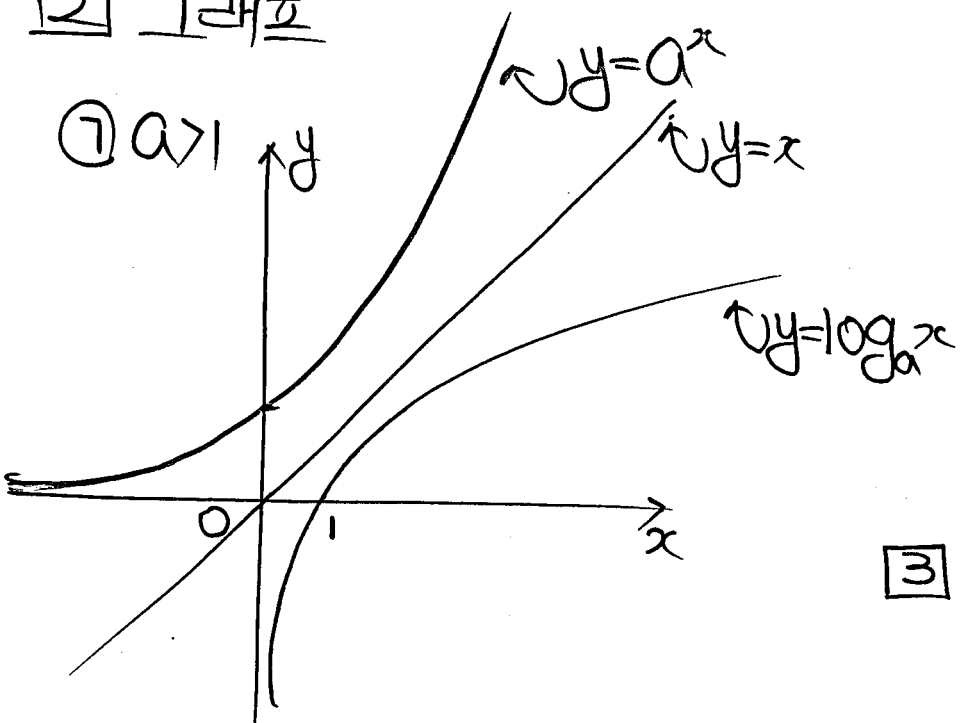
4. 로그함수

1 정의

$y=\log_a x \quad (a>0, a\neq 1, x>0)$

2 그래프

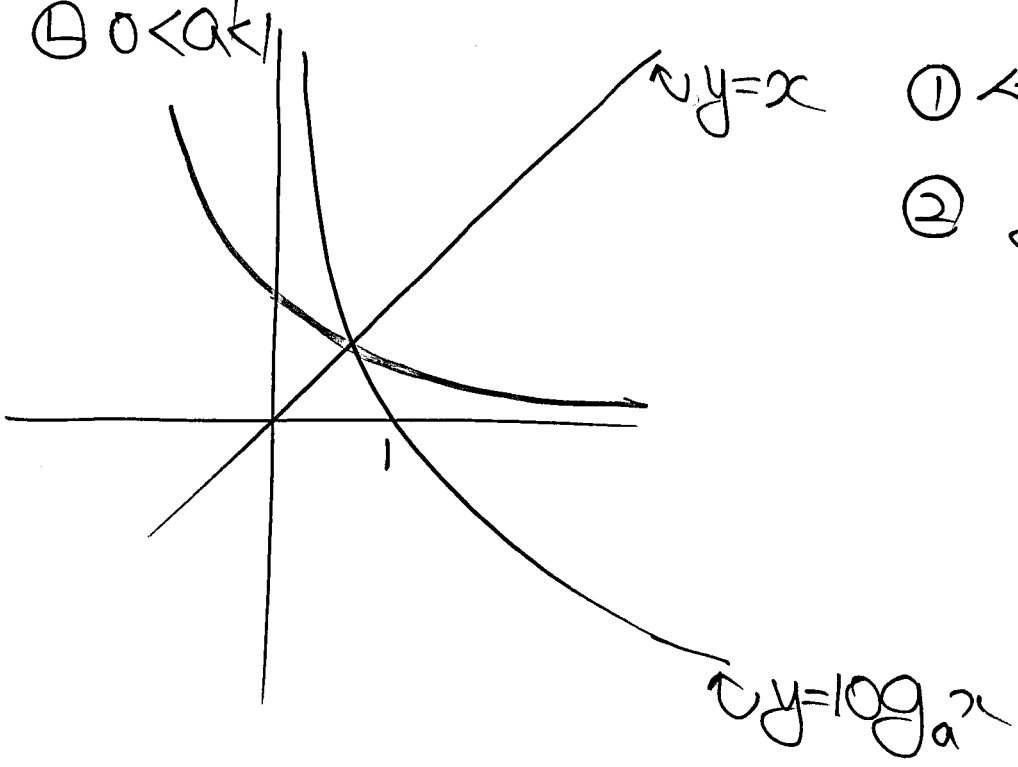
① $a>1$



점근선 : $x=0$

정점 $(1,0)$

② $0<a<1$



3 $y=\log_a x$ 와 $y=a^x$ 관계

① 서로 역함수이다.

