# 5. 로그방정식과 로그부등식

P195

EX, $\log_2 x = 3$

㉠ $x > 0$

㉡ $x = 8$

$\therefore x = 8$

P196

EX) (1) $\log_2 x = 4$

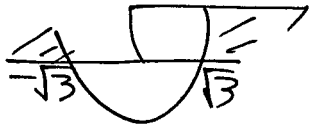
㉠ $x > 0$

㉡ $x = 16$

$\therefore x = 16$

(2) $\log_2 (x^2 - 3) = \log_2 2x$

㉠ $x^2 - 3 > 0$, $2x > 0$




$\therefore x > \sqrt{3}$

㉡ $x^2 - 3 = 2x$

$x^2 - 2x - 3 = 0$

$x = 3$ 또는 $x = -1$

㉠, ㉡에서 $x = 3$

P197

EX) $(\log_3 x)^2 = 3\log_3 x + 4$

㉠ $x > 0$

㉡ $\log_3 x = t$ (t는 실수)

$t^2 = 3t + 4$

$t^2 - 3t - 4 = 0$

$(t-4)(t+1) = 0$

$t = 4$ 또는 $t = -1$

$\log_3 x = 4$ 또는 $\log_3 x = -1$

$x = 81$ 또는 $x = \frac{1}{3}$

㉠, ㉡에서 $\boxed{x = 81 \text{ 또는 } x = \frac{1}{3}}$

P198

EX) $x^{\log x} = 100x$

㉠ $x > 0$

㉡ $\log x^{\log x} = \log 100x$

$(\log x)^2 = 2 + \log x$

$\log x = t$ (t는 실수)

$t^2 = 2 + t$

$t^2 - t - 2 = 0$

$(t-2)(t+1) = 0$

$t = 2$ 또는 $t = -1$

$\log x = 2$ 또는 $\log x = -1$

$x = 10^2$ 또는 $x = 10^{-1}$

㉠, ㉡에서

$\boxed{x = 100 \text{ 또는 } x = \frac{1}{10}}$

EX2) $2^{2x}=5\cdot 5^{-2x}$

$2^{2x}\cdot 5^{2x}=5$

$10^{2x}=5$ $\quad 2x=\log 5$

$x=\frac{1}{2}\log 5$

P199

11

(1) $\log_3 x=\frac{1}{2}$ $\quad x=\sqrt{3}$ $(\because x>0)$

(2) $\log_{\frac{1}{2}}(x-1)=-2$

① $x-1>0$ $\quad x>1$

② $x-1=\left(\frac{1}{2}\right)^{-2}=4$ $\quad x=5$

①, ② $\therefore x=5$

(3) $x>0, x\neq 1$

$x^{-\frac{1}{2}}=4$ $\quad x=4^{-2}=\frac{1}{16}$ $\therefore x=\frac{1}{16}$

(4) $x-2>0, x-2\neq 1$

$\log_{(x-2)} 25=2$ $\quad (x-2)^2=25$

$x-2=5$ $\quad x=7$

(5) $2\log(1+x)=\log(1-2x)$

① $1+x>0$, $1-2x>0$ $\quad -1<x<\frac{1}{2}$

② $\log(1+x)^2=\log(1-2x)$

$x^2+2x+1=1-2x$

$x^2+4x=0$ $\quad x=0, -4$

①, ② $x=0$

(6) $(\log_3 x)^2 - \log_3 x - 6 = 0$

① $x > 0$

② $\log_3 x = t$ ; $t^2 - t - 6 = 0$

(t는실수) $(t-3)(t+2) = 0$

$t = 3$ 또는 $t = -2$

$\log_3 x = 3$ 또는 $\log_3 x = -2$

$x = 27$ 또는 $x = \frac{1}{9}$

①,② $\boxed{x = 27 \text{ 또는 } x = \frac{1}{9}}$

P201

1-11

(1) ㉠ $x+2>0$, $x-4>0$ $\therefore x>4$

㉡ $\log_2(x+2)(x-4)=\log_2 16$

$x^2-2x-8=16$

$x^2-2x-24=0$

$(x-6)(x+4)=0$ $\therefore x=6$ 또는 $x=-4$

㉠, ㉡에서 $\boxed{x=6}$

(2) ㉠ $x>0$, $x\neq 1$, $3x+4>0$ $\therefore 0<x<1$ 또는 $x>1$

㉡ $\log_x(3x+4)=\log_x x^2$

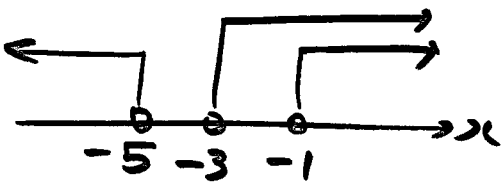
$x^2=3x+4$

$x^2-3x-4=0$

$(x-4)(x+1)=0$ $\therefore x=4$ 또는 $x=-1$

㉠, ㉡에서 $\boxed{x=4}$

(3) ㉠ $x^2+6x+5>0$, $x+3>0$




$(x+1)(x+5)>0$

$\therefore x>-1$

㉡ $\log_3(x^2+6x+5)=\log_3 3+\log_3(x+3)$

$\log_3(x^2+6x+5)=\log_3(3x+9)$

$x^2+6x+5=3x+9$

$x^2+3x-4=0$

$(x-1)(x+4)=0$ $\quad x=1$ 또는 $x=-4$

㉠, ㉡에서 $\boxed{x=1}$

(4) ㉠ $5x+5>0$, $2x-1>0$

$x>-1$, $x>\frac{1}{2}$ $\quad \therefore x>\frac{1}{2}$

㉡ $\frac{1}{2}\log(5x+5)+\frac{1}{2}\log(2x-1)=\log 10$

$\log(5x+5)(2x-1)=\log 100$

$10x^2+5x-5=100$

$2x^2+x-21=0$

$(x-3)(2x+7)=0$ $\quad \therefore x=3$ 또는 $x=-\frac{7}{2}$

㉠, ㉡에서 $\boxed{x=3}$

1-2)

(1) ㉠ $x+3>0$ $\quad \therefore x>-3$

㉡ $\log_4(x+3)^2=\log_4(x+3)+\log_4 4$

$\log_4(x+3)^2=\log_4 4(x+3)$

$x^2+6x+9=4x+12$

$x^2+2x-3=0$

$(x+3)(x-1)=0$ $\quad x=-3$ 또는 $x=1$

㉠, ㉡에서 $\boxed{x=1}$

(2) ㉠ $x+3>0$, $x+7>0$ $\therefore x>-3$

㉡ $\log_9(x+3)^2 = \log_9 9 + \log_9(x+7)$

$= \log_9(9x+63)$

$x^2+6x+9 = 9x+63$ $\therefore x=9$ 또는 $x=-6$

$x^2-3x-54=0$

$(x-9)(x+6)=0$

㉠, ㉡에서 $\boxed{x=9}$

1-3 ②

㉠ ① $x-3>0$, $2y+5>0$ $\therefore x>3$, $y>-\frac{5}{2}$

② $\log_4(x-3)^2 = \log_4(2y+5)$

$\therefore (x-3)^2 = 2y+5$

㉡ $y=x+12$

㉡, ㉡ 에서 $x^2-6x+9 = 2x+29$

$x^2-8x-20=0$

$(x-10)(x+2)=0$

$x=10$ ($\because x>3$)

$y=22$

$\alpha=10$, $\beta=22$ $\alpha+\beta=32$

2-11

(1) ㉠ $x>0$, $x\neq 1$

㉡ $\log_x 25 = 2\log_x 5$

$\log_5 x = t$ ($t$는 실수)

$t - \frac{2}{t} = 1$

$t^2 - t - 2 = (t-2)(t+1) = 0$

$t=2$ 또는 $t=-1$

$\log_5 x = 2$ 또는 $\log_5 x = -1$

$x = 5^2$ 또는 $x = 5^{-1}$

㉠, ㉡에서 $x=25$ 또는 $x=\frac{1}{5}$

(2) ㉠ $x>0$, $x\neq 1$

㉡ $\log_2 x = t$ ($t$는 실수)

$4t + \frac{1}{t} - 5 = 0$

$4t^2 - 5t + 1 = (t-1)(4t-1) = 0$

$t=1$ 또는 $t=\frac{1}{4}$

$\log_2 x = 1$ 또는 $\log_2 x = \frac{1}{4}$

$x=2$ 또는 $x=2^{\frac{1}{4}}$

㉠, ㉡에서 $x=2$ 또는 $x=2^{\frac{1}{4}}$

(3) ㉠ $x>0$

㉡ $\log_2 x=t$ ($t$는 실수)

$(t-6)^2+2t-11=0$

$t^2-10t+25=0$

$(t-5)^2=0$

$t=1$

$\log_2 x=5$

$x=2^5=32$

㉠, ㉡에서 $\boxed{x=32}$

(4) ㉠ $x>0$

㉡ $\log_2 x=t$ ($t$는 실수)

$t^3+4t=4t^2+t$

$t^3-4t^2+3t=t(t-1)(t-3)=0$

$t=0$ 또는 $t=1$ 또는 $t=3$

$x=1$ 또는 $x=2$ 또는 $x=8$

㉠, ㉡에서 $\boxed{x=1 \text{ 또는 } x=2 \text{ 또는 } x=8}$

## 2-2)

(1) ㉠ $x>0$

㉡ $5^{\log x}=t$ $(t>0)$

$t^2-6t+5=0$

$(t-1)(t-5)=0$

$t=1$ 또는 $t=5$

$5^{\log x}=5^0$ 또는 $5^1$

$\log x=0$ 또는 $1$

$x=1$ 또는 $x=10$

㉠, ㉡에서 $x=1$ 또는 $x=10$

(2) ㉠ $x>0$

㉡ $2^{\log x}=t$ $(t>0)$

$t^2-5t+4=(t-1)(t-4)=0$

$t=1$ 또는 $t=4$

$2^{\log x}=2^0$ 또는 $2^{\log x}=2^2$

$\log x=0$ 또는 $\log x=2$

$x=1$ 또는 $x=100$

㉠, ㉡에서 $x=1$ 또는 $x=100$

2-3 164

$x>0$

$\log_2 x=t$ (t는실수)

