등차수열

1 일반항

$a_1, \underbrace{a_2}_{d}, \underbrace{a_3}_{d}, \cdots\cdots, \underbrace{a_n}_{d}$ (d:공차)

$a_1 = a$

$a_2 = a+d$

$a_3 = a+2d$

$\vdots$

$\boxed{a_n = a+(n-1)d}$

$\boxed{a_{n+1} - a_n = d}$

$a_{n+2} - a_{n+1} = a_{n+1} - a_n$

$\boxed{2a_{n+1} = a_n + a_{n+2}}$

2 등차중항

a, x, b : 등차수열 $\Leftrightarrow 2x = a+b$

$x = \frac{a+b}{2}$

3 등차수열의 합

$S_n = a_1 + a_2 + a_3 + \cdots\cdots + a_n$

$S_n = a + (a+d) + (a+2d) + \cdots\cdots + (a+(n-1)d)$

$S_n = (a+(n-1)d) + (a+(n-2)d) + \cdots\cdots + a$

$2S_n = n(2a+(n-1)d)$ $\quad S_n = \frac{n(2a+(n-1)d)}{2} = \frac{n(a_1+a_n)}{2}$

4 a_n과 S_n의 관계

$S_n = a_1 + a_2 + \cdots\cdots + a_n$

$S_{n-1} = a_1 + a_2 + \cdots + a_{n-1}$

$\boxed{\begin{array}{l} a_1 = S_1 \\ a_n = S_n - S_{n-1} \ (n \geq 2) \end{array}}$

P343

EX1) 1, 3, 5, 7, 9, ...

제3항: 5, 제5항: 9

EX2) $a_n = \frac{n}{n+1}$ $a_{199} = \frac{199}{200}$

P344

EX2 $a_n = 2n-1$

P346

EX) $a_n = 5 + (n-1)\cdot(-3) = -3n+8$

P347

EX) $2x = 2+6$ $x = 4$

P348

EX) $\{a_n\}: \frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \cdots$

$\{\frac{1}{a_n}\}: 2, 5, 8, 11, \cdots$

$\frac{1}{a_n} = 2 + (n-1)\cdot 3 = 3n-1$

$a_n = \frac{1}{3n-1}$

P349

11

(1) 3, 7, 11, 15

(2) 2, 8, 26, 80

(3) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}$

(4) $-1, 1, -1, 1$

21

(1) $a_n = 1 + (n-1)\cdot 3 = 3n-2$

(2) $a_n = 20 + (n-1)\cdot(-2) = -2n+22$

31

$\{a_n\}$: 2, 6, 10, ... $d=4$

41

(1) $a_n = 3 + (n-1)d$

$a_3 = 3 + 2d = 15 \quad 2d = 12 \quad d = 6$

$a = a_2 = 3 + d = 9$

$b = a_4 = 3 + 3d = 21$

(2) $a_n = 21 + (n-1)d$

$a_4 = 21 + 3d = 9 \quad 3d = -12 \quad d = -4$

$a = a_2 = 21 + d = 17$

$b = a_3 = 21 + 2d = 13$

5| 3

$2 \cdot 4 = 3x + 2 + 6 - x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$ $\quad x = 3 \quad (\because x > 0)$