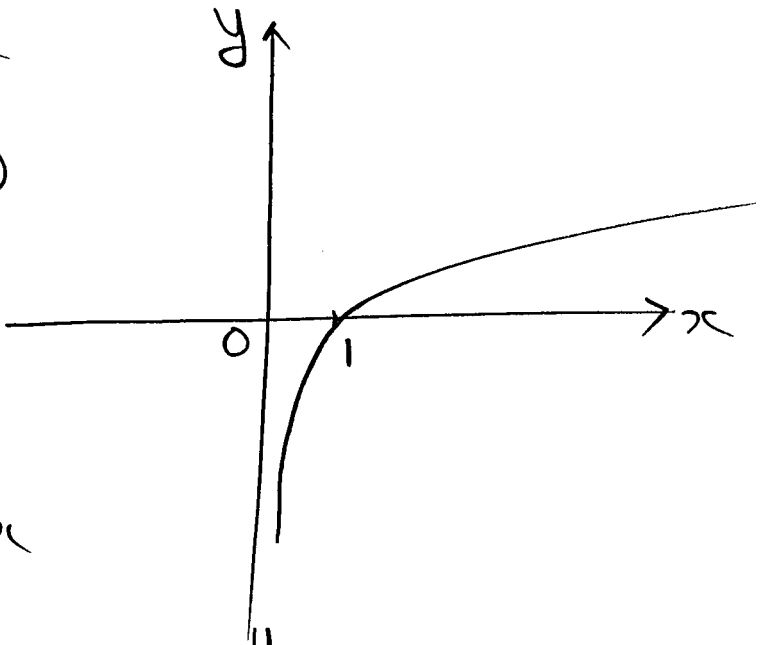
② $y=x$ 대칭

P151

EX) (1) $y=\log_5 x$

점근선 : $x=0$

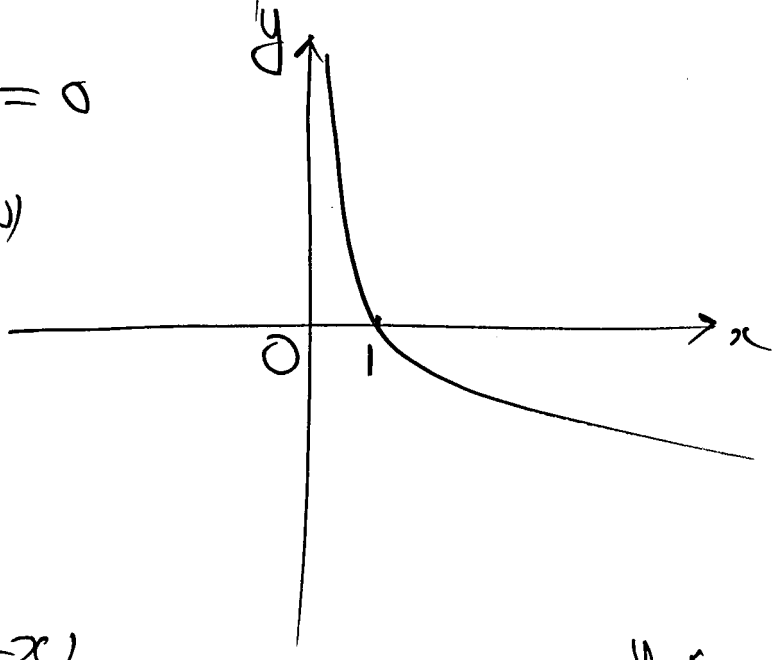
정점 $(1,0)$



(2) $y=\log_{\frac{1}{5}} x$

점근선 : $x=0$

정점 $(1,0)$



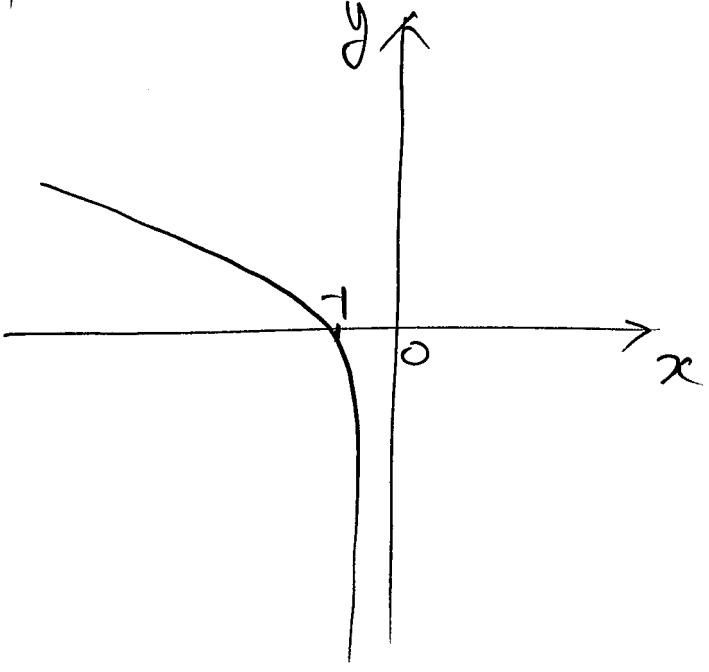
P155

EX1, (1) $y=\log_2(-x)$

정의역 : $-x>0$ $x<0$

점근선 : $x=0$

정점 $(-1,0)$

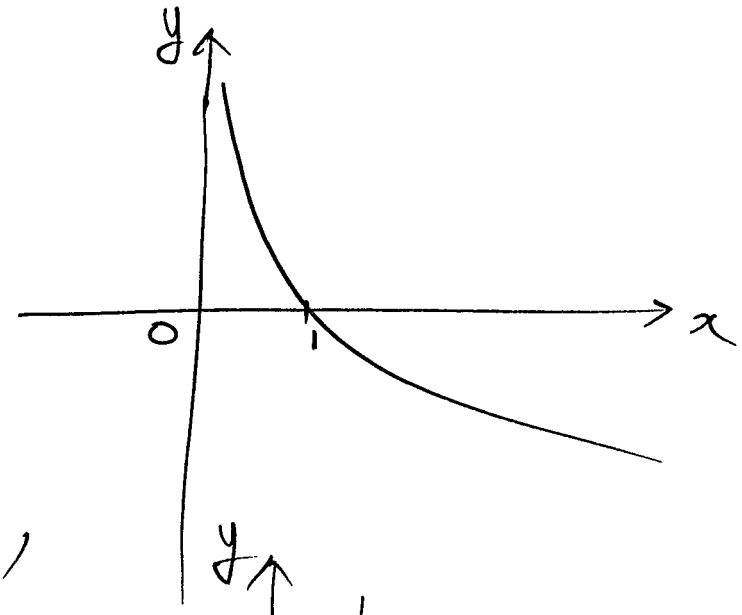


(2) $y = -\log_2 x$

정의역 : $x > 0$

점근선 : $x = 0$

정점 $(1, 0)$

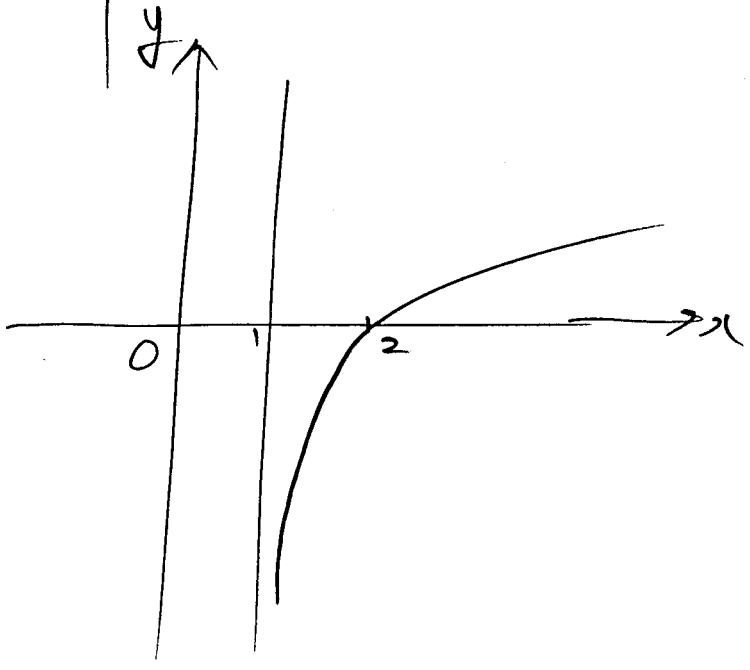


(3) $y = \log_2 (x-1)$

정의역 : $x > 1$

점근선 : $x = 1$

정점 $(2, 0)$

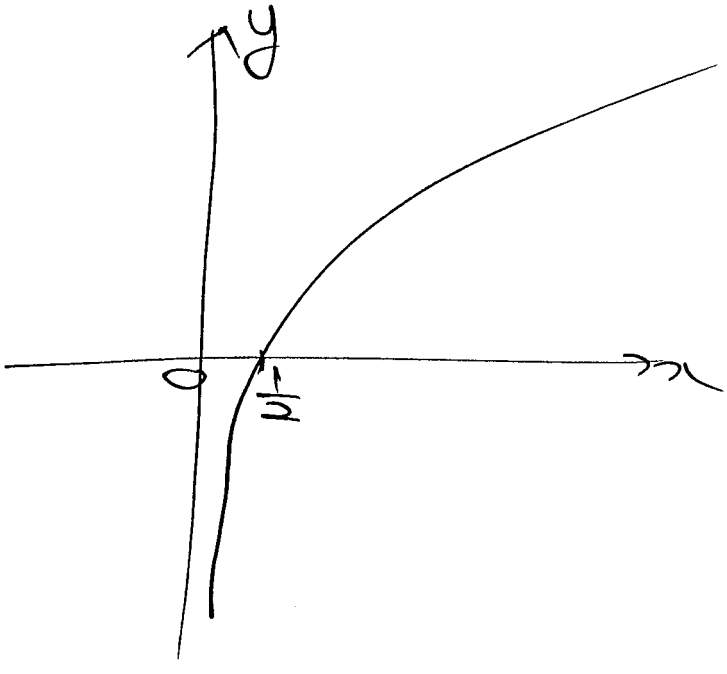


(4) $y = \log_2 2x$

정의역 : $x > 0$

점근선 : $x = 0$

정점 $\left(\frac{1}{2}, 0\right)$



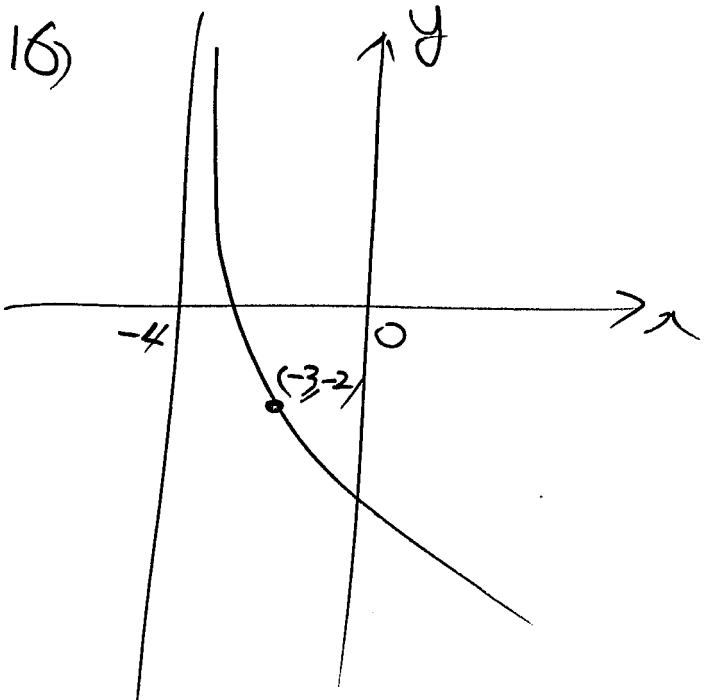
P156

EX) $y=\log_{\frac{1}{2}}(4x+16)$

정의역 : $x>-4$

점근선 : $x=-4$

$(-3,-2)$

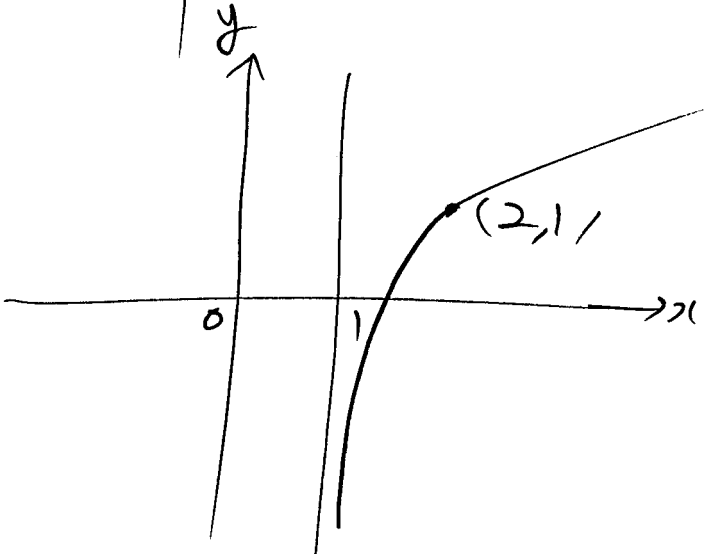


P157

EX) (1) $y=\log_2(2x-2)$

정의역 : $x>1$

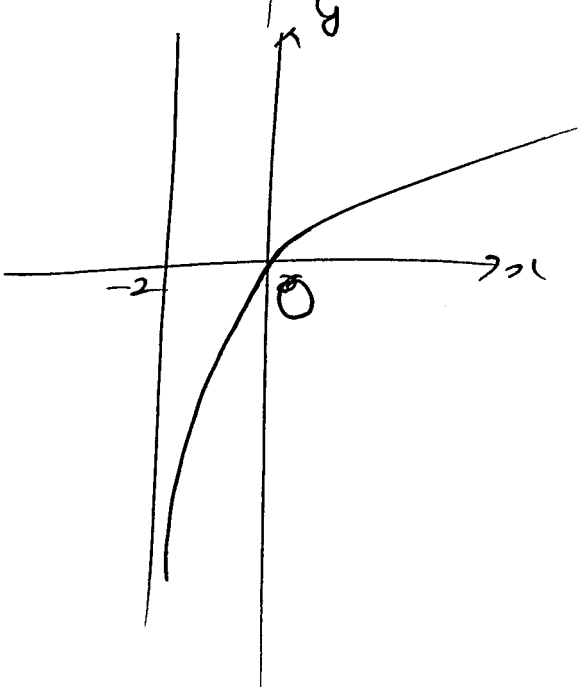
점근선 : $x=1$



(2) $y=\log_2\left(\frac{x+2}{2}\right)$

정의역 : $x>-2$

점근선 : $x=-2$



P159

EX) $y=2^{x-1}+3$

정의역: $\{x \mid x$는실수$\}$, 치역: $\{y \mid y>3$인실수$\}$

역함수 $\Longleftrightarrow$ $x=2^{y-1}+3$

$2^{y-1}=x-3$

$y-1=\log_2(x-3)$

$\therefore y=\log_2(x-3)+1$

정의역: $\{x \mid x>3$ 인실수$\}$

치역: $\{y \mid y$는 모든실수$\}$

P160

(1) $\sqrt{2}<\sqrt{3}<2$

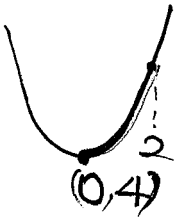
$\log_2\sqrt{2}<\log_2\sqrt{3}<\log_2 2$

(2) $\log_{\frac{1}{2}}2<\log_{\frac{1}{2}}\sqrt{3}<\log_{\frac{1}{2}}\sqrt{2}$

P162

EX)

(1) $f(x)=\log_2(x^2+4)$ $(0\le x\le 2)$



$M=f(2)=\log_2 8=3\log_2 2=3$

$m=f(0)=\log_2 4=2\log_2 2=2$

(2) $g(x)=\log_{\frac{1}{2}}(x^2+4)$ $(0\le x\le 2)$

$M=g(0)=\log_{\frac{1}{2}}4=\frac{2}{-1}\log_2 2=-2$

$m=g(2)=\log_{\frac{1}{2}}8=\frac{3}{-1}\log_2 2=-3$

P163

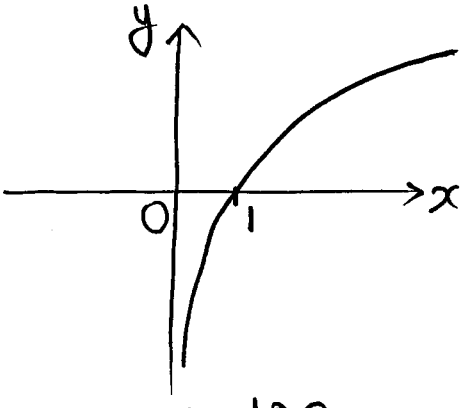
11

$y=\log_{(x-1)}(2-x)$

$x-1>0,\ x-1\neq 1,\ 2-x>0$

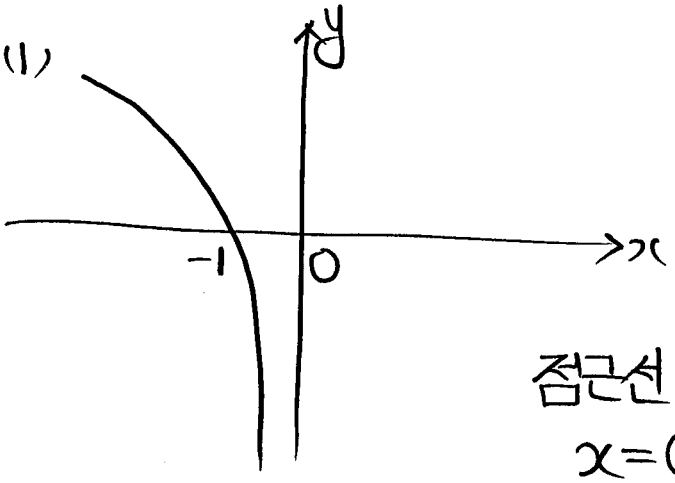
$x>1,\ x\neq 2,\ x<2$ $\therefore\ 1<x<2$

21.



$y=\log_3 x$

(1)