$t\times(4-t)=\frac{m}{16}$

$f(t)=4t-t^2=-(t-2)^2+4$

$\frac{m}{16}\leq 4$

$m\leq 64$

3-1

(1) ㉠ $x>0$

㉡ $\log x^{\log x}=\log 10000x^3$ $\log x=t$ (t는실수)

$\log x\cdot\log x=4+3\log x$

$t^2-3t-4=(t-4)(t+1)=0$

$t=4$ 또는 $t=-1$

$\log x=4$ 또는 $\log x=-1$

$x=10^4$ 또는 $x=10^{-1}$

㉠, ㉡에서 $x=10000$ 또는 $x=\frac{1}{10}$

(2) ㉠ $x>0$

㉡ $\log_3 x^{\log_3 x}=\log_3 \frac{x^3}{9}$ | $\log_3 x=t$ (t는 실수)

$\log_3 x\cdot\log_3 x=3\log_3 x-2$

$t^2-3t+2=(t-1)(t-2)=0$

$\log_3 x=1$ 또는 $\log_3 x=2$

$x=3$ 또는 $x=9$

㉠, ㉡에서 $x=3$ 또는 $x=9$

3-21

(1) ㉠ $x>0$

㉡ $2^{\log_2 x}=3^{\log 3x}$

$\log 2^{\log_2 x}=\log 3^{\log 3x}$

$(\log x+\log 2)\cdot\log 2=(\log x+\log 3)\cdot\log 3$

$(\log 2+\log 3)\cdot\log x=(\log 3+\log 2)(\log 3-\log 2)$

$\log x=-\log 6=\log\frac{1}{6}$ $\therefore x=\frac{1}{6}$ (㉠㉡)

(2) $\left(\frac{2}{x}\right)^{\log 2} = \left(\frac{3}{x}\right)^{\log 3}$

$\log\left(\frac{2}{x}\right)^{\log 2} = \log\left(\frac{3}{x}\right)^{\log 3}$

$\log 2 \times (\log 2 - \log x) = \log 3 \times (\log 3 - \log x)$

$(\log 3 - \log 2) \times \log x = (\log 3 - \log 2) \times (\log 3 + \log 2)$

$\log x = \log 3 + \log 2 = \log 6$

$\therefore x = 6$

3-31(1)

$\log 2^{x-1} = \log 5^{x+1}$

$(x-1)\log 2 = (x+1)\log 5.$

$x(\log 2 - \log 5) = \log 5 + \log 2 = \log 10 = 1$

$\lambda = \frac{1}{\log \frac{2}{5}}$ $\alpha = \frac{1}{\log \frac{2}{5}}$

$\alpha^{-1} = \log \frac{2}{5}$

$10^{\alpha^{-1}} = \frac{2}{5}$

## 4-11

$\log_3 x = t$ ($t$는 실수)

$t^2 - 3t - 1 = 0$ 두근 $\log_3\alpha$, $\log_3\beta$

$\log_3\alpha + \log_3\beta = 3$ ···㉠

$\log_3\alpha \cdot \log_3\beta = -1$ ··㉡

(1) ㉠에서 $\log_3\alpha\beta = 3$ $\alpha\beta = 27$

(2) $\log_\alpha\beta + \log_\beta\alpha = \frac{\log_3\beta}{\log_3\alpha} + \frac{\log_3\alpha}{\log_3\beta}$

$= \frac{(\log_3\beta)^2 + (\log_3\alpha)^2}{\log_3\alpha \cdot \log_3\beta} = \frac{9-(-2)}{-1} = -11$

## 4-21

(1) $\log x = t$ ($t$는 실수, $x > 0$)

$(t + \log 2) \times (t + \log 3) = 1$

$t^2 + (\log 3 + \log 2)\cdot t + (\log 2)(\log 3) - 1 = 0$

두근 $\log\alpha$, $\log\beta$

$\log\alpha + \log\beta = -\log 6$ ···㉠

$\log\alpha \cdot \log\beta = (\log 2)(\log 3) - 1$ ····㉡

㉠에서 $\log\alpha\beta = \log\frac{1}{6}$

$\therefore \alpha\beta = \frac{1}{6}$

(2) $\log_2 x = t$ ($t$는 실수, $x>0$)

$(2+t)\times t + (\log_2 3)t - 6 = 0$

$t^2 + (2+\log_2 3)t - 6 = 0$ 두근 $\log_2\alpha, \log_2\beta$

$\log_2\alpha + \log_2\beta = -(2+\log_2 3)$ ... ㉠

$\log_2\alpha \cdot \log_2\beta = -6$ .. ㉡

㉠ $\log_2 \alpha\beta = -\log_2 12 = \log_2 \frac{1}{12}$ $\therefore \alpha\beta = \frac{1}{12}$

4-31 ③

㉠ $2^{2x} - a\cdot 2^x + 8 = 0$ 두근 $\alpha, \beta$

$2^x = t$ ($t>0$)

$t^2 - at + 8 = 0$ 두근 $2^\alpha, 2^\beta$

$2^\alpha + 2^\beta = a$, $2^\alpha \cdot 2^\beta = 8$ $\therefore \alpha+\beta = 3$

$2^{\alpha+\beta} = 2^3$

㉡ $(\log_2 x)^2 - \log_2 x + b = 0$ 두근 $\alpha, \beta$

$\log_2 x = t$ ($t$는 실수)

$t^2 - t + b = 0$ 두근 $\log_2\alpha, \log_2\beta$

$\log_2\alpha + \log_2\beta = 1$, $\log_2\alpha \cdot \log_2\beta = b$

$\log_2 \alpha\beta = 1$ $\alpha=1, \beta=2$; $a=6$ $\alpha+b=6$

$\alpha\beta = 2$ $b=0$

P208

EX) $\log_2 x \geq \log_2(3x-6)$

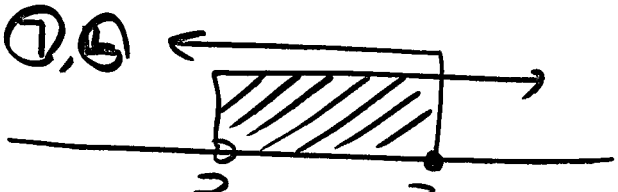
㉠ $x>0,\ 3x-6>0$ $\quad\therefore x>2$

㉡ $x \geq 3x-6$

$-2x \geq -6$

$x \leq 3$

㉠,㉡




$2<x \leq 3$

P209

EX1) (1) $\log_2 x \geq 4$

㉠ $x>0$

㉡ $\log_2 x \geq \log_2 16$ $\quad x \geq 16$

㉠,㉡ $\therefore x \geq 16$

(2) $\log_{\frac{1}{5}} x \geq 2$

㉠ $x>0$

㉡ $\log_{\frac{1}{5}} x \geq \log_{\frac{1}{5}}\left(\frac{1}{5}\right)^2$ $\quad\therefore x \leq \frac{1}{25}$

㉠,㉡에서 $\quad 0<x \leq \frac{1}{25}$

EX2)

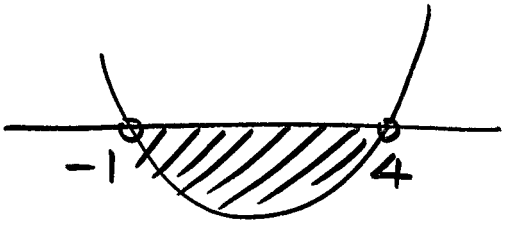
㉠ $x>0$

㉡ $\log_3 x = t$ ($t$는 실수)

$t^2 < 3t+4$

$t^2-3t-4<0$

$(t+1)(t-4)<0$




$-1<t<4$

$\log_3 3^{-1} < \log_3 x < \log_3 3^4$

$\frac{1}{3} < x < 81$

㉠, ㉡에서 $\boxed{\frac{1}{3} < x < 81}$

P.10

EX1)

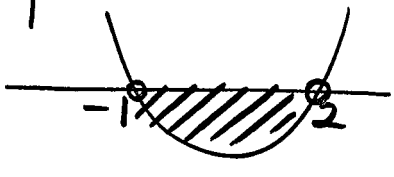
㉠ $x>0$

㉡ $\log x^{\log x} < \log 100x$

$(\log x)\cdot(\log x) < 2 + \log x$

$\log x = t$ ($t$는 실수)

$t^2 - t - 2 < 0$




$(t+1)(t-2)<0$

$-1<t<2$

$\log\frac{1}{10} < \log x < \log 100$ $\therefore \frac{1}{10} < x < 100$

㉠, ㉡에서 $\boxed{\frac{1}{10} < x < 100}$

(EX2) $2^{2x} > 5^{1-2x}$

$\log 2^{2x} > \log 5^{1-2x}$

$2x\log 2 > (1-2x)\log 5$

$2(\log 2+\log 5)\cdot x > \log 5$

$\boxed{x > \frac{1}{2}\log 5}$

P211

II

(1) $\log_3 x > \frac{1}{2}$

㉠ $x>0$

㉡ $\log_3 x > \log_3 \sqrt{3}$ $\therefore x > \sqrt{3}$

㉠, ㉡에서 $x>\sqrt{3}$

(2) $\log_{\frac{1}{2}}(x-1) < -2$

㉠ $x-1>0$ $\therefore x>1$

㉡ $\log_{\frac{1}{2}}(x-1) < \log_{\frac{1}{2}} 4$

$x-1>4$ $\therefore x>5$

㉠, ㉡에서 $\boxed{x>\sqrt{5}}$

(3) ㉠ $2x+1>0,\ 3x-2>0$ $\therefore x>\frac{2}{3}$

㉡ $2x+1 \geq 3x-2$

$x \leq 3$

㉠,㉡ $\boxed{\frac{2}{3}<x \leq 3}$

(4) ㉠ $x>0$

㉡ $\log x=t$ (t는 실수)

$t^2-3t<0$

$t(t-3)<0$


$0<t<3$

$\log 1<\log x<\log 1000$

$1<x<1000$

㉠,㉡ $\therefore \boxed{1<x<1000}$

5-11

(1) ㉠ $2x-1>0 \quad \therefore x>\frac{1}{2}$

㉡ $\log_{\frac{1}{2}}(2x-1) > \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-2}$

