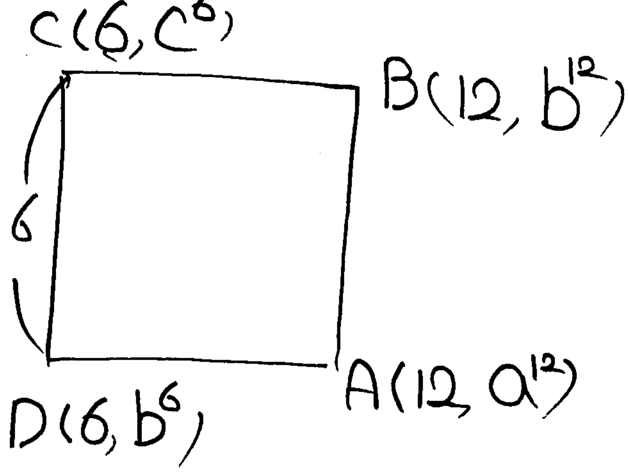
$y=4^x$	$y=a^{-x+4}$	x
1	a^4	0
4	a^3	1
16	a^2	2
64	a	3
4^4	1	4

$a \backslash x$	0	1	2	3	4	
2	1	4	4	2	1	
3	1	4	9	3	1	← 18
4	1	4	16	4	1	← 26
5	1	4	16	5	1	
⋮						
n	1	4	16	n	1	

$n+22 \le 40$

$4 \le n \le 18$ $\boxed{\therefore 15개}$

3-28 | 93



$b^{12} - b^6 = 6$

$b^{12} - b^6 - 6 = 0$

$(b^6 - 3)(b^6 + 2) = 0$

$b^6 = 3$

$\boxed{b^{12} = 9}$

$c^6 = 9$

$\boxed{c^{12} = 81}$

$\boxed{a^{12} = 3}$

$3 + 9 + 81 = 93$

3-29 | 27

$C(a, 2^{-a})$

$B(a, 4^a)$

$A(-2a, 4^a)$

$2^{-x} = 4^a = 2^{2a}$

$x = -2a$

$\overline{AB} = 3a = 2$ $\qquad a = \frac{2}{3}$

$l = 4^a - 2^{-a} = 4^{\frac{2}{3}} - 2^{-\frac{2}{3}} = 2^{\frac{4}{3}} - 2^{-\frac{2}{3}}$

$= 2^{-\frac{2}{3}}(2^2 - 1) = 2^{-\frac{2}{3}} \cdot 3$

$4l^3 = 4 \cdot 2^{-2} \cdot 3^3 = 27$