점근선

$x=0$

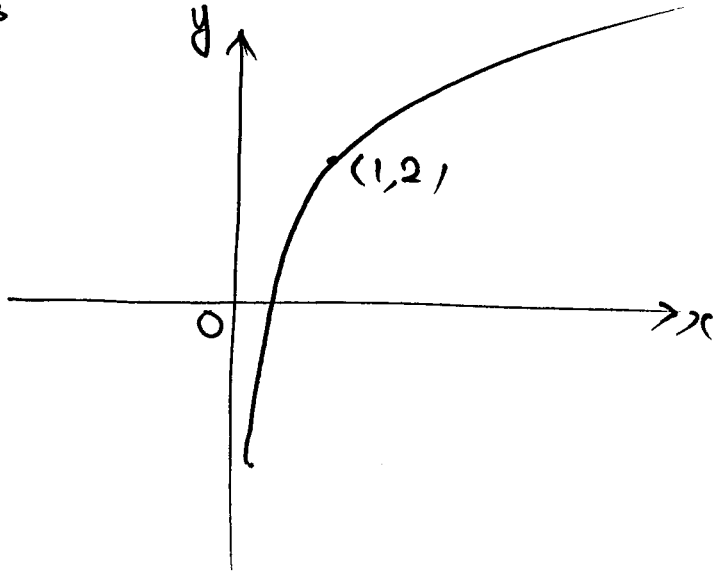
$y=\log_3(-x)$

(2) $y=\log_3 9+\log_3 x$

$y-2=\log_3 x$

$y=2+\log_3 x$

점근선 : $x=0$



P165

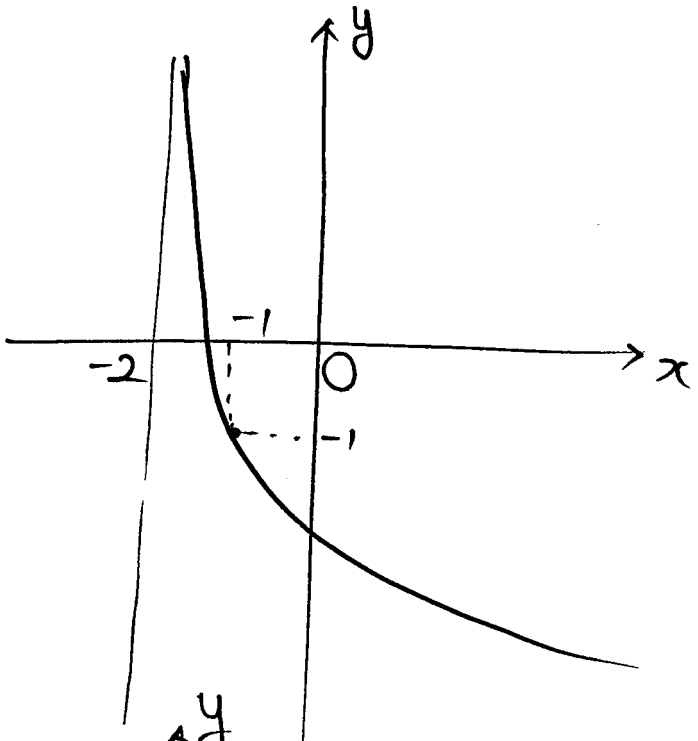
1-11

(1) $y=\log_{\frac{1}{2}}(x+2)-1$

$x+2>0 \quad x>-2$

점근선: $x=-2$

$(-1,-1)$

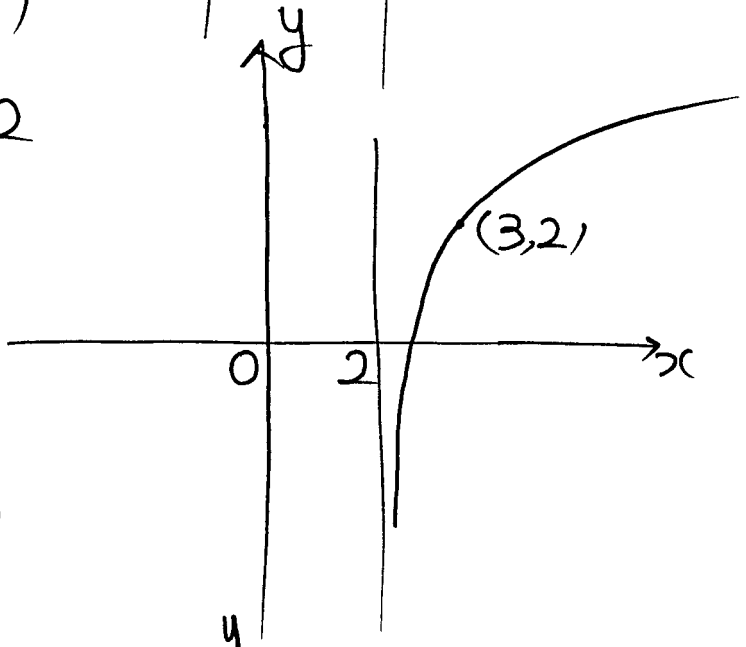


(2) $y=\log_2(4x-8)$

$4x-8>0 \quad x>2$

점근선: $x=2$

$(3,2)$



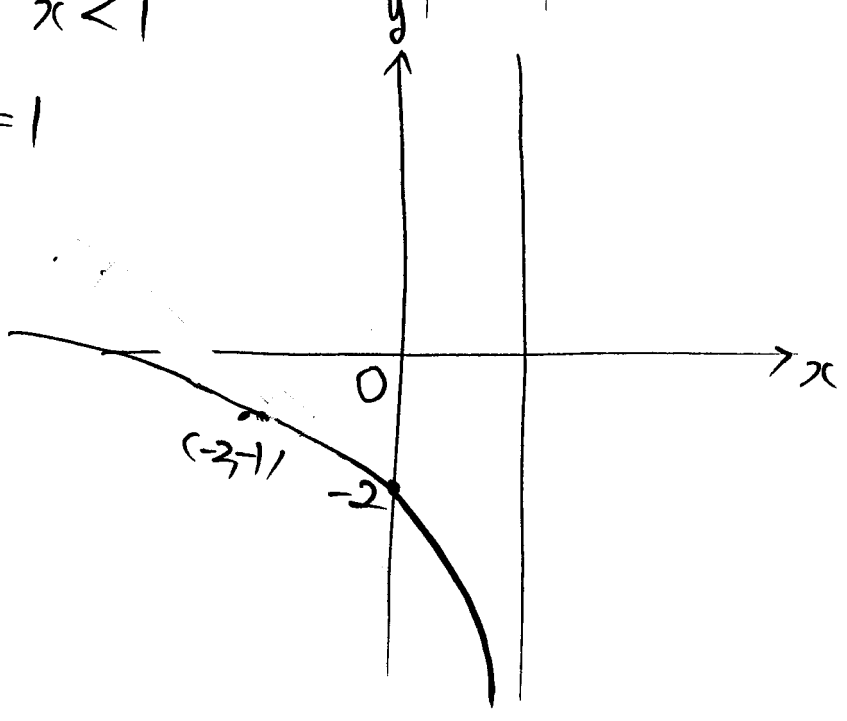
(3) $y=\log_3(1-x)-2$

$1-x>0 \quad x<1$

점근선: $x=1$

$(0,-2)$

$(-2,-1)$

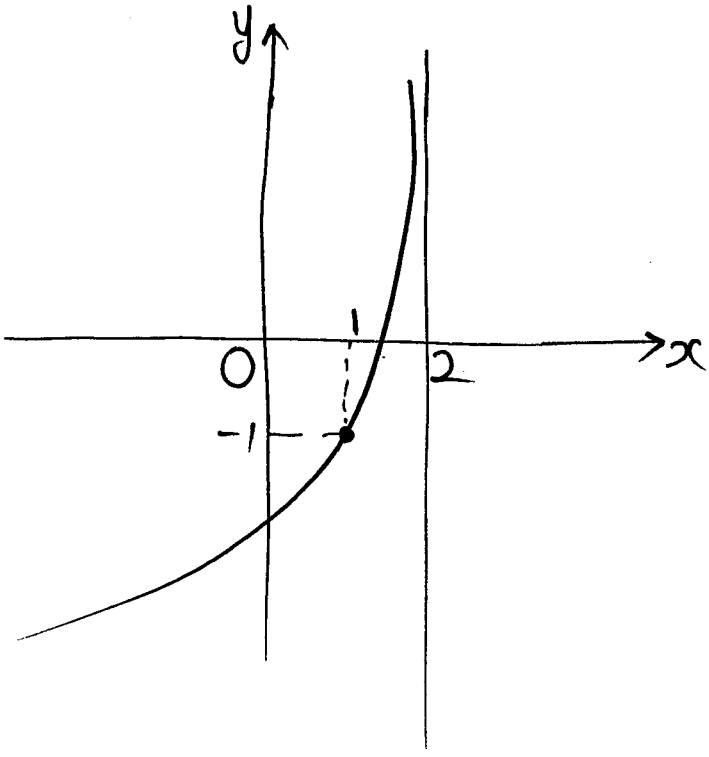


(4) $y=\log_{\frac{1}{3}}(-9x+18)+1$

$-9x+18>0 \quad x<2$

점근선: $-9x+18=0 \quad x=2$

$(1,-1)$



1-21 ④

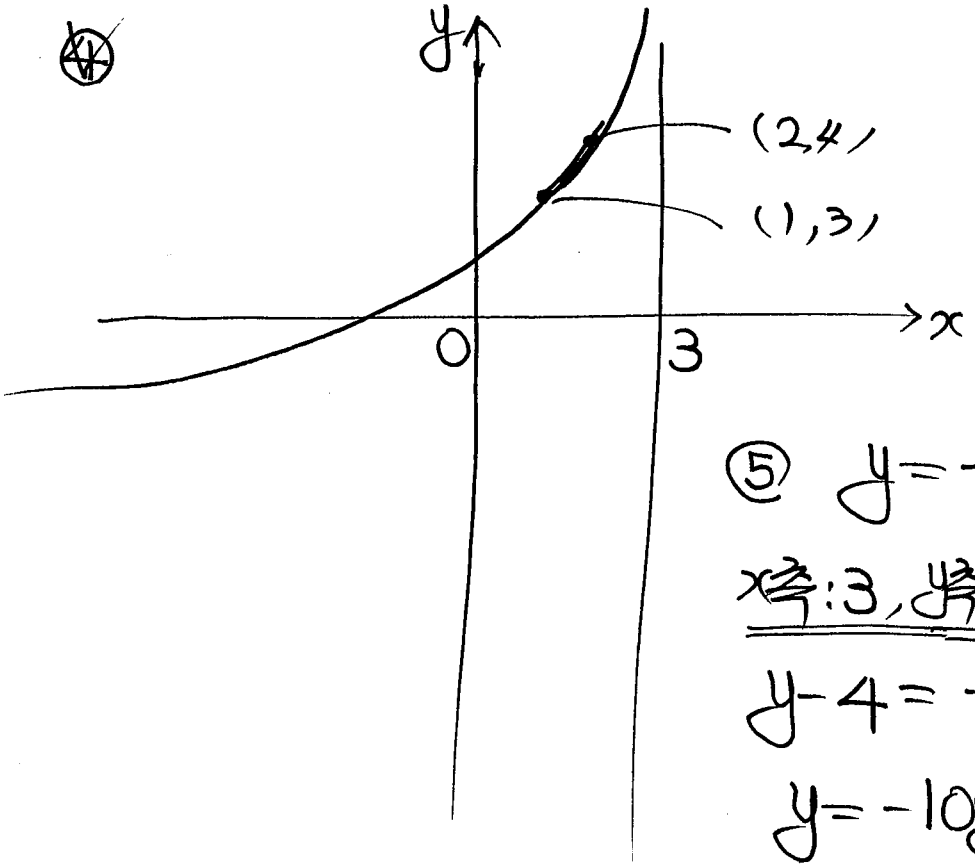
$y=-\log_2(3-x)+4$

① $3-x>0 \quad \therefore x<3$

② $3-x=0$ 점근선 : $x=3$

③ $y=-\log_2 1+4=4 \quad (2,4)$

④



⑤ $y=-\log_2(-x)$

x축: 3, y축: 4

$y-4=-\log_2(-(x-3))$

$y=-\log_2(3-x)+4$

1-318

점근선 : $a-x=0$ $x=a$ $a=4$

$$y=f(x)=k\cdot\log_{\frac{1}{2}}(4-x)+b$$

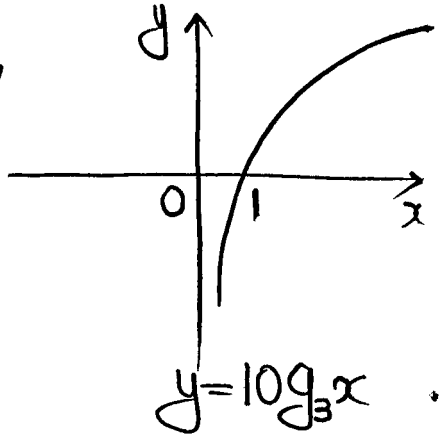
$(2,0)$; $f(2)=k\log_{\frac{1}{2}}2+b=-k+b=0$

$(0,-2)$; $f(0)=k\log_{\frac{1}{2}}4+b=-2k+b=-2$

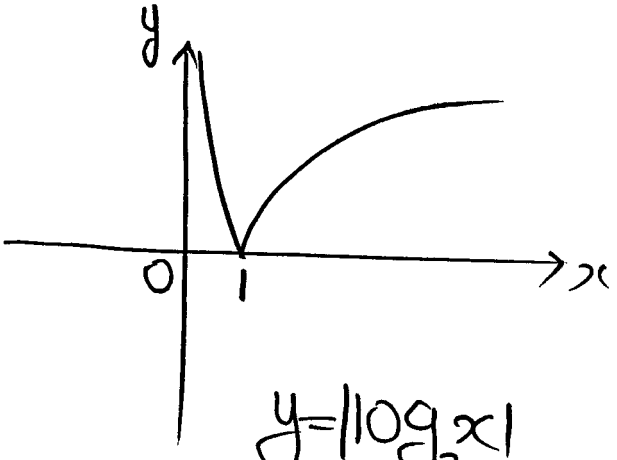
$k=2$, $b=2$ $\quad a+k+b=8$

2-11

(1)



$y=\log_3 x$

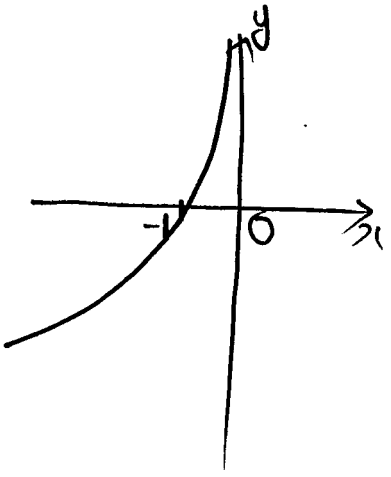


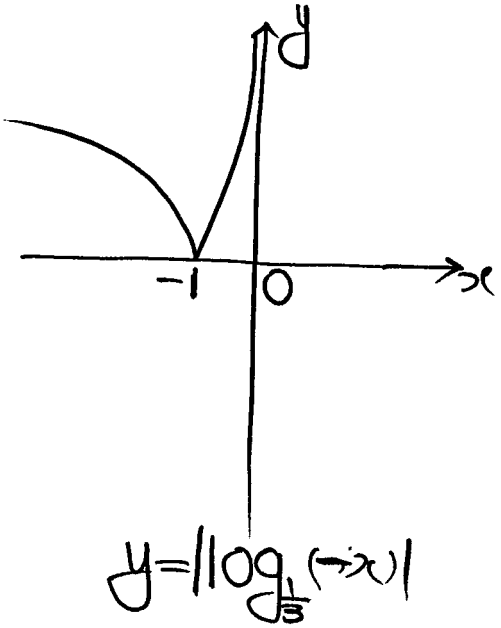
$y=|\log_3 x|$

(2) $y=|\log_{\frac{1}{3}}(-x)|$

$y=\log_{\frac{1}{3}}(-x)$

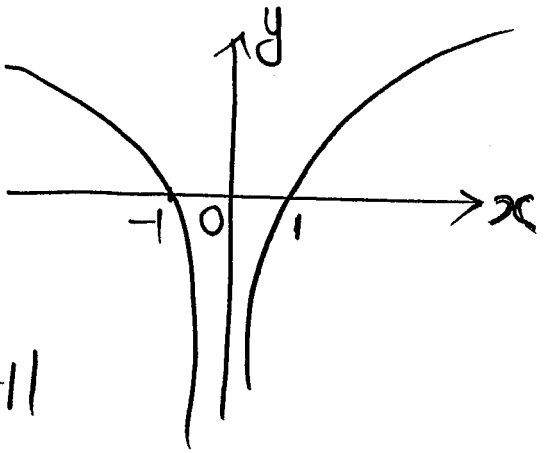
$y=-\log_3(-x)$



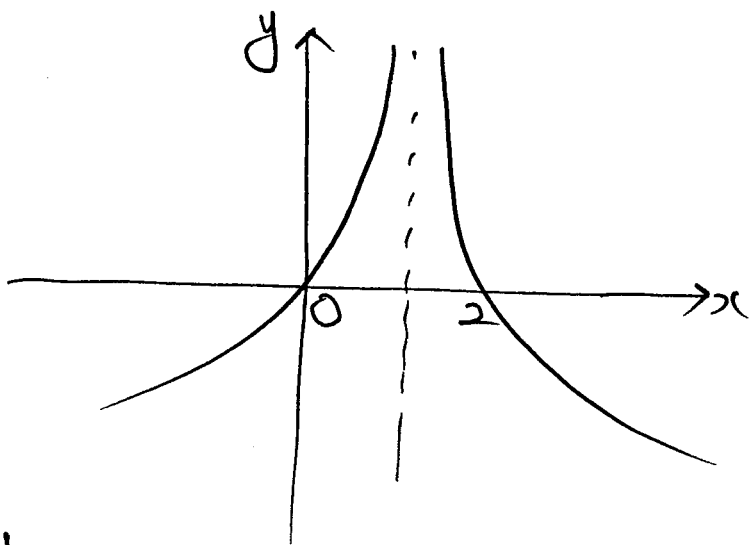


$y=|\log_{\frac{1}{3}}(-x)|$

(3) $y=\begin{cases}\log_3(-x) & (x<0)\\ \log_3 x & (x>0)\end{cases}$

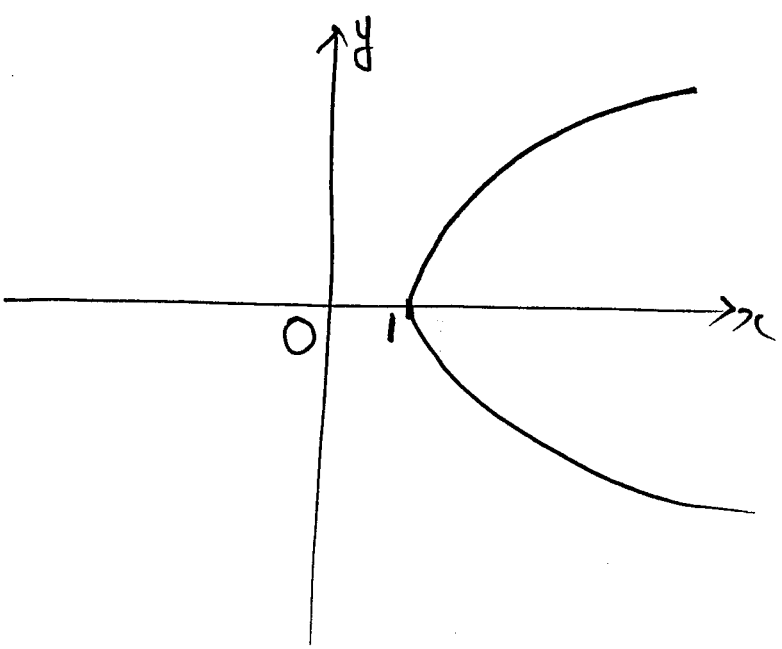


(4) $y=\log_{\frac{1}{3}}|x-1| = -\log_3|x-1|$



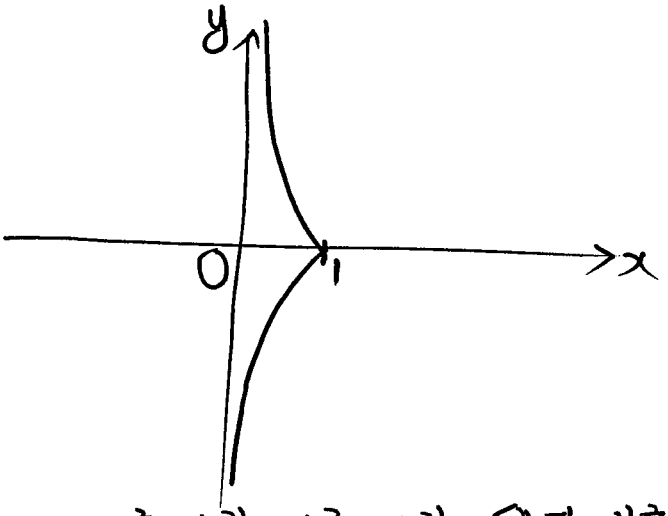
2-21

(1) $y=\begin{cases}\log_2 x & (y\geq 0)\\ -\log_2 x & (y<0)\end{cases}$ x축 대칭



(2) $|y| = \log_{\frac{1}{2}} x$ $\Longleftrightarrow$ x축 대칭

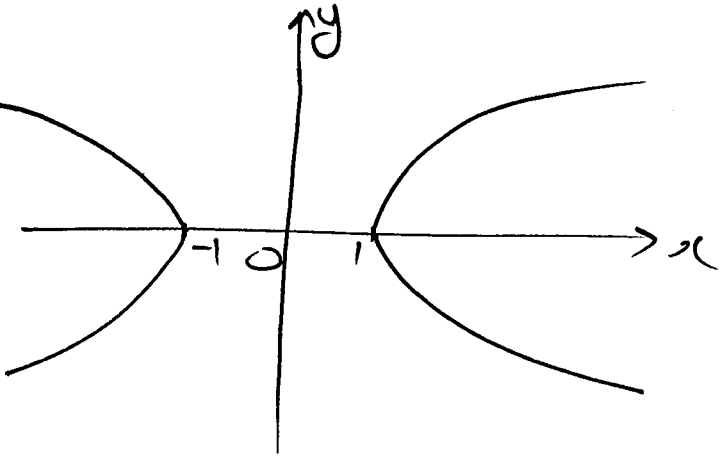
$y \geq 0$; $y = \log_{\frac{1}{2}} x$



(3) $|y| = \log_2 |x|$ $\Longleftrightarrow$ x축대칭, y축대칭, 원점 대칭

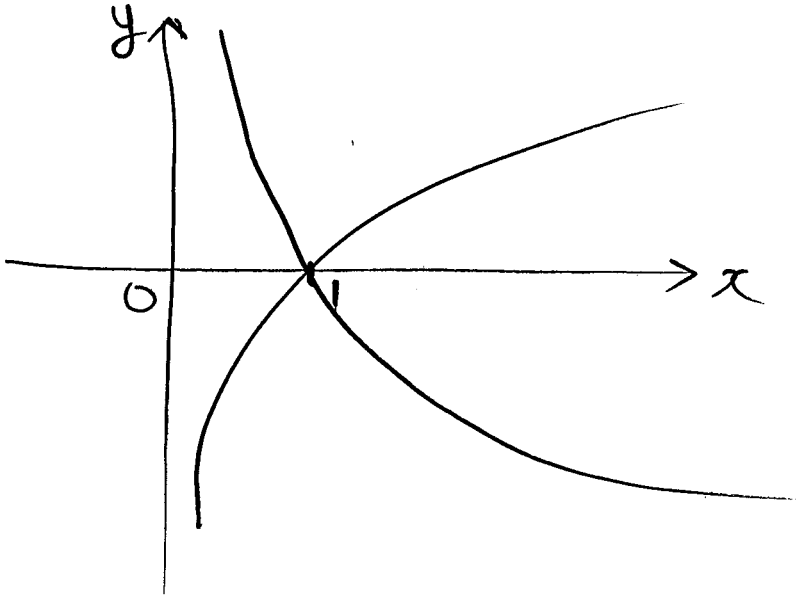
$x > 0$, $y \geq 0$

$y = \log_2 x$



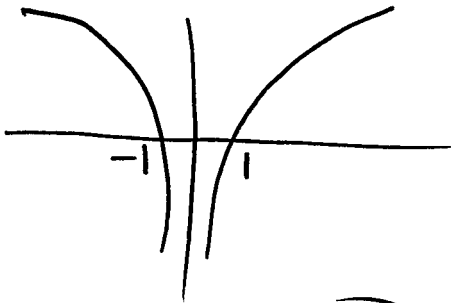
(4) $|A| = |B| \Longleftrightarrow A = B$ 또는 $A = -B$