$2x-1<4$

$x<\frac{5}{2}$

㉠, ㉡에서 $\frac{1}{2}<x<\frac{5}{2}$

(2) ㉠ $x>0$ , $\log_2 x>0$

$\log_2 x>\log_2 1$

$\therefore x>1$

$\therefore x>1$

㉡ $\log_2 1 \le \log_2(\log_3 x) < \log_2 2$

$1 \le \log_3 x < 2$

$\log_3 3 \le \log_3 x < \log_3 9$

$3 \le x < 9$

㉠, ㉡에서 $3 \le x < 9$

(3) ㉠ $x-4>0$, $x-1>0$

$x>4$, $x>1$

$\therefore x>4$

㉡ $\log_5 10 < \log_5(x-1) + \log_5(x-4)$

$\log_5 10 < \log_5(x-1)(x-4)$

$x^2-5x+4>10$

$x^2-5x-6=(x-6)(x+1)>0$


$x<-1$ 또는 $x>6$

㉠, ㉡에서 $x>6$

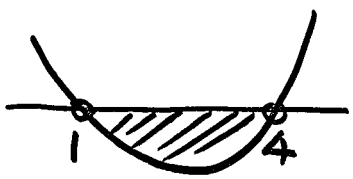
(4)

㉠ $x-3>0,\ 5-x>0$ $\therefore\ 3<x<5$

㉡ $\log_{\frac{1}{2}}(x-3)^2>\log_{\frac{1}{2}}(5-x)$

$(x-3)^2<5-x$

$x^2-5x+4=(x-1)(x-4)<0$




$\therefore\ 1<x<4$

㉠, ㉡에서 $3<x<4$

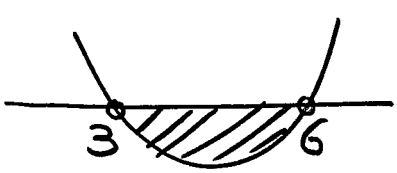
5-2|

(1) ㉠ $x-4>0,\ x-2>0$ $\therefore\ x>4$

㉡ $\log_4(x-4)^2<\log_4(x-2)$

$x^2-8x+16<x-2$

$x^2-9x+18<0$



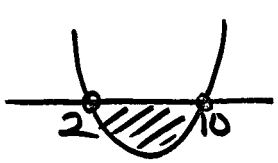

$(x-3)(x-6)<0$

$\therefore\ 3<x<6$

㉠, ㉡에서 $4<x<6$

(2) ㉠ $x-5>0,\ 2x+5>0$ $\therefore\ x>5$

㉡ $\log_{\frac{1}{9}}(x-5)^2>\log_{\frac{1}{9}}(2x+5)$




$x^2-10x+25<2x+5$ $\therefore\ 2<x<10$

$x^2-12x+20<0$

㉠, ㉡에서 $5<x<10$

$(x-2)(x-10)<0$

## 5-31

(1) ① $\log_3|x-3|<4$

㉠ $|x-3|>0$ $\therefore x\neq 3$

㉡ $\log_3|x-3|<\log_3 3^4$

$|x-3|<81$

$-81<x-3<81$

$-78<x<84$

㉠, ㉡에서

$-78<x<3$ 또는 $3<x<84$

② $\log_2 x+\log_2(x-2)\geq 3$

㉠ $x>0$, $x-2>0$ $\therefore x>2$

㉡ $\log_2(x^2-2x)\geq\log_2 8$

$x^2-2x\geq 8$

$x^2-2x-8\geq 0$

$(x-4)(x+2)\geq 0$


$x\leq -2$ 또는 $x\geq 4$

㉠, ㉡에서 $x\geq 4$

①, ②에서 $4\leq x<84$

(2) ① $2^{x+3}>4$

$2^{x+3}>2^2$

$x+3>2$ $\therefore x>-1$

$x>-1$

② $2\log(x+3)<\log(5x+15)$

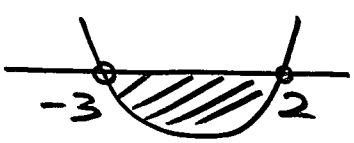
㉠ $x+3>0$, $5x+15>0$ $\therefore x>-3$

㉡ $\log(x+3)^2<\log(5x+15)$

$x^2+6x+9<5x+15$

$x^2+x-6<0$ ㉠, ㉡에서

$(x+3)(x-2)<0$ $-3<x<2$




$-3<x<2$

①, ② 에서 $\therefore -1<x<2$

6-11

(1) ① $x>0$

② $\log_2 x=t$ (t는실수)

$t^2-5t+6<0$

$(t-2)(t-3)<0$


$2<t<3$ ㉠, ㉡ $4<x<8$

$\log_2 2^2<\log_2 x<\log_2 2^3$

$\therefore 4<x<8$

(2) ㉠ $x>0$

㉡ $\log x=t$ ($t$는 실수)

$t^2<3t$

$t^2-3t<0$

$t(t-3)<0$ 


$0<t<3$

$\log 1<\log x<\log 10^3$

$\therefore 1<x<1000$

㉠, ㉡에서

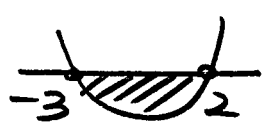
$1<x<1000$

(3)

㉠ $\frac{x}{3}>0$, $9x>0$ $\therefore x>0$

㉡ $\log_3 x=t$ ($t$는 실수)

$(t-1)(t+2)\le 4$

$t^2+t-6=(t+3)(t-2)\le 0$ 


$-3\le t\le 2$

$\log_3 3^{-3}\le \log_3 x\le \log_3 3^2$

$\frac{1}{27}\le x\le 9$

㉠, ㉡에서

$\therefore \frac{1}{27}\le x\le 9$

(4)

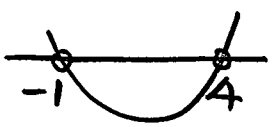
㉠ $x>0$

㉡ $\log_2 x=t$ ($t$는 실수) $\quad \log_{\frac{1}{2}} x=-\log_2 x=-t$

$t(3-t)>-4$

$t^2-3t-4<0$

$(t-4)(t+1)<0$




$-1<t<4$

$\log_2 2^{-1}<\log_2 x<\log_2 2^4$

$\therefore \frac{1}{2}<x<16$

㉠, ㉡에서

$\frac{1}{2}<x<16$

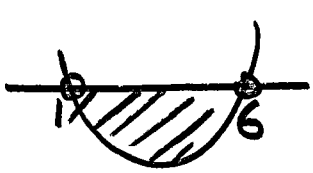
6-21

(1) ㉠ $x>0, x\neq 1$

㉡ $\log_2 x=t$ ($t$는 실수) $\quad \log_x 4=2\log_x 2=\frac{2}{t}$

$t+\frac{6}{t}-7<0$

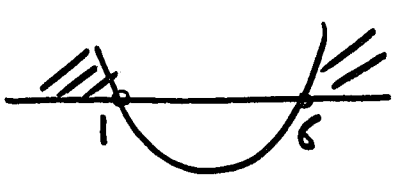
① $\begin{cases} t^2-7t+6<0 \\ t>0 \end{cases}$




$1<t<6$

$\therefore 1<t<6$

② $t<0$ : $t^2-7t+6>0$




$t<1$ 또는 $t>6$

$\therefore t<0$

①,② $t<0$ 또는 $1<t<6$

$\log_2 x<\log_2 1$ 또는 $\log_2 2<\log_2 x<\log_2 2^6$

$x<1$ 또는 $2<x<64$

①,ㄴ에서 $0<x<1$ 또는 $2<x<64$

(2) ㉠ $x>0$, $x\neq 1$

㉡ $\log_x 2=t$ ($t$는실수)

$\frac{3}{t}+t>4$

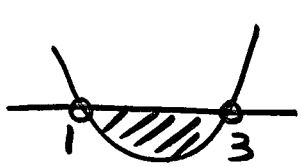
① $t>0$; $t^2-4t+3>0$ 


$(t-1)(t-3)>0$ $t<1$ 또는 $t>3$

$\therefore 0<t<1$ 또는 $t>3$

$\log 1<\log x<\log 10$ 또는 $\log x>\log 1000$

$1<x<10$ 또는 $x>1000$

② $t<0$; $t^2-4t+3<0$ 


$(t-1)(t-3)<0$ $1<t<3$

(X)

㉠,㉡ $1<x<10$ 또는 $x>1000$

6-3)

① $2^{2x} - 2^{x+1} - 8 < 0$ | $2^x = t \quad (t>0)$

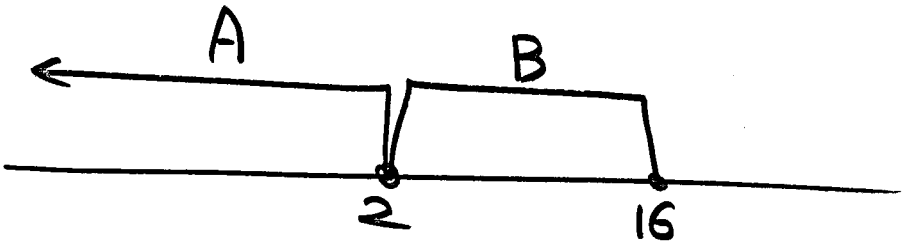
$t^2 - 2t - 8 < 0$

$(t+2)(t-4) < 0$

$t - 4 < 0$

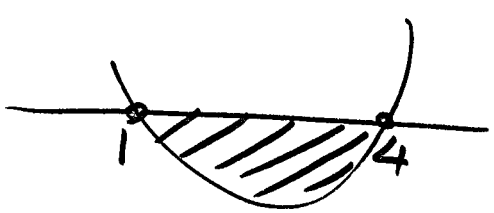
$t < 4$

$2^x < 2^2 \qquad \therefore x < 2$




② $2 \le x \le 16$

$1 \le \log_2 x \le 4$




$(\log_2 x - 1)(\log_2 x - 4) \le 0$

$a = 5, \; b = 4$

$a^2 + b^2 = 25 + 16 = 41$

7-1

(1) ㉠ $x>0$

㉡ $\log_2 x^{\log_2 x} < \log_2 4x$

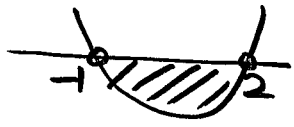
$\log_2 x = t$ (t는실수)