$y = \log_2 x$ 또는 $y = -\log_2 x$

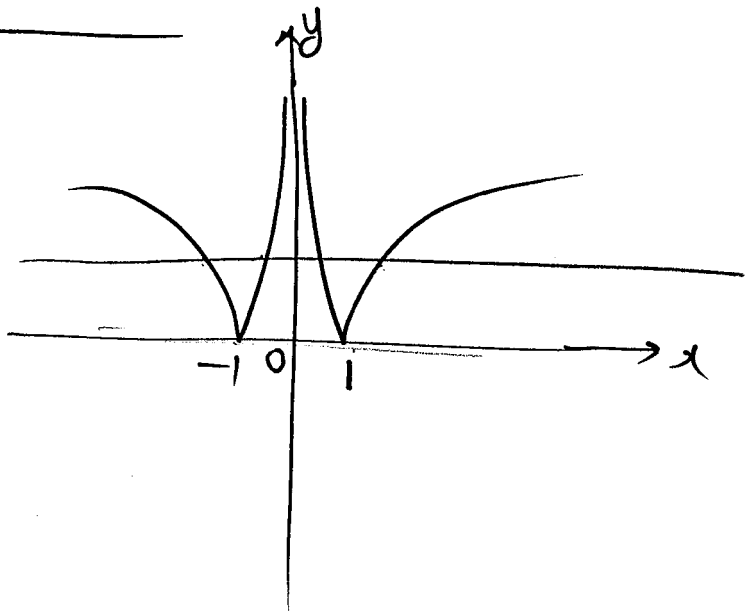


2-3 ④

$y = \log_2 x^2 = 2\log_2 |x|$ | $x^2 > 0$, $x \neq 0$



$y = |\log_2 x^2|$



↗

ㄴ

ㄷ

3-11

(1) $y = 10^{x-1} + 2$ ⟸ 역함수 ⟹

$x = 10^{y-1} + 2$

$10^{y-1} = x - 2$

$y - 1 = \log_{10}(x-2)$

$\therefore y = \log(x-2) + 1$

(2) $y = 2^{-x+3} - 1$ ⟸ 역함수 ⟹

$x + 1 = 2^{-y+3}$

$-y + 3 = \log_2(x+1)$

$\therefore y = -\log_2(x+1) + 3$

(3) $y=\log_4(x+1)-3$ $\Longleftrightarrow$ 역함수

$\log_4(y+1)=x+3$

$y+1=4^{x+3}$

$\therefore y=4^{x+3}-1$

(4) $y=\log_{\frac{1}{2}}\frac{1}{x+1}$ $\Longleftrightarrow$ 역함수

$\log_{\frac{1}{2}}\frac{1}{y+1}=x$

$\frac{1}{y+1}=2^x$

$y+1=2^{-x}$

$\therefore y=2^{-x}-1$

3-21

(1) $y=10^{ax}$ $\Longleftrightarrow$ 역함수

$x=10^{ay}$

$ay=\log_{10}x$

$y=\frac{1}{a}\log x \iff y=\frac{a}{100}\log x$

$\frac{1}{a}=\frac{a}{100}$ $a^2=100$

$a=10$ $(\because a>0)$

(2) $y=\left(\frac{1}{2}\right)^{2x-1}$ $\Longleftrightarrow$ 역함수

$\left(\frac{1}{2}\right)^{2y-1}=x$

$2y-1=\log_{\frac{1}{2}}x$

$y=-\frac{1}{2}\log_2 x+\frac{1}{2}$

$a=-\frac{1}{2}$, $b=\frac{1}{2}$

$a+b=0$

3-3번

$f(x)=\log_2 x-3$

$f^{-1}(x)=g(x)$

$\Longleftrightarrow f(g(x))=g(f(x))=x$

$y=f(x-1)$ $\Longleftarrow$ 역함수 $\Longrightarrow$

$x=f(y-1)$

$f^{-1}(x)=(f^{-1}\circ f)(y-1)$

$g(x)=y-1$

$\boxed{y=g(x)+1}$

4-11번

$(f\circ f\circ f\circ f\circ f\circ g\circ g\circ g\circ g\circ g)(x)=(f\circ f\circ f\circ f\circ f)(-3)$

$x=(f\circ f\circ f\circ f\circ f)(-3)$

$=18$

$f(-3)=\frac{71}{5}+\frac{19}{5}=18$

$(f\circ f)(-3)$

$=f(18)$

$=1-2\log_3 9$

$=-3$

4-2|

(1) $y = f(x) = 1 + 3\log_2 x$ $\quad g(x) = f^{-1}(x)$

$\Longleftrightarrow$ 역함수 $\quad x - 1 = 3\log_2 y \quad g(x) = 2^{\frac{x-1}{3}}$

$\log_2 y = \frac{x-1}{3}$

$y = 2^{\frac{x-1}{3}} \quad g(13) = 2^4 = 16$

(2) $y = f(x) = 5 \times 2^x$ $\Longleftrightarrow$ 역함수

$x = 5 \times 2^y$

$2^y = \frac{x}{5} \quad y = \log_2 \frac{x}{5} = g(x)$

$g(3) = \log_2 \frac{3}{5}$, $g\left(\frac{1}{3}\right) = \log_2 \frac{1}{15}$

$2^{g(3)+g\left(\frac{1}{3}\right)} = 2^{\log_2 \frac{1}{25}} = \frac{1}{25}$

4-3|3

$f^{-1}(x) = g(x) \iff f(g(x)) = g(f(x)) = x$

① $g(-12) = f^{-1}(12) = a$

$f(a) = -12$ $\quad$ i) $a < 1$; $f(a) = -a + 1 = 12$

$a = 13$ (x)

ii) $a \geq 1$ $f(a) = -2^{a+1} + 4 = -12$

$2^{a+1} = 16 \quad a = 3$

$g(-12)=3$

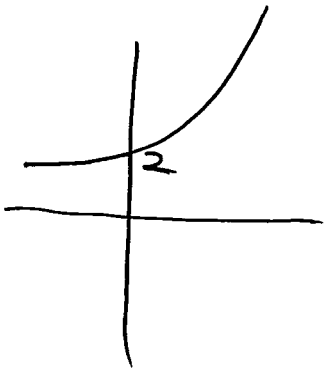
$g(k)=-2 \iff f(g(k))=f(-2)$

$k=3$

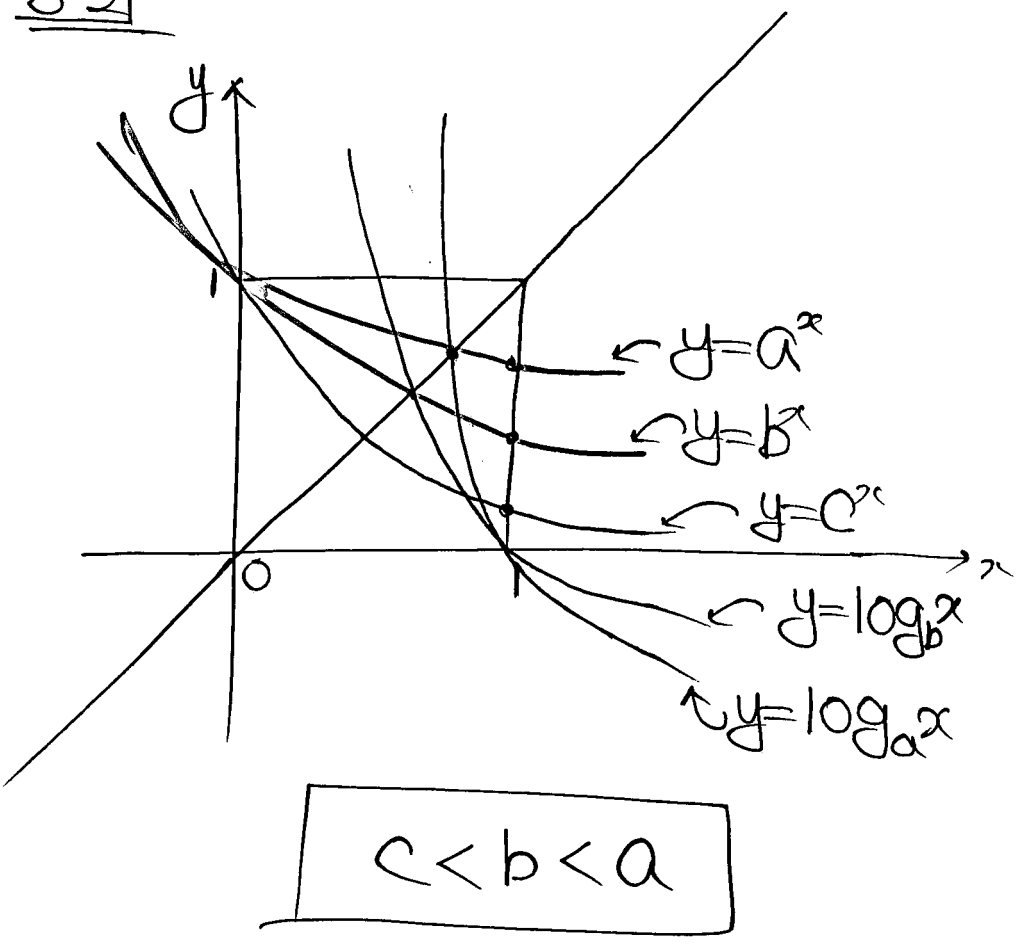
5-1 ①

$0<a<1$, $\log_a b=0$ $b=1$

$\frac{1}{a}>1$ $y=(\frac{1}{a})^x+1$ $(0,2)$



5-2



$c<b<a$

5-3 ②

㉠ $y=3^x \xrightarrow{x\text{축}: -\log_3\frac{1}{2}} y=3^{x+\log_3\frac{1}{2}}$

$y=\frac{1}{2}\cdot 3^x$

$\frac{1}{2}=3^{\log_3\frac{1}{2}}$

✗ $y=9^x+1$

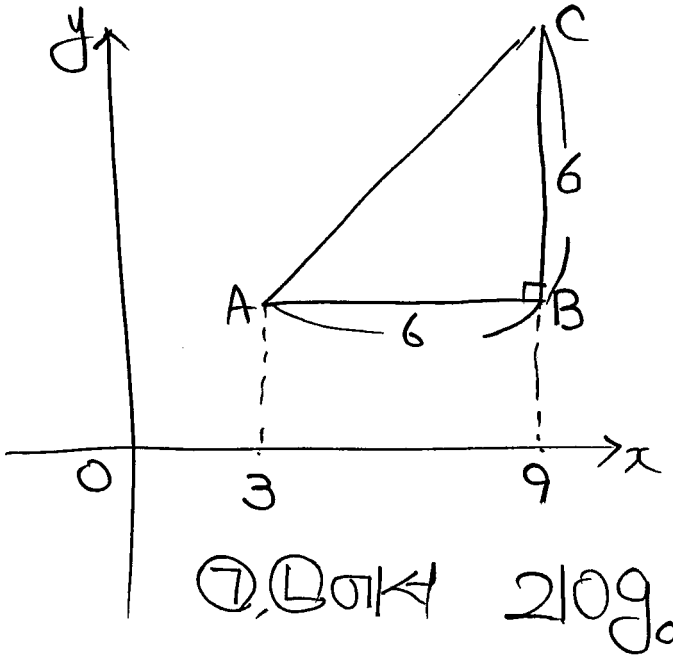
㉡ $y=3^x \xrightarrow{y=x\text{대칭}} y=\log_3 x$

$\xrightarrow{y\text{축}: -1} y+1=\log_3 x$

$\therefore y=\log_3 x-1$

✗ $y=\log_9 x^2=\log_3|x|$

6-11 12



$A(3, \log_a 3)$

$B(9, \log_b 9)$

$C(9, \log_a 9)$

㉠ $\log_a 3=\log_b 9$

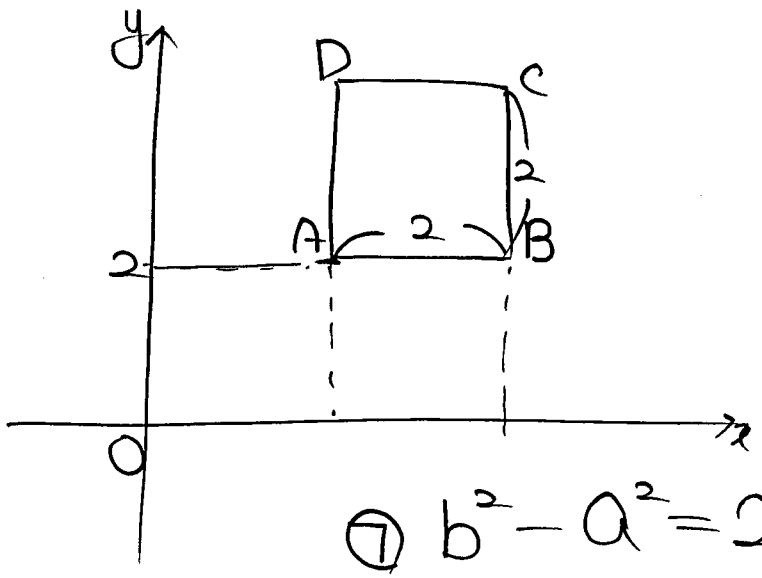
㉡ $\log_a 9=\log_b 9+6$

㉠, ㉡에서 $2\log_a 3=\log_a 3+6$

$\log_a 3=6 \quad a^6=3 \quad a^6+b^6=12$

$\log_b 9=6 \quad b^6=9$

6-2 ③



$A(a^2, 2)$

$B(b^2, 2)$

$C(b^2, 4)$

㉠ $b^2 - a^2 = 2$

㉡ $\log_a b^2 = 4$ $\quad b^2 = a^4$

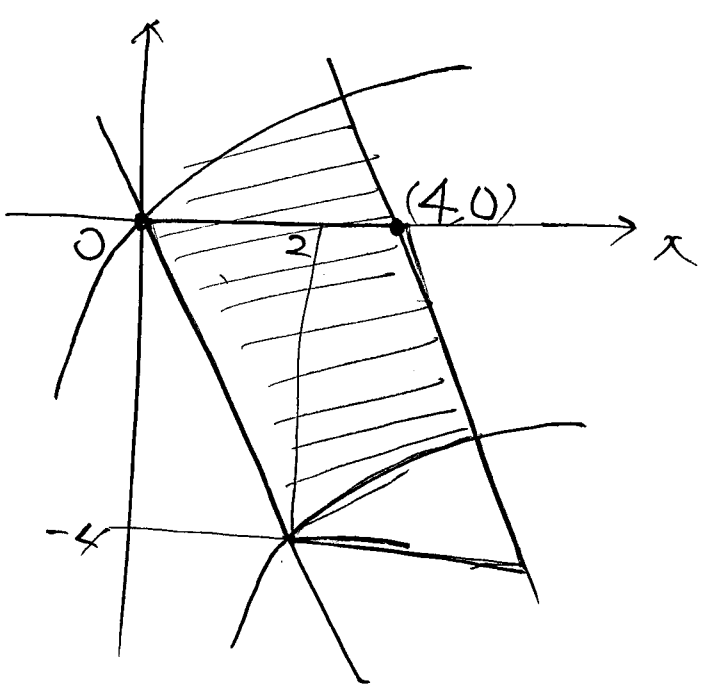
$b = a^2$

㉠, ㉡ 에서 $b^2 - b - 2 = 0$

$(b-2)(b+1) = 0$ $\quad \boxed{b=2}$

6-3

$y = \log_6(x+1)$ $\xrightarrow[y\text{축}: -4]{x\text{축}: 2}$ $y+4 = \log_6(x-1)$



$S = 16$

7-1 16

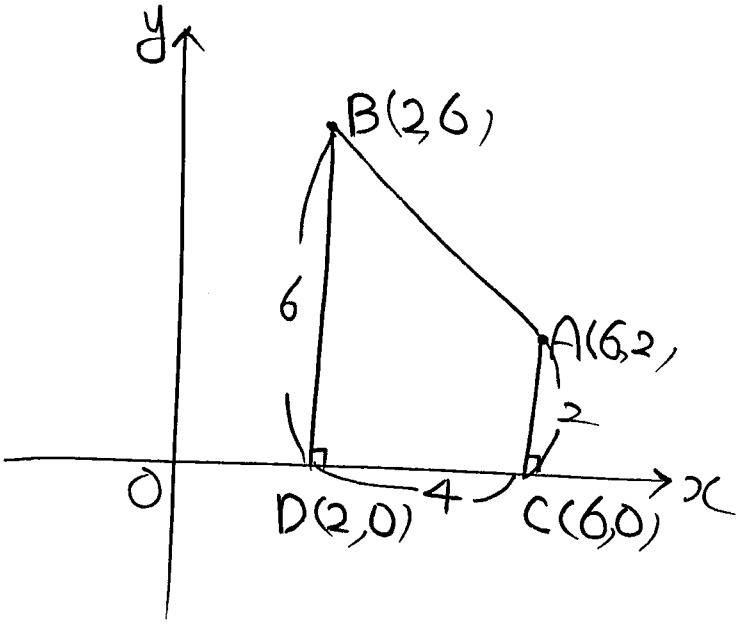
$y=\log_3(x+3)$

反解 $\Leftrightarrow x=\log_3(y+3)$

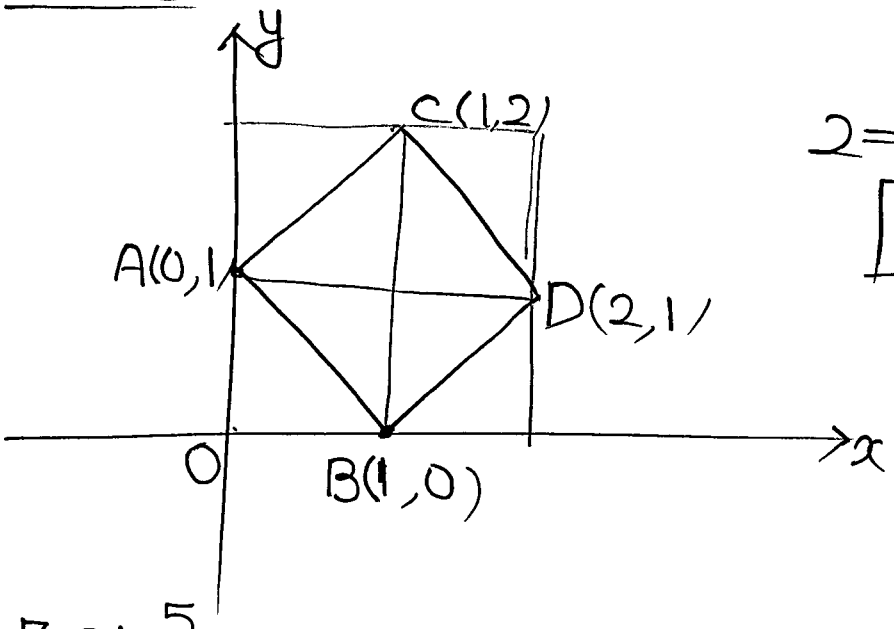
$y+3=3^x$

$y=3^x-3$

$S=\frac{1}{2}\times 8\times 4=16$



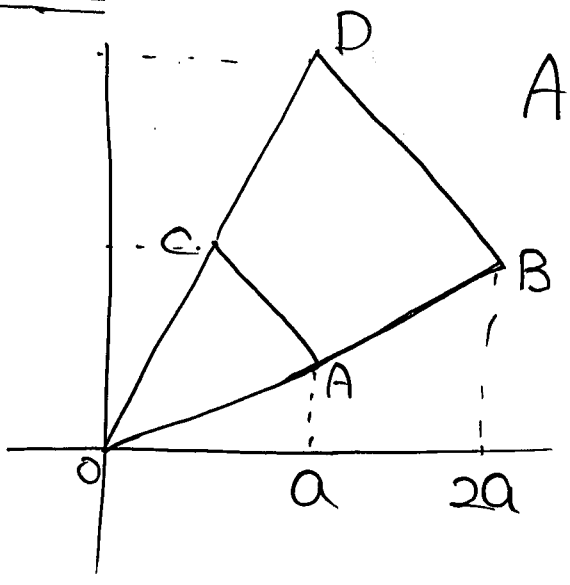
7-2 3.