$\log_2 x \cdot \log_2 x < 2 + \log_2 x$

$t^2 < 2 + t$

$t^2 - t - 2 < 0$

$(t+1)(t-2) < 0$




$-1 < t < 2$

$-1 < t < 2$

$\log_2 2^{-1} < \log_2 x < \log_2 2^2$ $\therefore \frac{1}{2} < x < 4$

㉠,㉡에서 $\therefore \frac{1}{2} < x < 4$

(2) ㉠ $x>0$

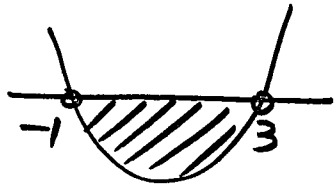
㉡ $\log_3 x^{\log_3 x} < \log_3 27x^2$

$\log_3 x = t$

$\log_3 x \cdot \log_3 x < 3 + 2\log_3 x$

$t^2 - 2t - 3 < 0$

$(t+1)(t-3) < 0$




$-1 < t < 3$

$\log_3 3^{-1} < \log_3 x < \log_3 3^3$

$\therefore \frac{1}{3} < x < 27$

㉠,㉡에서 $\frac{1}{3} < x < 27$

7-21

(1) $2^{2x} \geq \frac{10^{2x}}{10}$ $\Rightarrow$ $\log_5 5^{2x} \leq \log_5 10$

$\left(\frac{10}{2}\right)^{2x} \leq 10$ $\quad 2x \leq \log_5 10$

$5^{2x} \leq 10$ $\quad x \leq \frac{1}{2}\log_5 10$

(2) $2^x < \frac{3}{3^x}$ $\quad x < \frac{\log 3}{\log 6}$

$6^x < 3$ $\quad \boxed{x < \log_6 3}$

$\log 6^x < \log 3$

$x \log 6 < \log 3$

7-31

(1) $\log_3 x^{\log_3 x} \geq \log_3 \frac{x^4}{a}$

$\log_3 x \cdot \log_3 x \geq 4\log_3 x - \log_3 a$

$t^2 - 4t + \log_3 a \geq 0$

$t^2 - 4t + \log_3 a = 0$

$D/4 = 4 - \log_3 a \leq 0$

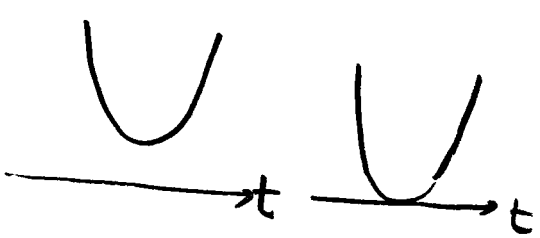
$\log_3 a \geq 4$

$\log_3 a \geq \log_3 81$

$a \geq 81$

$\log_3 x = t$

($t$는실수)




a의최솟값 : 81

(2) $\log_4 ax^{\log_4 x} \geq \log_4 x^4$

$\log_4 x = t$ (t는실수)

$\log_4 a + \log_4 x \cdot \log_4 x \geq 4\log_4 x$

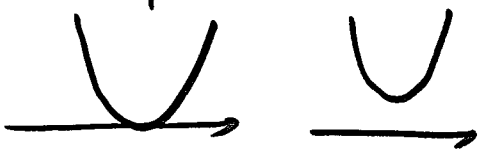
$t^2 - 4t + \log_4 a \geq 0$

$t^2 - 4t + \log_4 a = 0$

$D/4 = 4 - \log_4 a \leq 0$

$\log_4 a \geq \log_4 4^4$

$a \geq 256$



a의 최솟값 : 256

8-1 ②

$-7 = 10\log\frac{B}{A}$

$\log\frac{B}{A} = -\frac{7}{10}$

$\frac{B}{A} = 10^{-\frac{7}{10}} = \frac{1}{10^{\frac{7}{10}}} = \frac{10^{\frac{3}{10}}}{10^1} = \frac{1}{5}$

8-2

$L = 20\log mf - 48$ ... ①

$L + a = 20\log(5mf) - 48$ ... ②

② - ① $a = 20\log 5$

$= 14$

$\log 5 = \log\frac{10}{2}$

$= \log 10 - \log 2$

$= \frac{7}{10}$

8-3180

$$K=\frac{2.3Q}{2\pi LH}\times\log\frac{L}{r} \quad \cdots ①$$

$$\frac{K}{2}=\frac{2.3\times 2Q}{2\pi LH}\times\log\frac{L}{4r} \quad \cdots ②$$

$$①\div②;\ 2=\frac{1}{2}\times\frac{\log\frac{L}{r}}{\log\frac{L}{4r}}$$

$$4\left(\log\frac{L}{4r}\right)=\log\frac{L}{r}$$

$$4\left(\log\frac{L}{r}-\frac{6}{10}\right)=\log\frac{L}{r}$$

$$4\log\frac{L}{r}-\frac{24}{10}=\log\frac{L}{r}$$

$$3\log\frac{L}{r}=\frac{24}{10}$$

$$\log\frac{L}{r}=\frac{8}{10} \Rightarrow \frac{L}{r}=10^{\frac{8}{10}}$$

$$n=\frac{8}{10} \quad \therefore 100n=80$$

$$\log\frac{L}{4r}$$
$$=\log\frac{L}{r}-\log 4$$
$$=\log\frac{L}{r}-\frac{6}{10}$$

9-11 7년

$$A \longrightarrow A\cdot\frac{9}{10} \longrightarrow A\cdot\left(\frac{9}{10}\right)^2$$

$$\longrightarrow \boxed{A\left(\frac{9}{10}\right)^n \le A\cdot\frac{1}{2}}$$

$$\left(\frac{10}{9}\right)^n \ge 2$$

$$\log\left(\frac{10}{9}\right)^n \ge \log 2$$

$$n\log\frac{10}{9} \ge \log 2$$

$$n \geq \frac{301}{46} = 6.\times\times$$

$$\boxed{n = 7}$$

$$\log\frac{10}{9} = \log 10 - \log 9$$
$$= 1 - 2\log 3$$
$$= 1 - 0.954$$
$$= 0.046$$
$$\log 2 = 0.301$$

9-2 | 15

$$A\left(1 - \frac{10}{100}\right)^n \leq A \cdot \frac{1}{10}$$

$$\left(\frac{9}{10}\right)^n \leq \frac{1}{10}$$

$$\left(\frac{10}{9}\right)^n \geq 10$$

$$n \log\frac{10}{9} \geq \log 10$$

$$n \geq \frac{1}{0.04} = 25$$

$$\therefore 25 \times \frac{6}{10} = 25 \times \frac{3}{5}$$

$$= 15 \text{ (m)}$$

$$\log\frac{10}{9}$$
$$= \log 10 - \log 9$$
$$= 1 - 2\log 3 = 1 - 0.96$$
$$= 0.04$$

9-3 ③

$$\frac{A\left(1+\frac{r}{100}\right)-A}{A}\times 100 = 50$$

$$\frac{r}{100} = \frac{50}{100} \qquad r=50$$

$$A\left(1+\frac{50}{100}\right)^n > A\cdot 10$$

$$\left(\frac{3}{2}\right)^n > 10$$

$$\log\left(\frac{3}{2}\right)^n > \log 10$$

$$n\log\frac{3}{2} > 1$$

$$n > \frac{1000}{176} = 5.\times\times$$

2001년 → (6년후) → 2007년

$$\log\frac{3}{2} = \log 3 - \log 2$$
$$= 0.477 - 0.301$$
$$= 0.176$$

10-11

올해연봉 : $A$ $\xrightarrow{n\text{년후}}$ 연봉 : $A\left(1+\frac{10}{100}\right)^n$

물가지수 : 1 물가지수 : $(1.05)^n$

실질연봉 : $\frac{A}{1}=A$ 실질연봉 : $\frac{A(1.1)^n}{(1.05)^n}$

$$\frac{A(1.1)^n}{(1.05)^n} \geq 2A$$

$$\frac{(1.1)^n}{(1.05)^n} \geq 2$$

$$\log\left(\frac{1.1}{1.05}\right)^n \geq \log 2$$

$$n\log\frac{1.1}{1.05} \geq \log 2$$

$$n\times(0.0202) \geq 0.3010$$

$$n \geq \frac{3010}{202} = 14.\times\times$$

15년

$$\log\frac{1.1}{1.05}$$

$$= \log 1.1 - \log 1.05$$

$$= 0.0414 - 0.0212$$

$$= 0.0202$$

$$202\overline{)3010}\quad 14$$

$$202$$

$$990$$

10-21 ④

$$\frac{50\cdot(1.04)^n}{1000\cdot(1.003)^n} \geq \frac{20}{100}$$

$$\frac{(1.04)^n}{(1.003)^n} \geq 4$$

$$\log\left(\frac{1.04}{1.003}\right)^n \geq \log 4$$

$$n(\log 1.04 - \log 1.003) \geq 2\log 2$$

$$n \times 0.0157 \geq 0.6020$$

$$n \geq \frac{6020}{157} = 38.xx$$

$$\begin{array}{r} 38.xx \\ 157\overline{)6020} \\ \underline{471} \\ 1310 \end{array}$$