$2=-1+k$

$\boxed{k=3}$

7-3 $\frac{5}{2}$



$A(a,\log_2 a)$

$B(2a,\log_2 2a)$

$2\log_2 a=\log_2 2a$

$\log_2 a^2=\log_2 2a$

$a^2=2a \quad a=2$

$A(2,1) \quad m=\frac{1}{2}$

$C(1,2) \quad n=2$

$m+n=\frac{5}{2}$

8-11

(1) $-2 = \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-2} = \log_{\frac{1}{2}} 4 \qquad 3 < \sqrt{10} < 4$

$\therefore \log_{\frac{1}{2}} 4 < \log_{\frac{1}{2}}\sqrt{10} < \log_{\frac{1}{2}} 3$

$$\boxed{-2 < \log_{\frac{1}{2}}\sqrt{10} < \log_{\frac{1}{2}} 3}$$

(2) $0 < a^2 < b < a < 1$

㉠ $\log_a a^2 > \log_a b > \log_a a > \log_a 1$

$\therefore 1 < \log_a b < 2$

㉡ $\log_b a^2 > \log_b b > \log_b a > \log_b 1$

$2\log_b a > 1 > \log_b a > 0$

$\therefore \frac{1}{2} < \log_b a < 1$

$\log_a \frac{a}{b} = 1 - \log_a b \qquad (-2 < -\log_a b < -1)$

$-1 < \log_a \frac{a}{b} < 0$

㉢ $\log_b \frac{b}{a} = 1 - \log_b a \qquad (-1 < -\log_b a < -\frac{1}{2})$

$0 < \log_b \frac{b}{a} < \frac{1}{2}$

$\log_{\frac{a}{b}} a < \log_b \frac{b}{a} < \frac{1}{2} < \log_b a < \log_a b$

8-21 ⑤

$0<a<1<b$, $ab<1$

$\log_a a > \log_a 1 > \log_a b$ $\therefore \log_a b < 0$

$\log_b a < \log_b 1 < \log_b b$ $\therefore \log_b a < 0$

$\log_a ab > \log_a 1$

$1 + \log_a b > 0$ $\therefore -1 < \log_a b < 0$

$\log_b ab < \log_b 1$

$\log_b a + 1 < 0$ $\therefore \log_b a \leq -1$

$A = \frac{1}{2}\log_a b$, $B = 2\log_b a$

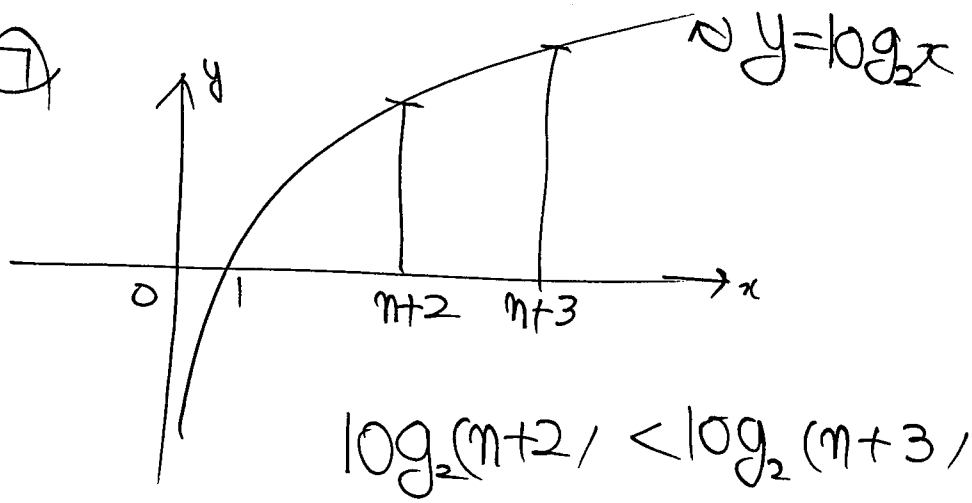
㉠ $A = \frac{1}{2}\log_a b < 0$

㉡ $AB = 1$

㉢

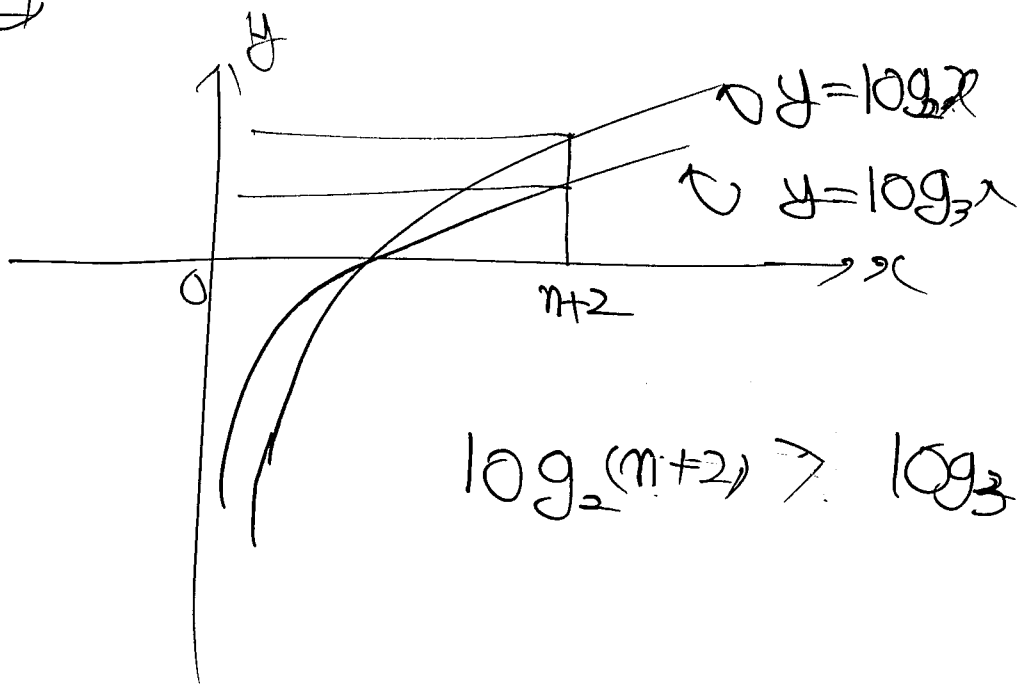
8-3 ⑤

㉠



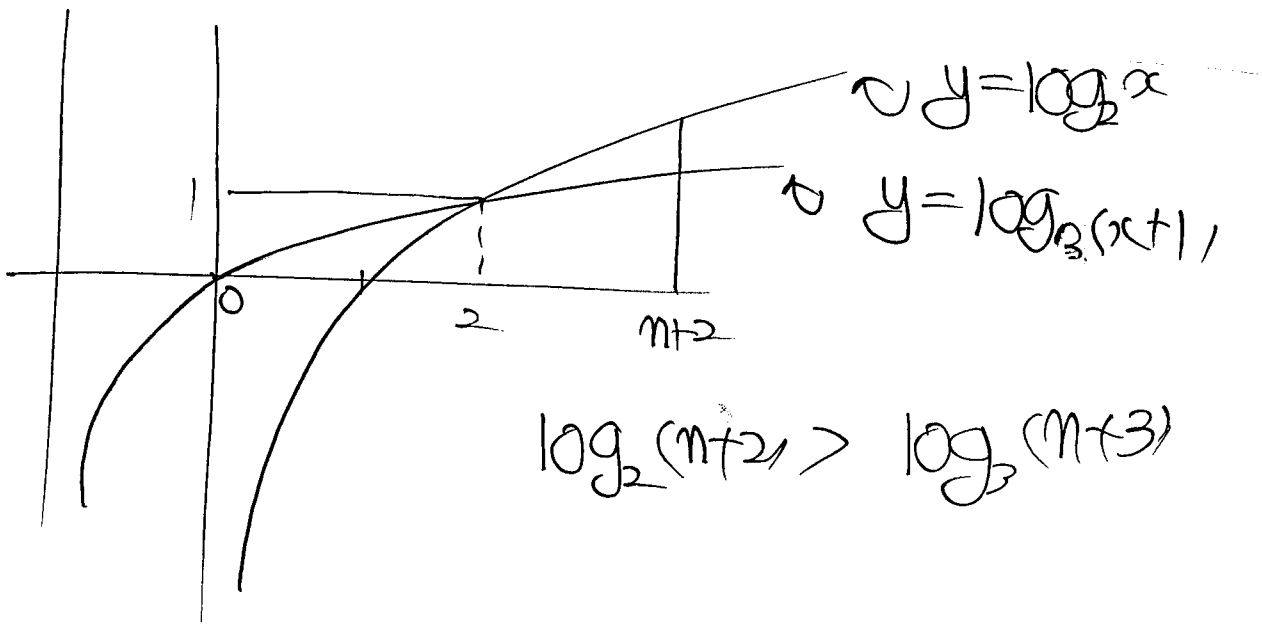
$\log_2(n+2) < \log_2(n+3)$

㉡



$\log_2(n+2) > \log_3(n+2)$

㉢



$\log_2(n+2) > \log_3(n+3)$

9-1

(1) $f(x)=\log_{\frac{1}{3}}(-2x+5)$ $(-2\le x\le 2)$

$M=f(2)=\log_{\frac{1}{3}}1=0$

$m=f(-2)=\log_{\frac{1}{3}}9=-2$

(2) $f(x)=\log_5((x-3)^2+25)$

M : 없다, $m=f(3)=\log_5 25=2$

(3) $f(x)=\log_2((x-1)^2+2)$ $(0\le x\le 3)$

$M=f(3)=\log_2 6$, $m=f(1)=\log_2 2=1$

(4) $f(x)=\log_3(-(x+2)^2+27)$ $(-3\le x\le 3)$

$M=f(-2)=\log_3 27=3$, $m=f(3)=\log_3 2$

9-2

(1) $x>0, y>0$

$x+y=32$

$\log_4 2x+\log_4 2y$

$=\log_4 4xy \le \log_4 4\cdot 2^8$

$x+y\ge 2\sqrt{xy}$ $=\frac{10}{2}\log_2 2$

$16\ge\sqrt{xy}$

$2^8\ge xy$ $=5$

(2) $x>0, y>0$

$\log_3 xy = 5$

$xy = 3^5$

$3x+y \geq 2\sqrt{3x \cdot y}$

$= 2\sqrt{3^6} = 2 \cdot 3^3 = 54$

9-3]

(1) $5 \leq x \leq 8$; $f(x) = \log_{\frac{1}{2}}(x-a)$

$f(8) = \log_{\frac{1}{2}}(8-a) = -2$ $\quad 8-a=4$

$\boxed{a=4}$

(2) $f(x) = \log_a\{(x-1)^2+4\}$

$0<a<1$; $f(1) = \log_a 4 = -2$

$a^{-2} = 4$ $\quad a = 4^{-\frac{1}{2}} = \frac{1}{2}$

10-1]

(1) $\log_2 x = t$ (t는실수)

$f(t) = (1+t)^2 - 2t$

$= t^2 + 1$

M : 없다

m : $f(0) = 1$

$\log_2 2x = \log_2 2 + \log_2 x$

$= 1+t$

$\log_2 x^2 = 2\log_2 x$

(2) $\log_3 x = t$ (t는 실수)

$f(t) = 4t - t^2 + 1$

$= -(t-2)^2 + 5$

$M = f(2) = 5$

m : 없다.

$\log_3 x^4 = 4\log_3 x$

$= 4t$

(3) $\log_3 x = t$ ($1 \le x \le 81 \Rightarrow \log_3 1 \le \log_3 x \le \log_3 81$)

$(0 \le t \le 4)$

$f(t) = t^2 + 3 - 2t$

$= (t-1)^2 + 2$

$M = f(4) = 9 + 2 = 11$

$m = f(1) = 2$

$\log_3 \frac{27}{x^2}$

$= 3\log_3 3 - 2\log_3 x$

$= 3 - 2t$

(4) $\log_2 x = t$ ($1 \le x \le 16 \Rightarrow \log_2 1 \le \log_2 x \le \log_2 16$)

$(0 \le t \le 4)$

$f(t) = (1+t)(3-t)$

$= -t^2 + 2t + 3$

$= -(t-1)^2 + 4$

$M = f(1) = 4$

$m = f(4) = -5$

$\log_2 2x = \log_2 2 + \log_2 x$

$= 1 + t$

$\log_2 \frac{8}{x} = 3 - \log_2 x$

$= 3 - t$

10-2 ⑤

$\log_3 x = t$ (t는실수)

$\log_{3^{\frac{1}{2}}} = -2\log_3 x$

$= -2t$

$f(x) = g(t) = -t^2 + 2at + b$

$f(9) = g(2) = 6$

$g(t) = -(t-2)^2 + 6$

$f(3) = g(1) = -1 + 6 = 5$

$\log x = t$ (t는실수)

10-3

(1) $\log f(x) = \log \frac{1000x^4}{x^{\log x}}$

$= 3 + 4\log x - \log x^{\log x}$

$= 3 + 4t - t^2$

$= -(t-2)^2 + 7$

$f(x) = g(t) = 10^{-(t-2)^2+7}$ ∩

$M = g(2) = 10^7$

m : 최댓.

(2) $\log_2 g(x) = \log_2 (8x)^{5-\log_2 x}$

$= (5-\log_2 x)(3+\log_2 x)$

$= -t^2+2t+15$

$= -(t-1)^2+16$

$\log_2 x = t$

$(-1 \le t \le 2)$

$12 \le \log_2 g(x) \le 16$

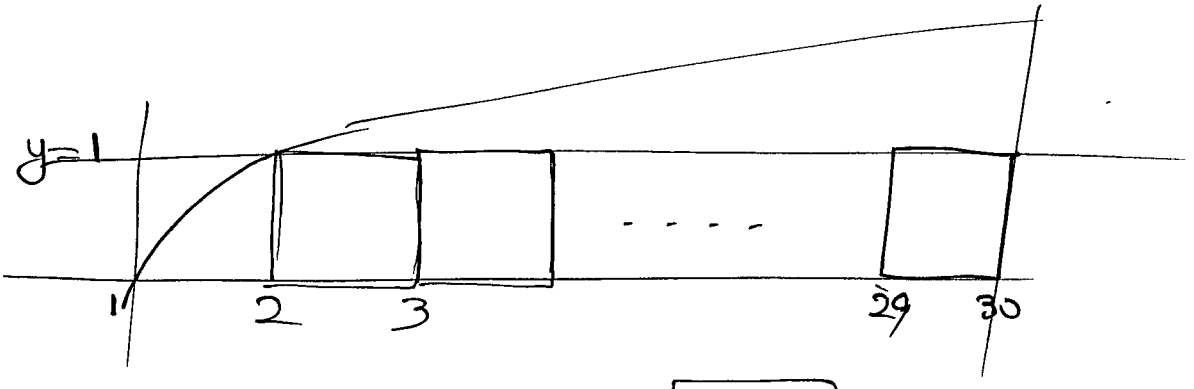
$2^{12} \le g(x) \le 2^{16}$

$M = 2^{16}$

$m = 2^{12}$

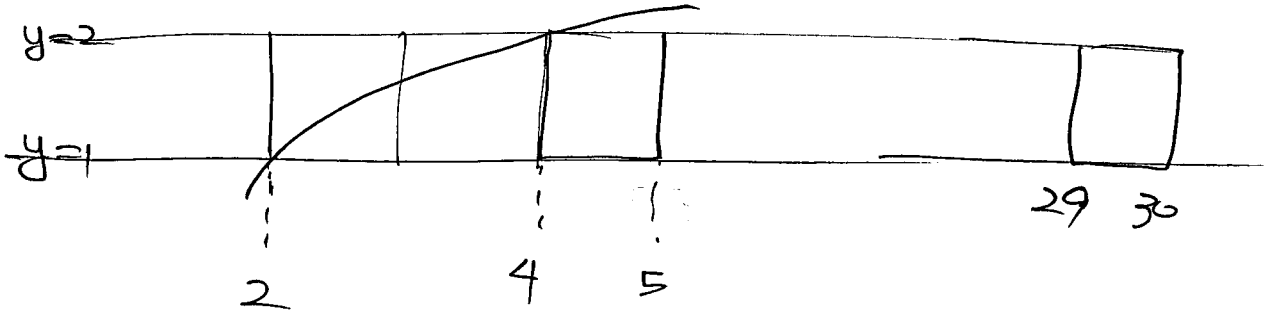
11-1190

㉠

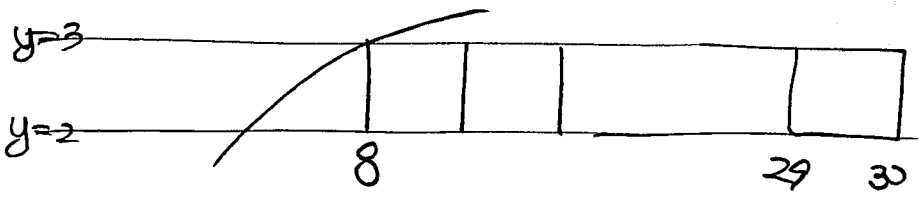


2~29 28개

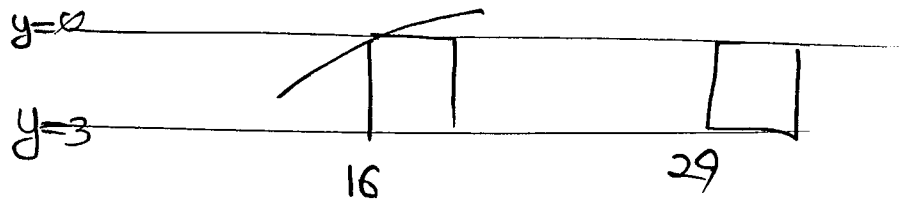
㉡



4~29 26개



8~29 22개



16~29 14개

$28+26+22+14=90$

11-21 143

x	2	4	8	16	32	64	128	256	512
$\log_2 x$	1	2	3	4	5	6	7	8	9
$\log_4 x$	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$

$2,\times\times\times 4,\times\times$ $3\triangle\triangle < \square < 6\triangle\triangle$

3개

$m=16$

$M=127$

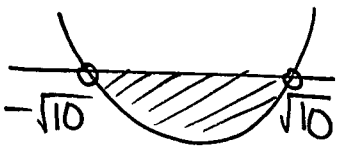
$M+m=143$

P186

4-112

㉠ $10-x^2>0$

$x^2-10<0$



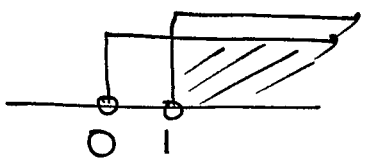
$-\sqrt{10}<x<\sqrt{10}$

$A=\{x \mid -\sqrt{10}<x<\sqrt{10}\}$

㉡ $x>0$, $\log x>0$

$\log x>\log 1$

$x>1$



$x>1$

$B=\{x \mid x>1\}$

$A\cap B=\{x \mid 1<x<\sqrt{10}\}$ $x=2, 3$ 2개

4-2116

$y=1+\log_2 x \xrightarrow{\text{평: } 1} y=1+\log_2(x-1)$

$\xrightarrow{y=x}$ $x=1+\log_2(y-1)$

$\log_2(y-1)=x-1$

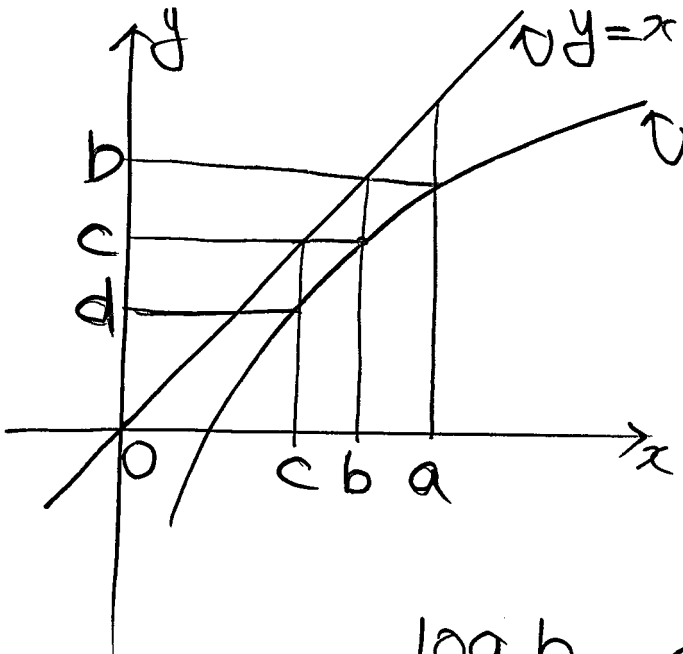
$y-1=2^{x-1}$

$y=2^{x-1}+1$

$\xrightarrow{y\text{평: } -1}$ $y+1=2^{x-1}+1$

$y=g(x)=2^{x-1}$ $g(5)=2^4=16$

4-3 ④



$\log_2 c = d \quad c = 2^d$

$\log_2 b = c \quad b = 2^c$

$\log_2 a = b \quad a = 2^b$

$$\log_a b = \frac{\log_2 b}{\log_2 a} = \frac{c}{b}$$

4-4 ①

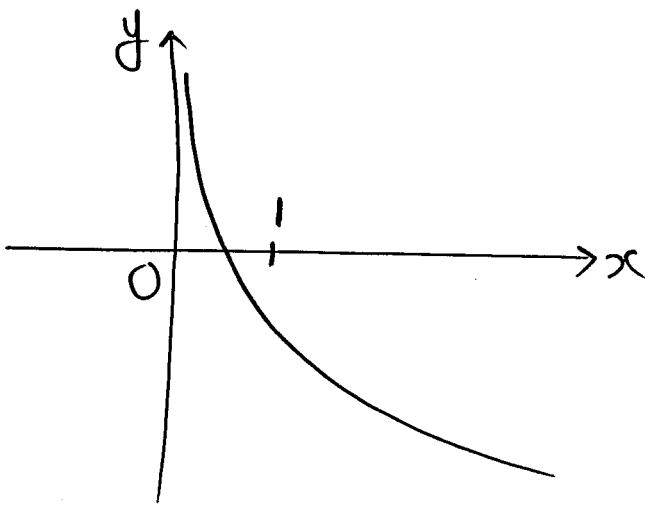
$y = \log_b ax$

$0 < b < 1$

$\log_b a < 0$

$\log_b a < \log_b 1$

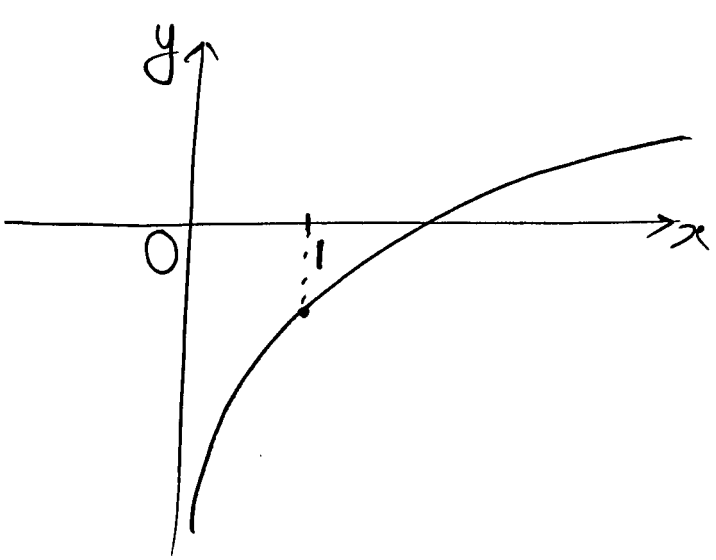
$a > 1$



$y = \log_a bx$

$\log_a b < 0$

$\log_a b < \log_a 1$



4-5)

(1) $y=\frac{1}{2}(3^x-3^{-x})$ $\Longleftrightarrow$ 역함수

$x=\frac{1}{2}(3^y-3^{-y})$

$3^y-3^{-y}=2x$

$(3^y)^2-2x\cdot 3^y-1=0$

$3^y=x\pm\sqrt{x^2+1}$

$3^y>0$ 이므로

$3^y=x+\sqrt{x^2+1}$

$y=\log_3(x+\sqrt{x^2+1})$

(2) $y=\log_2(x+\sqrt{x^2-1})$ $(x\geq 1,\ y\geq 0)$

$\Longleftrightarrow$ 역함수

$x=\log_2(y+\sqrt{y^2-1})$

$y+\sqrt{y^2-1}=2^x$

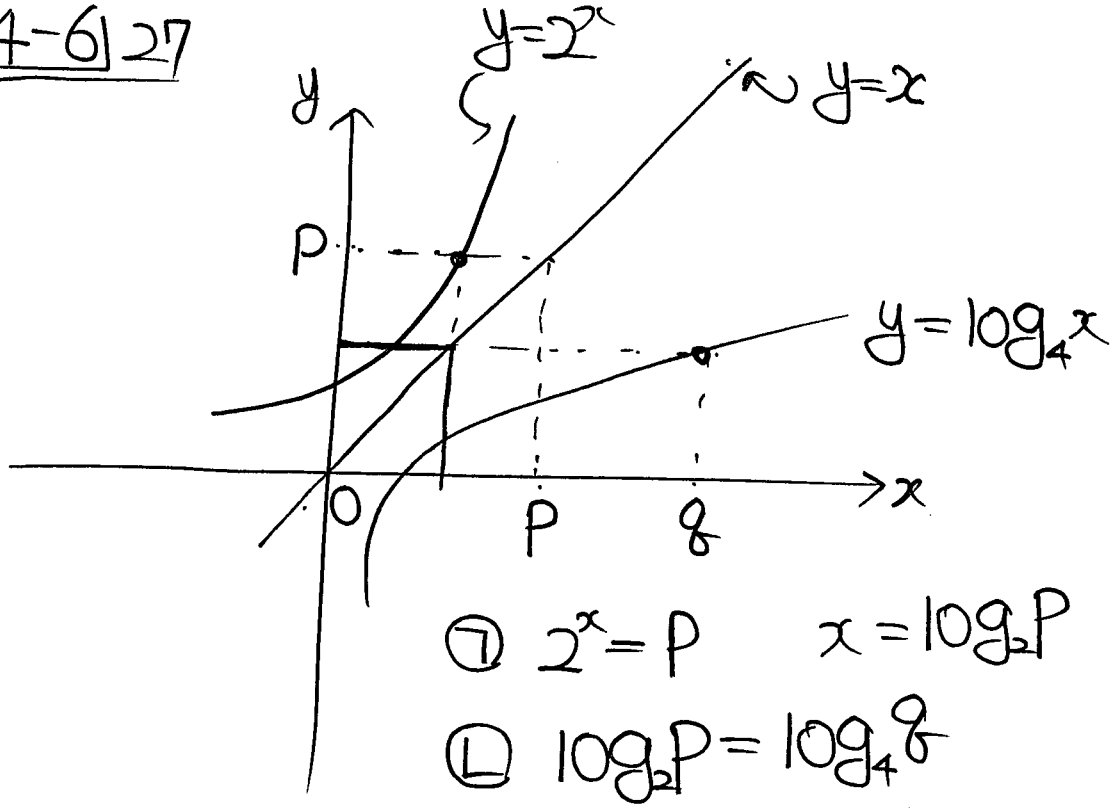
$\sqrt{y^2-1}=2^x-y$

$y^2-1=2^{2x}-2\cdot 2^x\cdot y+y^2$

$2\cdot 2^x\cdot y=2^{2x}+1$

$y=\frac{1}{2}(2^x+2^{-x})$ $(x\geq 0)$

4-6 | 27



① $2^x = P \quad x = \log_2 P$

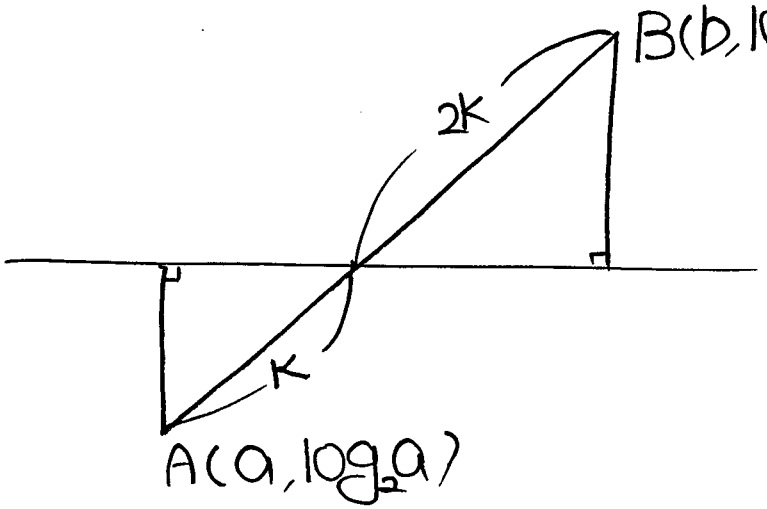
② $\log_2 P = \log_4 q$

$\log_4 P^2 = \log_4 q$

$\begin{cases} P^2 = q \\ P + q = 12 \end{cases}$ $P^2 + P - 12 = 0$ $P = 3, q = 9$