10-31 ②

| | A | B |
|---|---|---|
| 현재 | 24 | 16 |
| 3n개월후 | $24(0.9)^n$ | $16(0.95)^n$ |

$$24(0.9)^n - 16(0.95)^n \leq 16(0.95)^n \times \frac{2}{10}$$

$$24(0.9)^n \leq 16(0.95)^n \times \frac{12}{10}$$

$$\left(\frac{0.95}{0.9}\right)^n \geq \frac{2}{24} \times \frac{10}{16 \times 12}$$

$$\left(\frac{0.95}{0.9}\right)^n \geq \frac{5}{4}$$

$\log\left(\frac{0.95}{0.9}\right)^n \geq \log\frac{5}{4}$

$n(\log 0.95 - \log 0.9) \geq \log\frac{5}{4}$

$n(0.02) \geq 0.1$

$n \geq 5$

**15개월후**

$\log 0.9 = \log\frac{9}{10}$

$= 2\log 3 - 1$

$= -0.04$

$\log\frac{5}{4} = \log\frac{10}{8}$

$= 1 - 3\log 2$

$= 0.1$

## 5-11

(1) $2^x = t \quad (t>0)$

$t(t+5) = 24$

$t^2 + 5t - 24 = 0$

$(t-3)(t+8) = 0$

$t>0$ 이므로

$t = 2^x = 3$

$x = \log_2 3$

(2) $3^x + 3^{-x} = 2\sqrt{3^x \cdot 3^{-x}} = 2$

$3^x + 3^{-x} = t \quad (t \geq 2)$

$9^x + 2 + 9^{-x} = t^2$

$2(t^2-2) - t - 6 = 0$

$2t^2 - t - 10 = 0$

$(t+2)(2t-5) = 0$

$t \geq 2$ 이므로

$t = 3^x + 3^{-x} = \frac{5}{2}$

$3^x = p \quad (p>0)$

$p + \frac{1}{p} = \frac{5}{2}$

$2p^2 - 5p + 2 = 0$

$(2p-1)(p-2) = 0$

$p = \frac{1}{2}$ 또는 $p = 2$

$3^x = \frac{1}{2}$ 또는 $3^x = 2$

$x = \log_3\frac{1}{2}$ 또는 $x = \log_3 2$

(3) $2^{2x-1} = 3^{x+2}$

$\log 2^{2x-1} = \log 3^{x+2}$

$(2x-1)\log 2 = (x+2)\log 3$

$(2\log 2 - \log 3)\cdot x = 2\log 3 + \log 2$

$$x = \frac{2\log 3 + \log 2}{2\log 2 - \log 3}$$

(4) ㉠ $x>0$, $\log_3 x > 0$

$\log_3 x > \log_3 1$ $\therefore x>1$

$x>1$

㉡ $\log_2(\log_3 x) + \log_2(\log_7 9) = \log_2 4$

$\log_2\{(\log_3 x)\cdot(\log_7 9)\} = \log_2 4$

$(\log_3 x)\cdot(2\log_7 3) = 4$ ㉠,㉡

$\log_3 x = \frac{2}{\log_7 3} = 2\log_3 7 = \log_3 49$ $\therefore x=49$

5-2|16

㉠ $x>0$

㉡ $\log_2 x = t$ ($t$는 실수)

$t^3 + 3t = 4t^2 + t$

$t^3 - 4t^2 + 2t = 0$

$t(t^2 - 4t + 2) = 0$

$t=0$ 또는 $t = 2\pm\sqrt{2}$

$\log_2 x = 0$ 또는 $\log_2 x = 2\pm\sqrt{2}$

$x=1$ 또는 $x = 2^{2\pm\sqrt{2}}$

$1\cdot 2^{2+\sqrt{2}}\cdot 2^{2-\sqrt{2}} = 2^4 = 16$

5-3 ④

$\log_2 x = t$ ( $t$는 실수 )

$\frac{1}{4}t^2 - kt + 2 = 0$ 두근 $\log_2\alpha$, $\log_2\beta$

$t^2 - 4kt + 8 = 0$

$\log_2\alpha + \log_2\beta = 4k$ ... ㉠

$\log_2\alpha \cdot \log_2\beta = 8$ ... ㉡

㉠에서 $4k = \log_2\alpha\beta = \log_2 2^8 = 8$ $\boxed{k=2}$

5-4 ①

㉠ $\log_2 xy = \log_2 16$ ( $\log_2 x = 3$, $\log_2 y = 1$ $x=8, y=2$

$\boxed{\log_2 x + \log_2 y = 4}$

㉡ $\log_2 x^{\log_2 y} = \log_2 8$ ( $\log_2 x = 1$, $\log_2 y = 3$ $x=2, y=8$

$\boxed{\log_2 y \cdot \log_2 x = 3}$

$\alpha = 8, \beta = 2$ $|\alpha - \beta| = 6$

5-5 ②

㉠ $\log_2 x + \log_3 y = 2$

㉡ $\frac{\log x}{\log 3} \times \frac{\log y}{\log 4} = \frac{\log x}{2\log 2} \times \frac{\log y}{\log 3} = -\frac{3}{2}$

$\log_2 x \times \log_3 y = -3$

$\alpha = 8, \beta = \frac{1}{3}$

$3\alpha\beta = 8$

$\log_2 x = 3$, $\log_3 y = -1$ $x=8, y=\frac{1}{3}$

# 5-61

(1) ㉠ $x>0$, $\log_3 x>0$, $\log_2(\log_3 x)>0$

$\log_3 x>\log_3 1$ $\quad$ $\log_2(\log_3 x)>\log_2 1$

$x>1$ $\quad$ $\log_3 x>1$

$\log_3 x>\log_3 3$

$x>3$

$\therefore x>3$

㉡ $\log_{\frac{1}{3}}(\log_2(\log_3 x))>\log_{\frac{1}{3}}1$

$\log_2(\log_3 x)<1$

$\log_2(\log_3 x)<\log_2 2$

$\log_3 x<2$

$\log_3 x<\log_3 9$

$x<9$

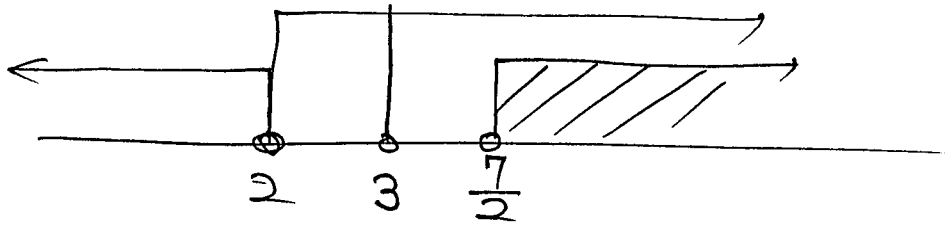
㉠, ㉡ 에서

$3<x<9$

☆

(2) ㉠ $x-2>0$, $x-2\neq 1$, $2x^2-11x+14>0$

$x>2$, $x\neq 3$ $\quad$ $(2x-7)(x-2)>0$




$x>\frac{7}{2}$

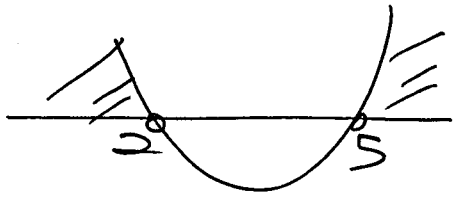
㉡(㉠)

$\log_{(x-2)}(2x^2-11x+14) > \log_{(x-2)}(x-2)^2$

$2x^2-11x+14 > x^2-4x+4$

$x^2-7x+10>0$

$(x-2)(x-5)>0$




$x<2$ 또는 $x>5$

㉠, ㉡에서 $\boxed{x>5}$

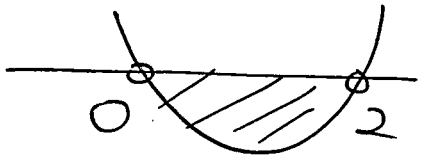
(3) ㉠ $x>0$

㉡ $\log x=t$ ($t$는실수)

$t^2<2t$

$t^2-2t<0$

$t(t-2)<0$




$0<t<2$

$\log 1<\log x<\log 100$

$\therefore 1<x<100$

㉠, ㉡에서 $1<x<100$

(4) $5^{2x+2} > 2^{3-2x}$

$10^{2x} > \frac{2^3}{5^2}$

$\log 10^{2x} > \log \frac{2^3}{5^2}$

$2x > 3\log 2 - 2\log 5 \quad \therefore x > \frac{3\log 2 - 2\log 5}{2}$

5-7)

(1) ㉠ $x > 0$ , $-2 + \log_2 x > 0$

$\log_2 x > \log_2 4 \quad \therefore x > 4$

$x > 4$

㉡ $\log_{16}(-2 + \log_2 x) < \log_{16} 16^{\frac{1}{2}}$

$-2 + \log_2 x < 4$

$\log_2 x < 6$

$\log_2 x < \log_2 2^6$

$x < 64$

㉠, ㉡에서

$4 < x < 64$

$\alpha = 4, \beta = 64$

$\alpha + \beta = 68$

(2) $\frac{1}{3} < x < 9$

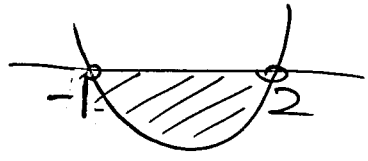
$\log_3 \frac{1}{3} < \log_3 x < \log_3 9$

$-1 < t < 2$

$(t+1)(t-2) < 0$

$\Leftrightarrow (t+1)(t-a) < 0$

$\log_3 x = t$




$a = 2$

5-8 ④

$|x| \le a \quad (a>0)$

$-a \le x \le a$

$|\log_2 x - a| \le 1$

$\Leftrightarrow -1 \le \log_2 x - a \le 1$ (∵ $x>0$)

$a-1 \le \log_2 x \le a+1$

$\log_2 2^{a-1} \le \log_2 x \le \log_2 2^{a+1}$

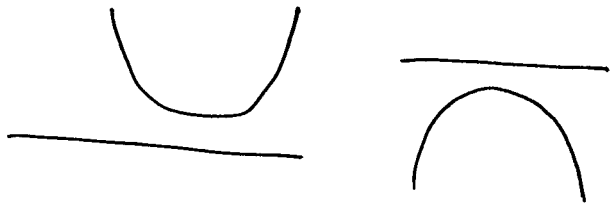
$2^{a-1} \le x \le 2^{a+1}$

$2 \cdot 2^a - \frac{2^a}{2} = \frac{3}{2} \cdot 2^a = 24$

$2^a = 16$

5-9

① $3 + \log a \neq 0$

② 

$D/4 = (1+\log a)^2 - (3+\log a)$

$= (\log a)^2 + \log a - 2 < 0$ (∵ $a>0$)