$(P-3)(P+4) = 0$ $Pq = 27$

4-7 | 1



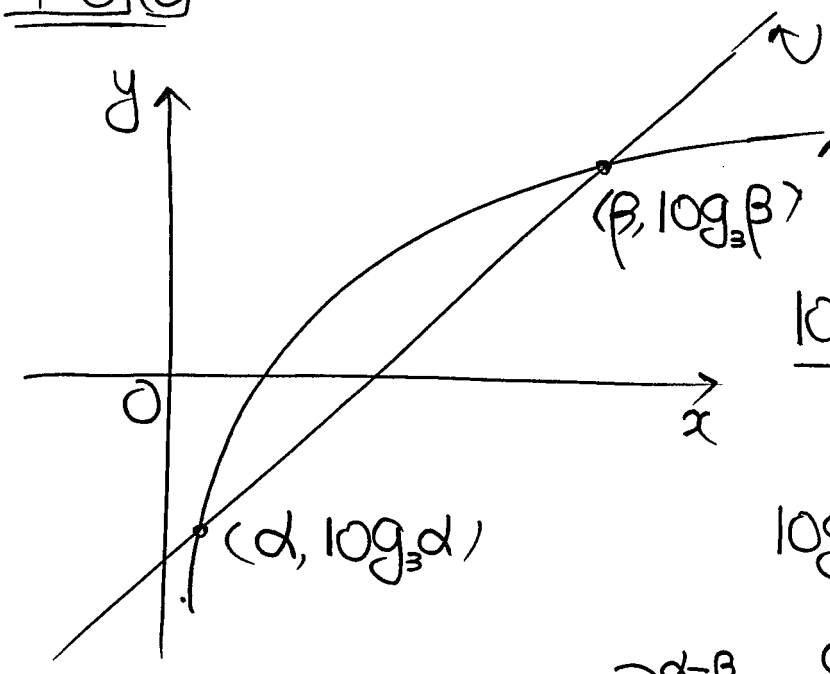
$\log_2 b = -2\log_2 a$

$2\log_2 a + \log_2 b = 0$

$\log_2 a^2 b = 0$

$a^2 b = 1$

4-8 ③

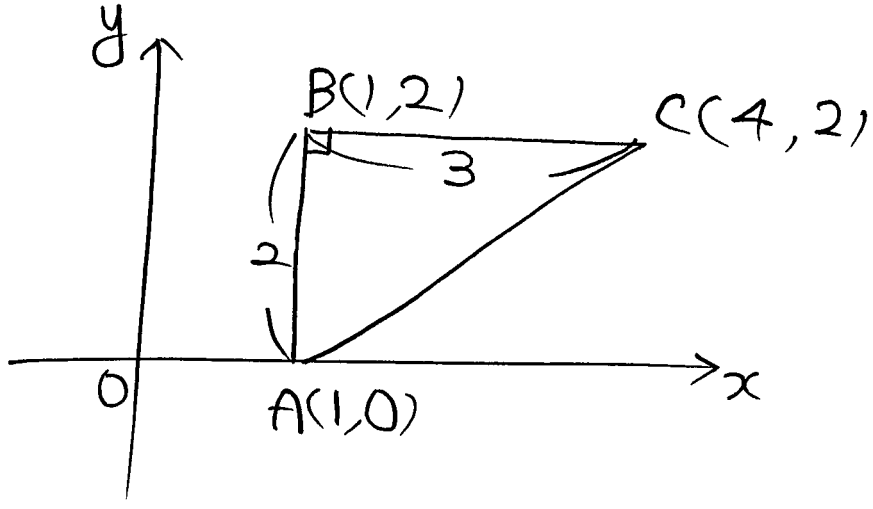


$$\frac{\log_3\alpha-\log_3\beta}{\alpha-\beta}=1$$

$$\log_3\frac{\alpha}{\beta}=\alpha-\beta$$

$$3^{\alpha-\beta}=\frac{\alpha}{\beta}=\frac{1}{2}$$

4-9 3



$S=3$

4-10 19

$\log_2 x=t \quad (0\le t\le 4)$

$\log_{\frac{1}{2}} x=-\log_2 x=-t$

$f(x)=g(t)=-t^2+2t+a$

$=-(t-1)^2+a+1$

$g(1)=a+1=b$

$g(4)=a-8=1$

$b=10$

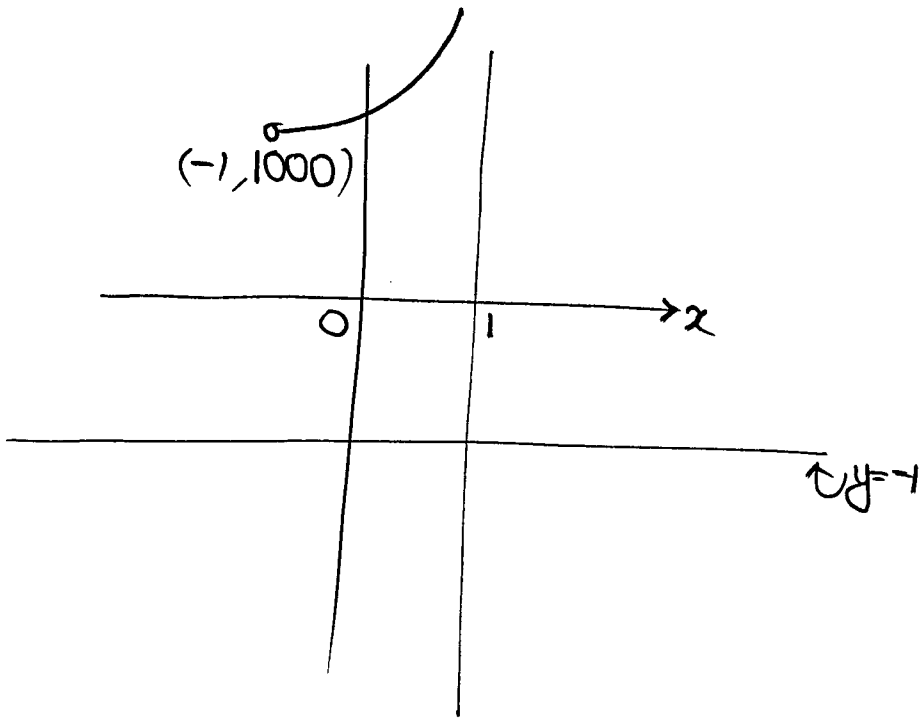
$a=9$

$a+b=19$

4-11 ③

$$f(x)=\frac{2000+x}{1-x}$$

$$f(-1)=1000$$



$$\{y \mid y>3\}$$

4-12 ⑤

$$f^{-1}(x)=g(x) \iff f(g(x))=g(f(x))=x$$

$$y=f(\log_2 x-1) \xleftrightarrow{\text{역함수}}$$

$$x=f(\log_2 y-1)$$

$$g(x)=g(f(\log_2 y-1))$$

$$\log_2 y-1=g(x)$$

$$\log_2 y=g(x)+1$$

$$y=2^{g(x)+1}$$

4-13 ③

$$A=\log_{0.2} 0.3=\frac{\log 0.3}{\log 0.2}=\frac{\log 3-1}{\log 2-1}=\frac{1-\log 3}{1-\log 2}$$

$$B=\log_2 3=\frac{\log 3}{\log 2}$$

$$C=\log_{20} 30=\frac{\log 30}{\log 20}=\frac{1+\log 3}{1+\log 2}$$

㉠ $C-B=\frac{1+\log 3}{1+\log 2}-\frac{\log 3}{\log 2}$

$=\frac{\log 2+\log 3\cdot\log 2-\log 3-\log 3\cdot\log 2}{(1+\log 2)\cdot\log 2}<0$

$\therefore\ C<B$

㉡ $C-A=\frac{1+\log 3}{1+\log 2}-\frac{1-\log 3}{1-\log 2}$

$=\frac{(1+\log 3)(1-\log 2)-(1-\log 3)(1+\log 2)}{(1+\log 2)(1-\log 2)}$

$=\frac{2(\log 3-\log 2)}{(1+\log 2)(1-\log 2)}>0\quad\therefore\ C>A$

㉠, ㉡ $\therefore$ $\boxed{A<C<B}$

4-14| ①

$f(x)=(\log x-\log 3)\cdot(\log x-\log 2)$ | $\log x=t$ (t는 실수)

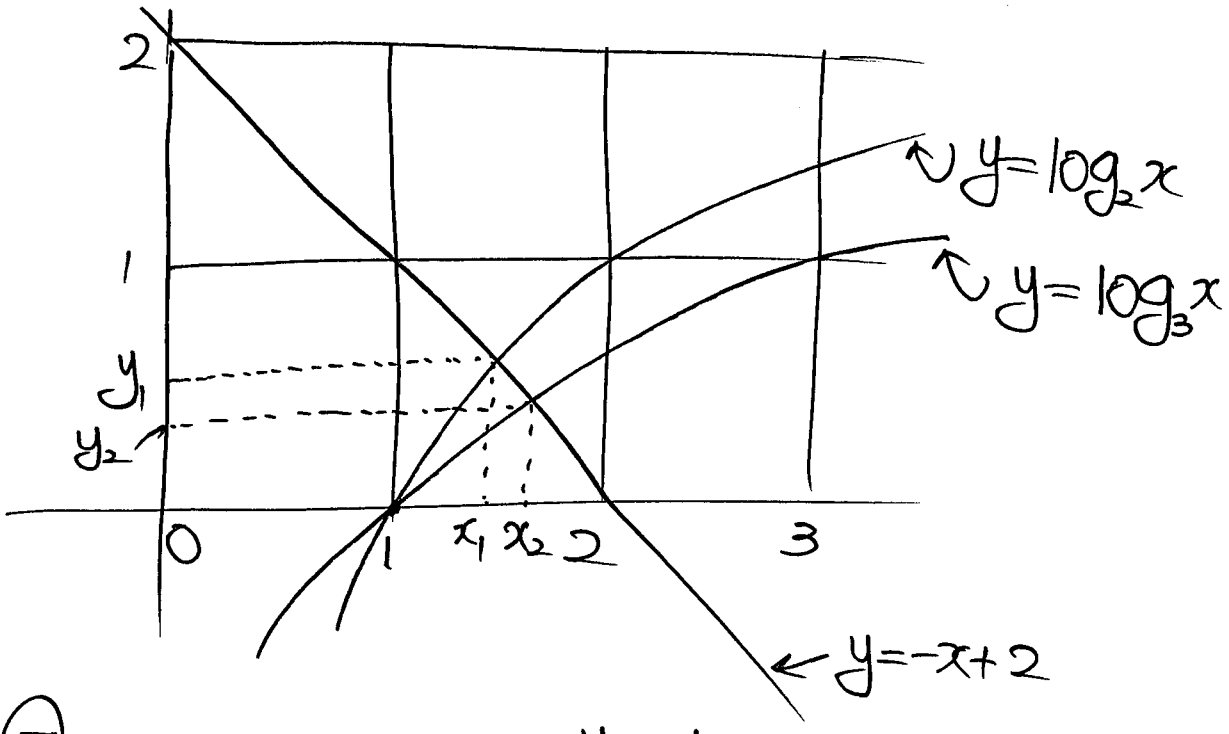
$g(t)=(t-\log 3)(t-\log 2)$

$=t^2-(\log 3+\log 2)\cdot t+(\log 3)(\log 2)$

$=\left(t-\frac{\log 3+\log 2}{2}\right)^2-\frac{(\log 3+\log 2)^2+(\log 3)\cdot(\log 2)}{4}$

최솟값 : $-\frac{(\log 3+\log 2)^2-4\log 3\cdot\log 2}{4}=-\frac{1}{4}(\log\frac{3}{2})^2$

4-15 ⑤



㉠ $1<x_1<2$, $0<y_2<1$

$x_1>y_2$

㉡ $y_1=-x_1+2$

$-)\ y_2=-x_2+2$

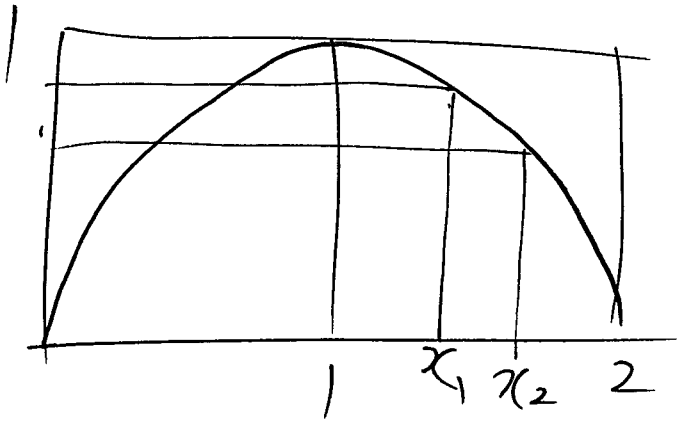
$y_1-y_2=x_2-x_1$

㉢ $x_1y_1=x_1(-x_1+2)$ $\quad x_1y_1>x_2y_2$

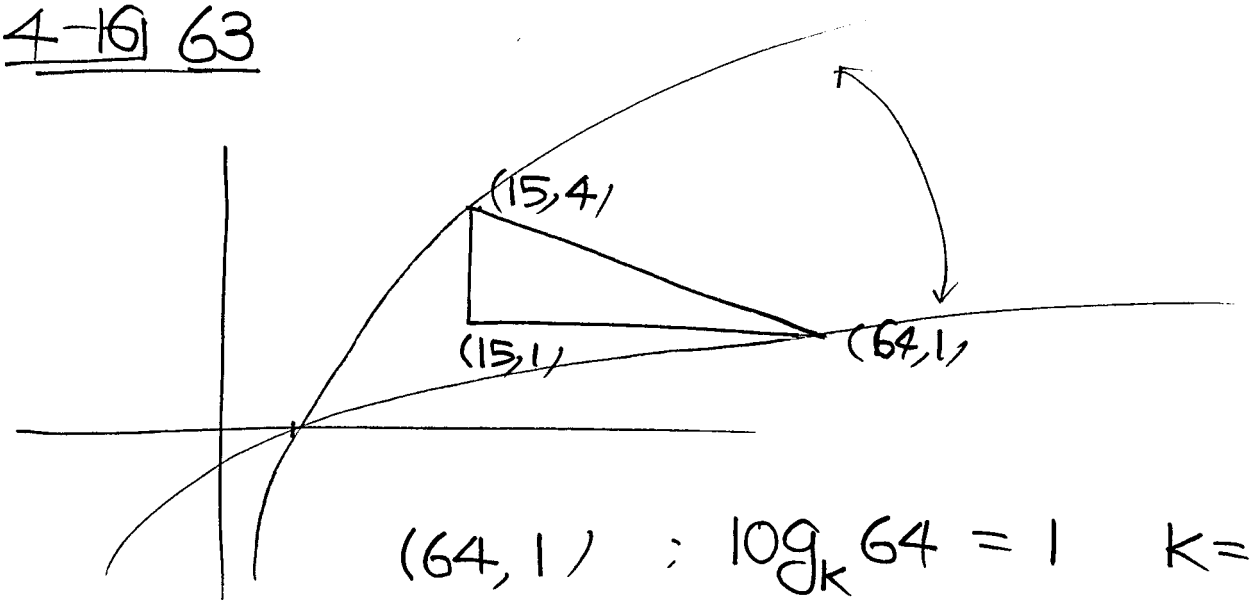
$x_2y_2=x_2(-x_2+2)$

$f(x)=x(-x+2)$

$=-x^2+2x$



4-16 63



$(64,1)$: $\log_k 64 = 1$ $\quad k=64$

$(15,4)$; $\log_k 15 = 4$ $\quad k^4=15$.

$1. \times\times < k \leq 64$

63개

4-17 10

① $\log_4(4+p)+q=1$

② $\log_{\frac{1}{2}}(4+p)+q=1$

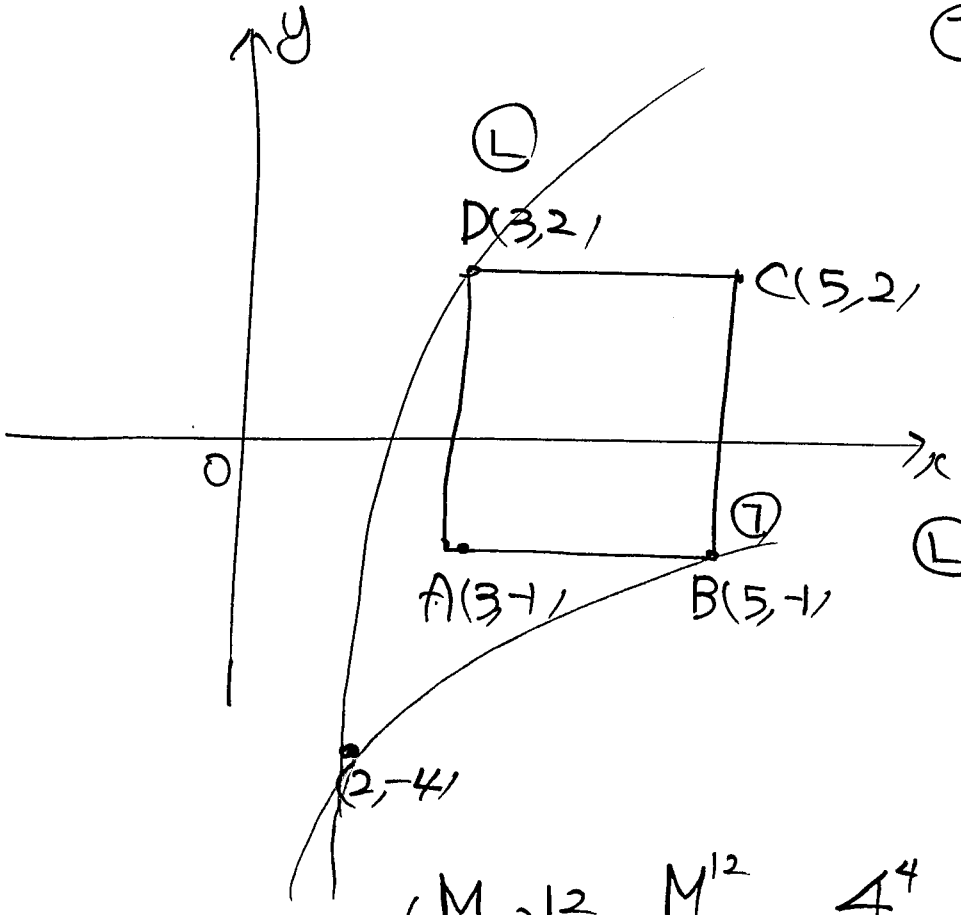
$\frac{1}{2}\log_2(4+p)+\log_2(4+p)=0$

$\frac{3}{2}\log_2(4+p)=0$

$p=-3,\ q=1$

$p^2+q^2=10$

4-18|64



㉠ $\log_a 4 - 4 = -1$

$\log_a 4 = 3$

$a^3 = 4$

$\boxed{M^3 = 4}$

㉡ $\log_a 2 - 4 = 2$

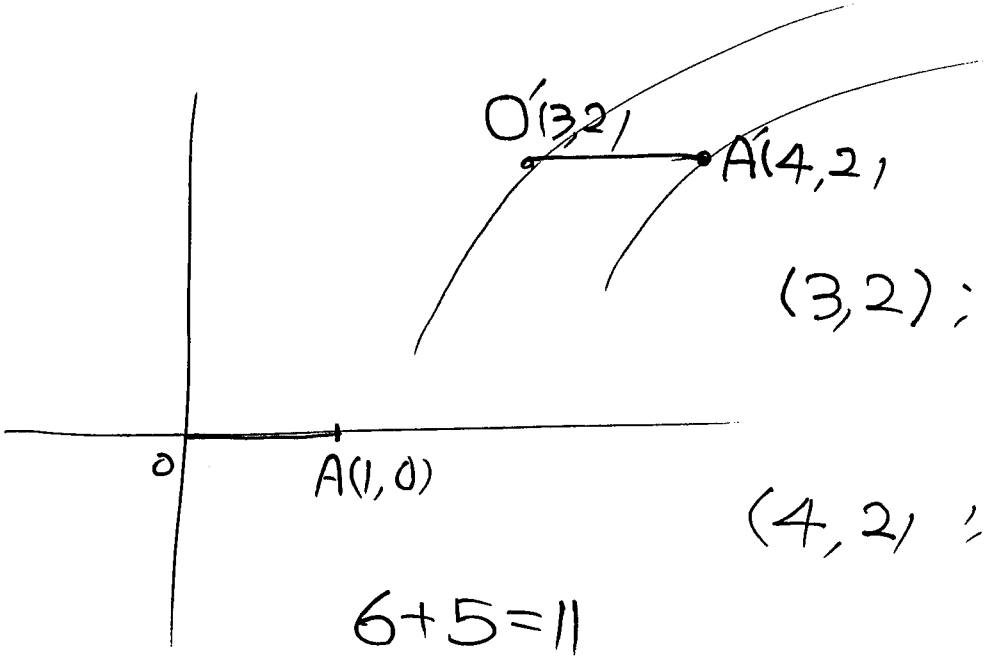
$\log_a 2 = 6$

$a^6 = 2$

$\boxed{N^6 = 2}$

$$\left(\frac{M}{N}\right)^{12} = \frac{M^{12}}{N^{12}} = \frac{4^4}{4} = 64$$

4-19|11



$(3,2)$: $\log_3(3+a) = 2$

$a = 6$

$(4,2)$: $\log_3(4+a) = 2$

$a = 5$

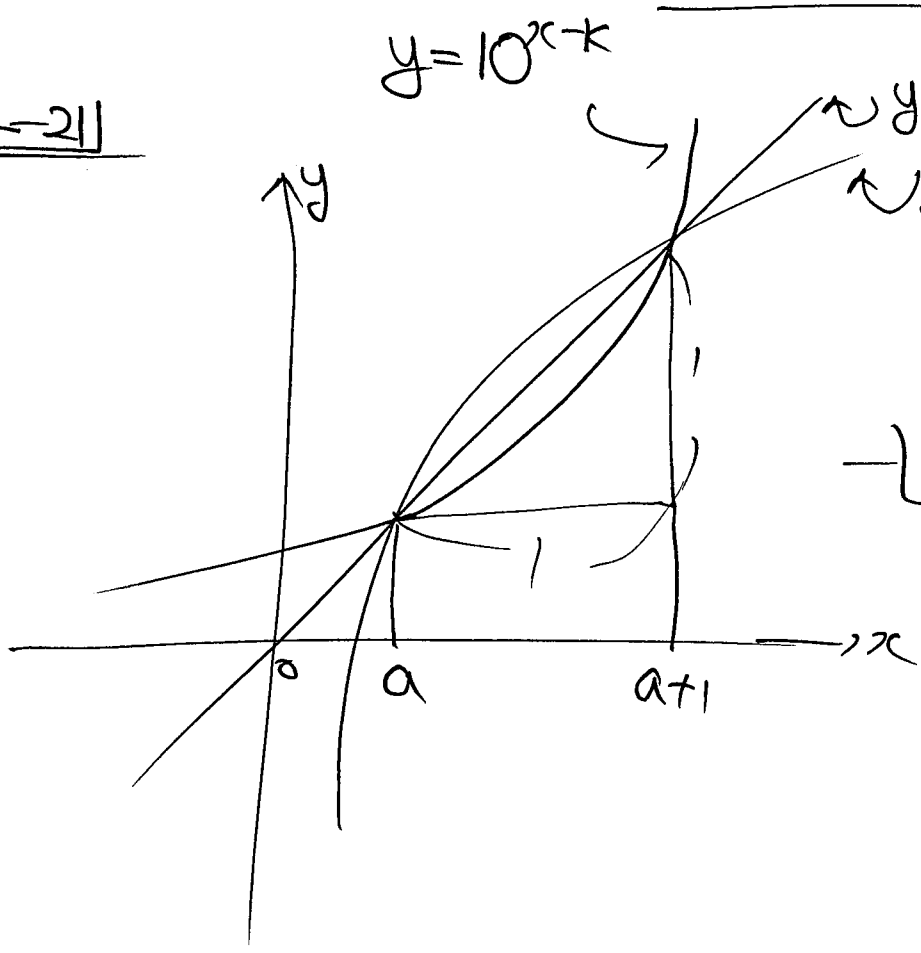
$6 + 5 = 11$

4-20)

$y=\log_2 x$ —(x方向:-2, y方向:1)→ $y-1=\log_2(x+2)$

$$\boxed{y=\log_2(x+2)+1}$$

4-21)



$\log(a+1)+k=a+1$

$\log a+k=a$

$\log\left(\frac{a+1}{a}\right)=1$

$\frac{a+1}{a}=10$

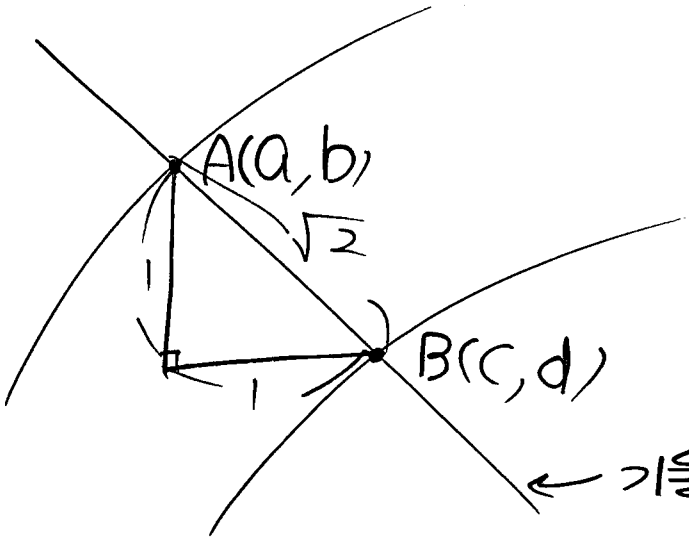
$a=\frac{1}{9}$

$k=a-\log a$

$=\frac{1}{9}-\log\frac{1}{9}$

$=\frac{1}{9}+2\log 3$

4-22|13



$c = a+1$

$A(a, \log_2 a)$, $B(-a+1, \log_4(a+3))$

$\log_4 a^2 - \log_4(a+3) = 1$

$\log_4 a^2 = \log_4(a+3) + \log_4 4$

$= \log_4(4a+12)$

$a^2 = 4a+12$ $\quad a^2 - 4a - 12 = 0$

$(a-6)(a+4) = 0$ $\quad a = 6$

$c = 7$

$\boxed{a+c = 13}$

4-23|18

$$\frac{\square OQPR}{\triangle ABP} = \frac{9}{4} = \frac{ab}{\frac{1}{2}(a-1)\cdot b}$$

$\frac{9}{2}(a-1) = 4a$

$9a - 9 = 8a$ $\quad a = 9,\ b = 2$ $\quad ab = 18$

4-24 | 18

$Q(a, \log_{\frac{1}{2}} a)$ $R(b, \log_2 b)$

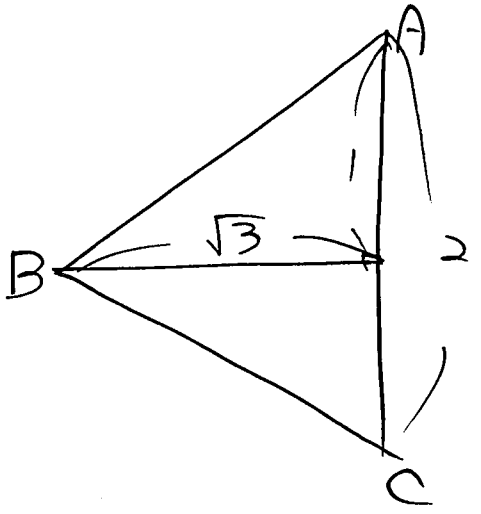
$\log_2 \frac{1}{a} = \log_2 b$ $ab = 1$

$b - a = 4$

$a^2 + b^2 = (b-a)^2 + 2ab = 18$

4-25 | $12\sqrt{3}$

$y = \log_2 4x = 2 + \log_2 x$



$B(p, q)$

$A(p+\sqrt{3}, q+1)$

$q = 2 + \log_2 p$

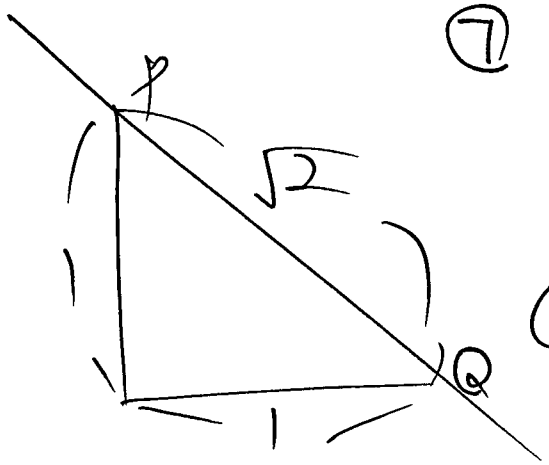
$q+1 = 2 + \log_2 (p+\sqrt{3})$

$\log_2 \left(\frac{p+\sqrt{3}}{p}\right) = 1$ $\frac{p+\sqrt{3}}{p} = 2$ $p = \sqrt{3}$.

$q = 2 + \log_2 \sqrt{3} = \log_2 4\sqrt{3}$ $2^q = 4\sqrt{3}$

$p^2 \times 2^q = 12\sqrt{3}$

4-26 | 7



$\overline{AB}=7$

① $y=\log_2(\log_3 x)$

$A(3,0)$

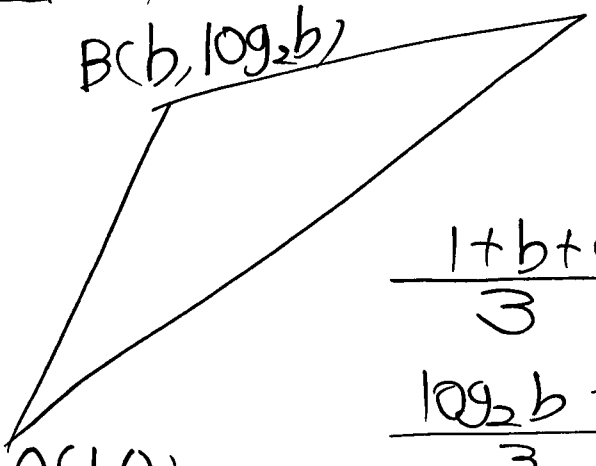
② $y+1=\log_2(\log_3(x-1))$

$\log_3(x-1)=2$

$x=10$

$B(10,0)$

4-27 | $2\sqrt{37}$



$\frac{1+b+c}{3}=7$ $\quad b+c=20$

$\frac{\log_2 b+\log_2 c}{3}=2$

$\log_2 bc=6$ $\quad bc=2^6=64$

$b=4,\ c=16$

$B(4,2),\ C(16,4)$

$\overline{BC}=\sqrt{144+4}=2\sqrt{37}$

4-28 | 9

A(2, 1)

$h(2) = a$

$h^{-1}(a) = 2$

$g(a) = \log_3 f(a) = 2$

$f(a) = 9$

$\log_2 a = 9 \quad a = 2^9$

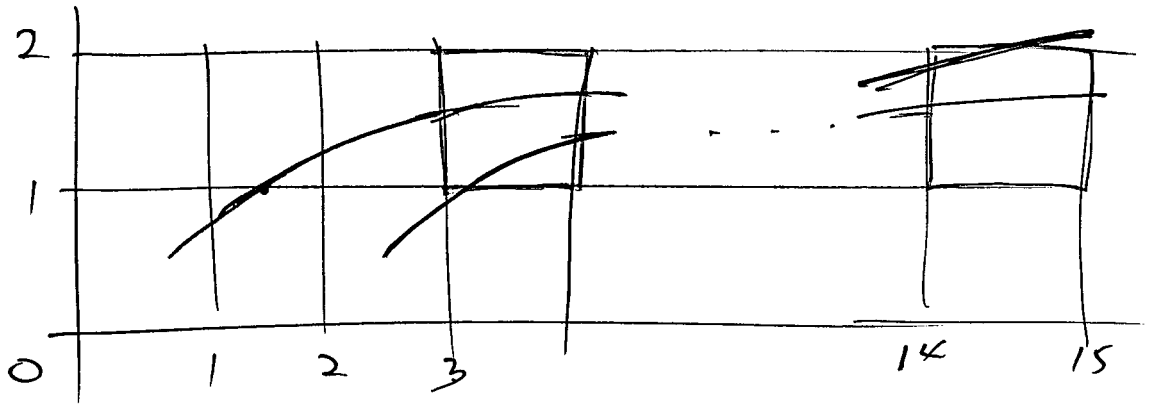
$Q(2^9, 2^9)$

$C(2^9, 9)$

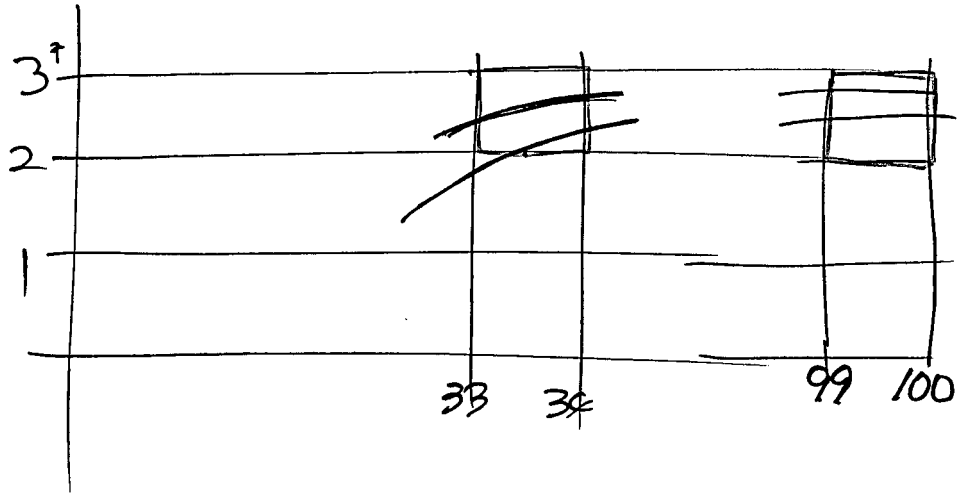
$\therefore \overline{CD} = 9$

$D(2^9, 0)$

4-29 | 79



3~14 12개



33~99 67개

12+67 = 79