$(\log a + 2)(\log a - 1) < 0$

$-2 < \log a < 1$

$\log 10^{-2} < \log a < \log 10$

$\frac{1}{100} < a < 10$

## 5-10|20

$x+1=t$

$2^{t+1} \le t^3$

$\log_2 2^{t+1} < \log_2 t^3$

$t+1 < 3\log_2 t$

$2 < t < 8$

$2 < x+1 < 8$

$1 < x < 7$

$\therefore 2+3+4+5+6=20$

## 5-11|①

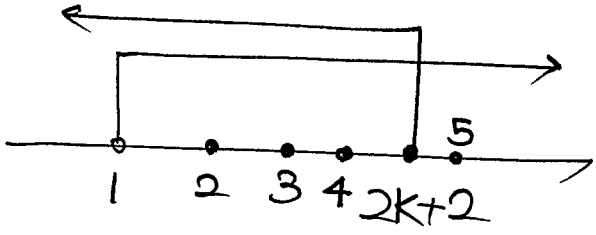
㉠ $x-1>0$, $\frac{1}{2}x+k>0$

$x>1$, $x>-2k$ $\quad \therefore x>1$

㉡ $x-1 \le \frac{1}{2}x+k$

$\frac{1}{2}x \le k+1$

$x \le 2k+2$




$4 \le 2k+2 < 5$

$1 \le k < \frac{3}{2}$ $\quad \therefore k=1$

## 5-12|99

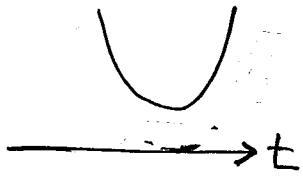
$\log x = t$ ( $t$는 실수)

$(t+\log a)(t+2\log a)+1>0$

$t^2+(3\log a)t+2(\log a)^2+1>0$

$t^2+(3\log a)t+2(\log a)^2+1=0$

$D=9(\log a)^2-8(\log a)^2-4<0$



$(\log a)^2 - 4 < 0$

$(\log a + 2)(\log a - 2) < 0$

$\therefore \ -2 < \log a < 2$

$\frac{1}{100} < a < 100 \quad (\because a > 0)$

$a = 1, 2, \cdots, 99$ **99개**

5-13 ④

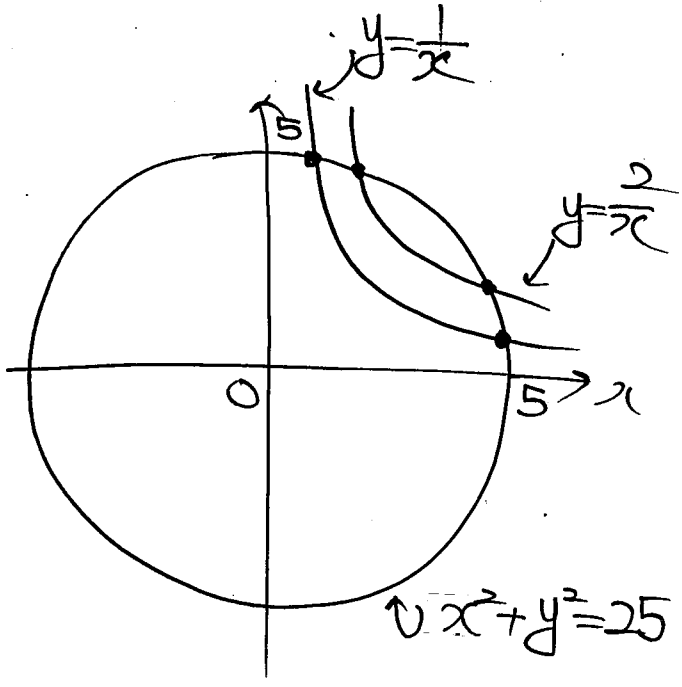
$x > 0, y > 0$

$\log_2 xy = (\log_2 xy)^2$

$\log_2 xy = 0$ 또는 $\log_2 xy = 1$

$xy = 1$ 또는 $xy = 2$

$y = \frac{1}{x}$ 또는 $y = \frac{2}{x}$




4개

5-14 ①

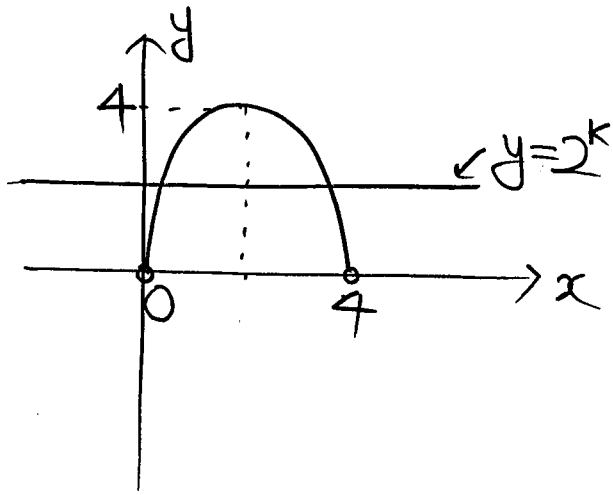
㉠ $x > 0, 4 - x > 0 \quad \therefore 0 < x < 4$

㉡ $\log_2 x + \log_2 (4 - x) = k$

$\log_2 x(4 - x) = \log_2 2^k$

$x(4 - x) = 2^k$

$\begin{cases} y = x(4 - x) = -x^2 + 4x = -(x - 2)^2 + 4 \\ y = 2^k \end{cases}$




$0 < 2^k < 4$

$2^k < 2^2 \quad \therefore k < 2$

$k = 1 \quad \therefore 1$개

5-15 ②

① $x > 0,\ x \neq 1$

② $30\log_x 2 = n$ $\quad \log_2 x = \frac{30}{n}$

$\frac{30}{\log_2 x} = n$ $\quad \therefore 8$

$30 = 2 \cdot 3 \cdot 5$

$n = \pm 1, \pm 2, \pm 3, \pm 5, \pm 15, \pm 30, 10, 6$

5-16 ②

$20\{1 - a^{-0.7(t+0.4)}\} = 16$

$a^{0.7(t+0.4)} = 5$

$0.7(t+0.4) = \log_a 5 = 1.4$

$t + 0.4 = 2$

$t = 1.6$

## 5-17

㉠ $x^2-4x+4>0$, $x^2-4x+4\neq 1$, $6-x>0$

$x\neq 2$ , $x\neq 1, x\neq 3$, $x<6$

㉡ ① $x^2-4x+4=4$

$x^2-4x=0$

② $6-x=1$

$\therefore x=0$ 또는 $x=4$

$\therefore x=5$

## 5-18 $\frac{5}{3}$

㉠ (i) $x>0$, $x\neq 1$

(ii) $1+\log_x 3-\log_x 5<0$

$\log_x x<\log_x 5-\log_x 3$

$\log_x x<\log_x \frac{5}{3}$

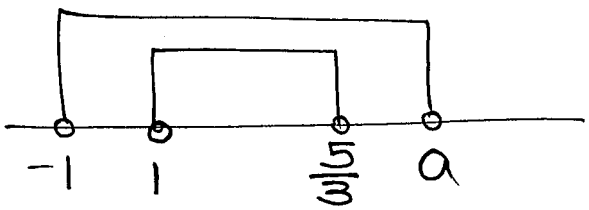
$0<x<1$ ; $x>\frac{5}{3}$ (X)

$x>1$ ; $x<\frac{5}{3}$ $\therefore 1<x<\frac{5}{3}$

㉡ $a>x(x-a+1)$

$x^2-(a-1)x-a<0$

$(x-a)(x+1)<0$ $-1<x<a$



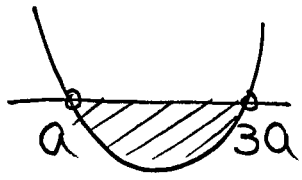

$a\geq\frac{5}{3}$

a의 최솟값: $\frac{5}{3}$

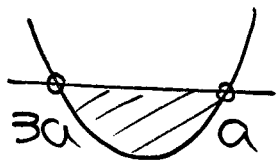
5-19]

㉠ $x(x-3a)<a(x-3a)$

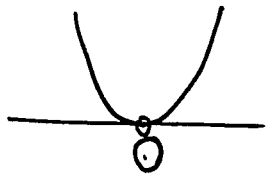
$(x-a)(x-3a)<0$




$a>0$




$a<0$




$a=0$

㉡ i) $x^2-2x+6$

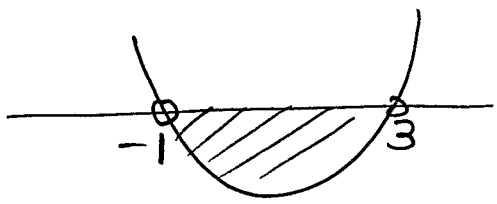
$=(x-1)^2+5>0$

ii) $\log_3(x^2-2x+6)<\log_3 3^2$

$x^2-2x+6<9$

$x^2-2x-3<0$

$(x+1)(x-3)<0$

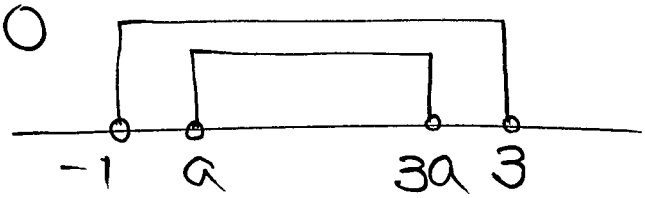



$-1<a<3$

① $a=0$ : $A=\emptyset$ $A\subset B$ $\therefore a=0$

이므로

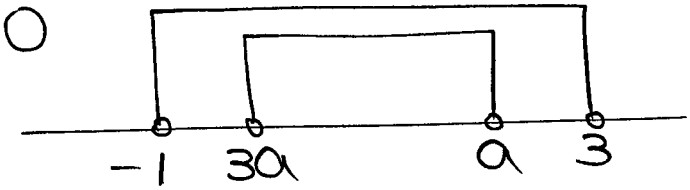
② $a>0$




$3a\le 3$

$a\le 1$ $\therefore 0<a\le 1$

③ $a<0$



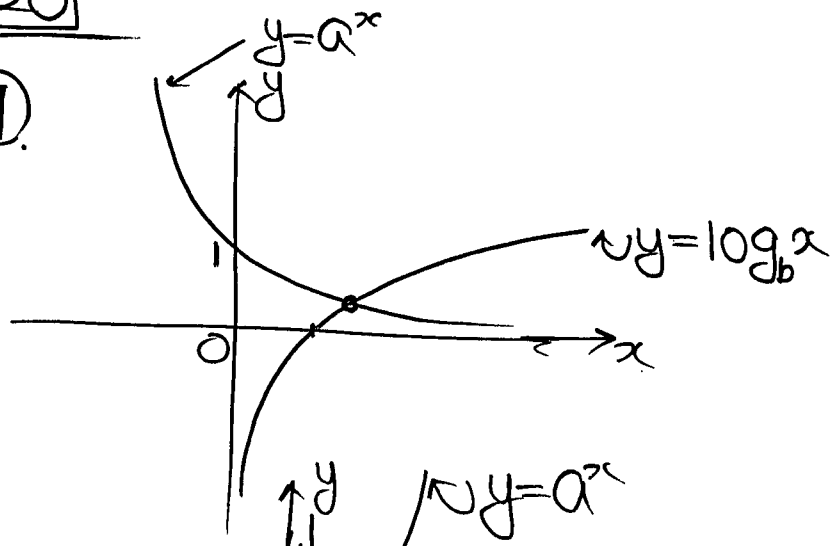

$-1\le 3a$

$a\ge -\frac{1}{3}$ $\therefore -\frac{1}{3}\le a<0$

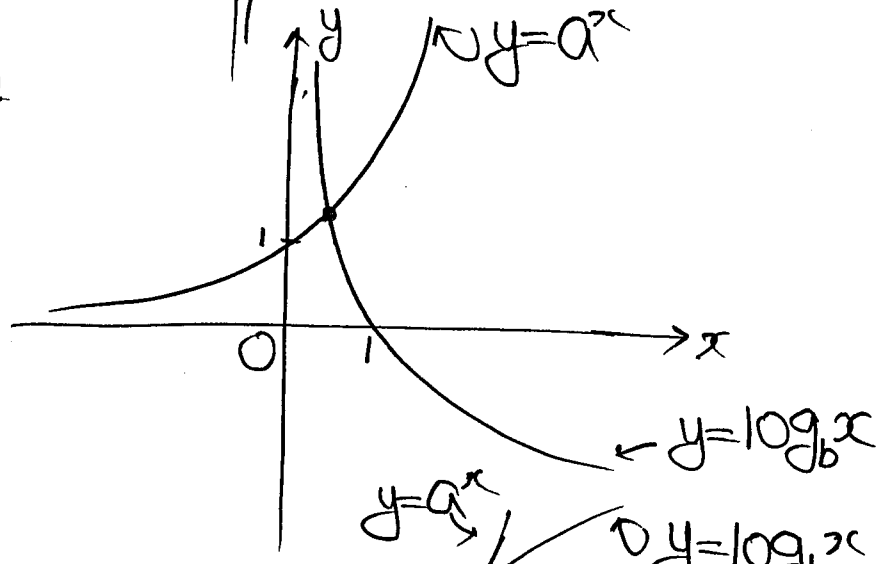
①, ②, ③ 에서 $\boxed{-\frac{1}{3}\le a\le 1}$

5-20

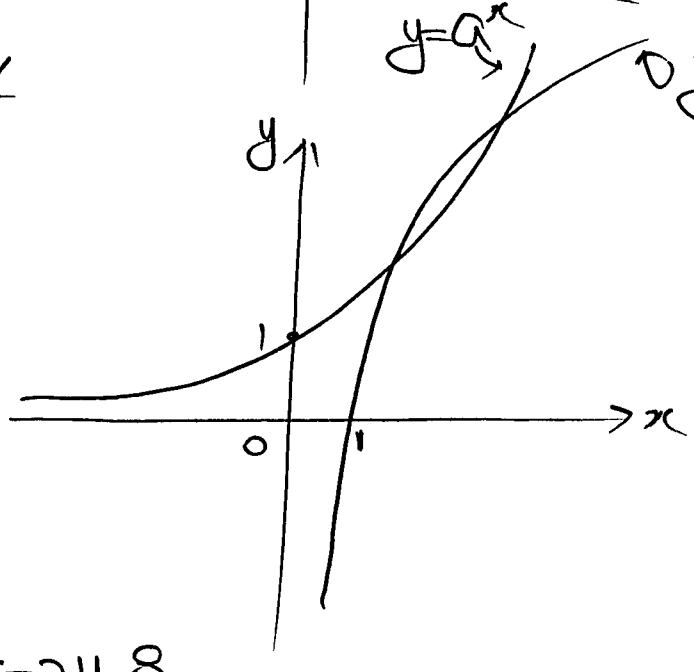
㉠



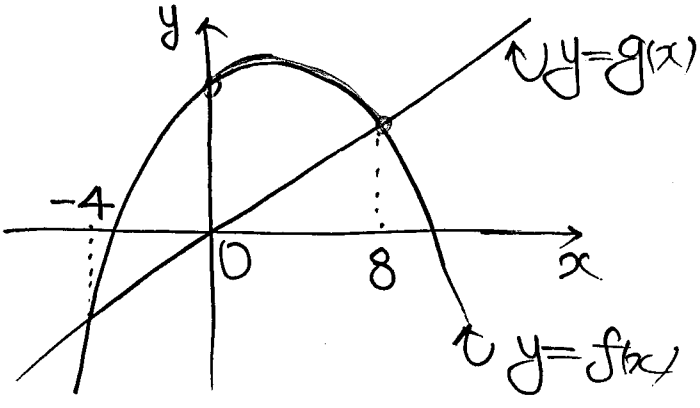

㉡




㉢




5-21 8




① $f(x)>0$, $g(x)>0$

② $f(x)>g(x)$

①, ②에서 $0<x<8$


$x(x-8)<0$

$x^2-8x<0$

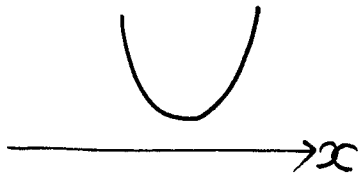
$a=8$, $b=0$ $a+b=8$

5-22|30

$$\log_{32} 32^{x^2+\log_{32} a} > \log_{32} a^{-2x}$$

$$x^2+\log_{32} a > -2x\log_{32} a$$

$$x^2+2(\log_{32} a)\cdot x+\log_{32} a>0$$




$$x^2+2(\log_{32} a)\cdot x+\log_{32} a=0$$

$$D/4=(\log_{32} a)^2-(\log_{32} a)<0$$

$$\log_{32} a\cdot(\log_{32} a-1)<0$$

$$0<\log_{32} a<1$$

$$\log_{32} 1<\log_{32} a<\log_{32} 32$$

$$1<a<32 \qquad \therefore 30\text{개}$$

5-23|3

$$x^2+2\geq 2$$

$$\log_2(x^2+2)\geq \log_2 2=1$$

$$\log_2(x^2+2)=t \quad (t\geq 1)$$

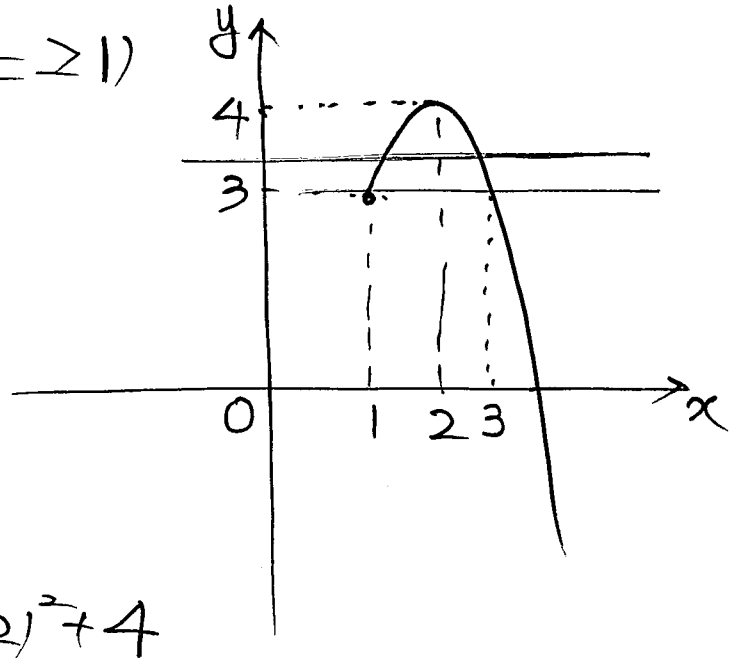
$$x^2+2=2^t$$

$$t^2-4t+a=0$$

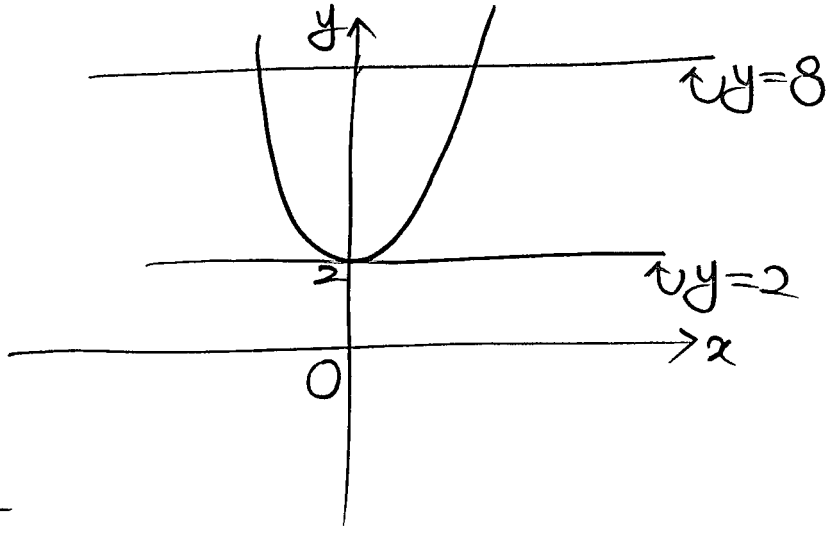
$$a=-t^2+4t.$$

$$y=a$$

$$y=-t^2+4t.=-(t-2)^2+4$$



$a=3$ ; $t=1$, $t=3$




5-24|17

① $a \geq 10$

$\log_2 a - \log_2 10 + \log_2 b \leq \log_2 2$

$\log_2 ab \leq \log_2 20$

$ab \leq 20$

$b=1$ ; $a \leq 20$ $\therefore 10 \leq a \leq 20$ $\therefore$ 11개 ) 12개

$b=2$ ; $a \leq 10$ $\therefore a=10$ 1개

② $a < 10$

$-\log_2 a + \log_2 10 + \log_2 b \leq \log_2 2$

$\log_2 10b \leq \log_2 2a$

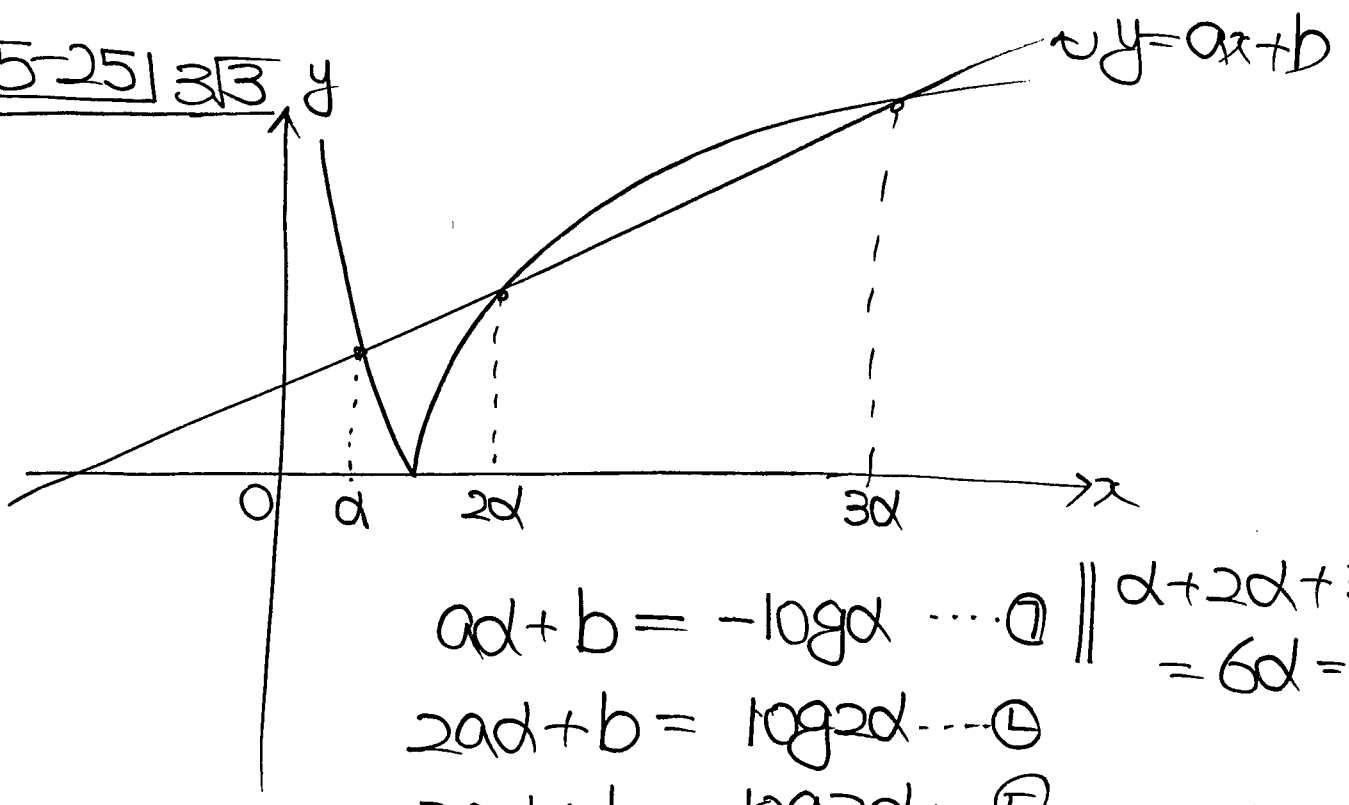
$10b \leq 2a$

$b=1$ ; $10 \leq 2a$ $5 \leq a$ $\therefore 5 \leq a < 10$ 5개

$b=2$ ; $20 \leq 2a$ $10 \leq a$ (x)

$12+5=17$

## 5-25 | 3√3




$a\alpha+b=-\log\alpha$ ···①

$2a\alpha+b=\log 2\alpha$ ···②

$3a\alpha+b=\log 3\alpha$ ···③

$\alpha+2\alpha+3\alpha=6\alpha=?$

② − ① $\quad a\alpha=\log 2\alpha+\log\alpha=\log 2\alpha^2$

③ − ② $\quad a\alpha=\log 3\alpha-\log 2\alpha=\log\frac{3}{2}$

$2\alpha^2=\frac{3}{2}$

$\alpha^2=\frac{3}{4}$

$\alpha=\frac{\sqrt{3}}{2}$ $(\because \alpha>0)$

$6\alpha=3\sqrt{3}$

## 5-26 | 18

$f(x+10)=[\log_2(x+10)]$

| $x$ | $f(x)$ |
|---|---|
| $1\le x<2$ | 0 |
| $2\le x<4$ | 1 |
| $4\le x<8$ | 2 |
| $8\le x<16$ | 3 |
| $16\le x<\infty$ | 4 |

| $x$ | $f(x)$ |
|---|---|
| $1\le x<6$ | 3 |
| $6\le x\le 20$ | 4 |
| | |

① $x=2$ or $x=3$

② $x=6$ or $x=7$

$2+3+6+7=18$

5-27 | 27

$$2\log_2 x + 2\log_2 y = 2\log_2 (x+y+3)$$

$$\log_2 xy = \log_2 (x+y+3)$$

$$xy = x + y + 3$$

$$xy - x - y = 3$$

$$(x-1)(y-1) = 4$$

| | |
|---|---|
| 1 | 4 ... ① |
| 2 | 2 ... ② |
| 4 | 1 ... ③ |

① $x=2, y=5$

$x^2+2y^2 = 4+50 = 54$

② $x=3, y=3$

$x^2+2y^2 = 9+18 = 27$

③ $x=5, y=2$

$x^2+2y^2 = 25+8 = 33$

$\therefore$ **27**

5-28 | 10

$$(\log_3 x)^2 + (\log_3 y)^2 = \log_3 x + \log_3 y$$

$\log_3 x = X$, $\log_3 y = Y$

$$X^2 + Y^2 = X + Y$$

$$X^2 + Y^2 - X - Y = 0$$

$$\left(X - \frac{1}{2}\right)^2 + \left(Y - \frac{1}{2}\right)^2 = \frac{1}{2}$$

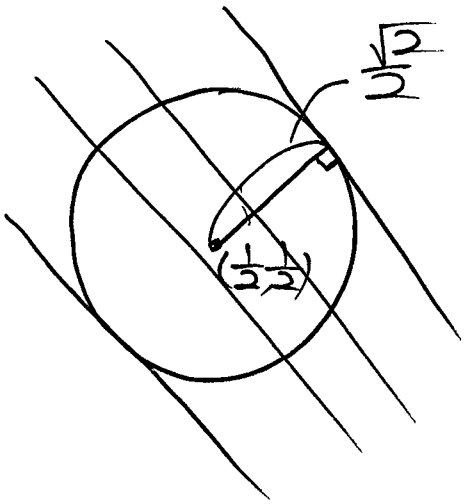
$$Y = -X + \log_3 k$$

$xy = k$

$\log_3 xy = \log_3 k$

$\log_3 x + \log_3 y = \log_3 k$

$X + Y = \log_3 k$




$$\begin{cases} x+y-\log_3 k=0 \\ (\frac{1}{2},\frac{1}{2}) \end{cases}$$

$$\frac{\left|\frac{1}{2}+\frac{1}{2}-\log_3 k\right|}{\sqrt{1+1}} \le \frac{\sqrt{2}}{2}$$

$$|\log_3 k-1| \le 1$$

$$-1 \le \log_3 k-1 \le 1$$

$$0 \le \log_3 k \le 2$$

$$\boxed{1 \le k \le 9}$$

$M=9,\ m=1$

$M+m=10$

5-29 17

가격 : P

무게 : 1

$$\frac{P}{(0.9)^n} \ge 2P$$

$$\left(\frac{10}{9}\right)^n \ge 2$$

$$\log\left(\frac{10}{9}\right)^n \ge \log 2$$

$$n\log\frac{10}{9} \ge \log 2$$

$$n \ge \frac{3010}{458} = 6.\times\times$$

$\therefore$ 7번

$$\log\frac{10}{9} = 1-2\log 2$$

$$= 1-0.9542$$

$$= 0.0458$$

$\log 2 = 0.301$

$$458\overline{)3010}\ 6.\times\times$$

$2748$

$262$

5-3018

남자: 3a

여자: a

$$\frac{a\times(1.2)^n}{40a\times(1.1)^n} \geq \frac{1}{2}$$

$$\left(\frac{1.2}{1.1}\right)^n \geq 2$$

$$\log\left(\frac{1.2}{1.1}\right)^n \geq \log 2$$

$$n(\log 1.2 - \log 1.1) \geq \log 2$$

$$n \times 0.038 \geq 0.301$$

$$n > \frac{301}{38} = 7.xx$$

$$\therefore 8\text{년후}$$

$$\begin{array}{r} 7.xx \\ 38\overline{)301} \\ 266 \\ \hline 35 \end{array}$$