6|

$\left\{\frac{1}{a_n}\right\}$: $-5, -3, -1, 1, 3, 5, \cdots$

$\frac{1}{a_n} = -5 + (n-1) \cdot 2 = 2n - 7$ $\quad a_n = \frac{1}{2n-7}$

P351

1-1|

(1) $a_n = a + (n-1)d$

$a_2 = a + d = 8$ $\quad d = 2, \ a = 6$

$-) \ a_{10} = a + 9d = 24$ $\quad a_{20} = a + 19d = 6 + 38 = 44$

$8d = 16$

(2) $a_n = -2005 + (n-1) \cdot 4 = 4n - 2009 > 0$

$4n > 2009$

$n = 503$ $\quad$ 제 503항

1-21

(1) $a_n = a + (n-1)d$

$a_2 = a + d = 5$

$-\ a_{10} = a + 9d = -11$

$-8d = 16$

$d = -2,\ a = 7$

$a_n = 7 + (n-1)(-2) = -2n + 9 = -31$

$2n = 40$ $\quad \boxed{n = 20}$

(2) $a_n = a + (n-1)d$

① $a_3 = a + 2d = 11$

② $5a_{10} = 8a_6$

$5(a+9d) = 8(a+5d)$

$5a + 45d = 8a + 40d$

$3a = 5d$

$3a + 6d = 33$

$-\ 3a - 5d = 0$

$11d = 33$

$d = 3,\ a = 5$

$a_{20} = a + 19d = 5 + 57 = 62$

(3) $a_n = a + (n-1)d$

$a_1 + a_7 + a_{13} = a + (a+6d) + (a+12d)$

$= 3a + 18d = 12 \qquad a + 6d = 4$

$a_5 + a_{10} + a_{15} = (a+4d) + (a+9d) + (a+14d)$

$= 3a + 27d = 21 \qquad a + 9d = 7$

$a_7 + a_{10} = (a+6d) + (a+9d) = 11$

(4) $a_n = a + (n-1)d$

$a_5 + a_7 = (a+4d) + (a+6d)$

$= 2a + 10d = 12 \qquad a+5d = 6$

$a_3 + a_6 + a_9 = (a+2d) + (a+5d) + (a+8d)$

$= 3a + 15d = 3(a+5d) = 18$

1-3|17

$a_n = a + (n-1)\cdot 4$

$a_{23} = a + 88 = 23 \qquad a = -65$

$a_n = -65 + (n-1)\cdot 4 = 4n - 69$

$a_{17} = -1, \quad a_{18} = 3 \qquad \boxed{\therefore n = 17}$

2-11|83

$a, a+d, a+2d \qquad (d \geq 0)$

① $3a + 3d = 15 \qquad a + d = 5$

② $5-d, 5, 5+d$

$5(25 - d^2) = 105$

$25 - d^2 = 21 \qquad d^2 = 4 \qquad d = 2$

$\therefore 3, 5, 7$

$9 + 25 + 49 = 83$

2-21 ③

$x^3-15x^2+kx-75=0$ 세실근 $a, a+d, a+2d$ $(d>0)$

$a+(a+d)+(a+2d)=3a+3d=15 \quad a+d=5$

세실근 $5-d, 5, 5+d$

$(5-d)\cdot 5+5(5+d)+(5+d)(5-d)=k$

$(5-d)\cdot 5\cdot(5+d)=75$

$25-d^2=15$

$d^2=10$

$k=25-5d+25+5d+25-d^2$

$=75-d^2=65$

2-31 13

$a, a+d, a+2d, a+3d \quad (d>0)$

$a+a+d+a+2d+a+3d=4a+6d=28$

$\therefore 2a+3d=14$

$(a+3d)\cdot a=(a+d)(a+2d)-32$

$a^2+3ad=a^2+3ad+2d^2-32$

$d^2=16 \quad d=4, a=1$

$\therefore a+3d=1+12=13$

3-1)

$\left\{\frac{1}{a_n}\right\}$: $17, \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, 5$

$\frac{1}{a_n} = 17 + (n-1)d$

$\frac{1}{a_5} = 17 + 4d = 5 \quad d = -3$

$\frac{1}{x} = 17 + d = 14 \quad x = \frac{1}{14}$

$\frac{1}{y} = 17 + 2d = 11 \quad y = \frac{1}{11}$

$\frac{1}{z} = 17 + 3d = 8 \quad z = \frac{1}{8}$

3-2) 300

① $a, 10, b$: 등차수열 $\quad 20 = a + b$

② $a, 5, b$: 조화수열 $\quad \frac{2}{5} = \frac{1}{a} + \frac{1}{b}$

$\frac{1}{a}, \frac{1}{5}, \frac{1}{b}$: 등차수열 $\quad \frac{2}{5} = \frac{b+a}{ab}$

$2ab = 100 \quad ab = 50$

$a^2 + b^2 = (a+b)^2 - 2ab$

$= 400 - 100 = 300$

3-3) ④

$\left\{\frac{1}{a_n}\right\}$: 등차수열

: $\frac{1}{3}, \frac{1}{2}, \cdots$

$\frac{1}{a_n} = \frac{1}{3} + (n-1) \cdot \frac{1}{6} = \frac{n+1}{6}$

$a_n = \frac{6}{n+1} \quad a_{11} = \frac{6}{12} = \frac{1}{2}$

P357

EX) $a_1=-2,\ a_{10}=34$

$$S_{10}=\frac{10(-2+34)}{2}=160$$

P358

EX) $a_n=11+(n-1)(-3)=-3n+14$

$a_1=11,\ a_{20}=-46$

$$S_{20}=\frac{20(11+(-46))}{2}=-350$$

P359

EX)

$A_n=a_1+a_2+\cdots+a_n=n^2 \quad a_1=A_1=1$

$-)\ A_{n-1}=a_1+a_2+\cdots+a_{n-1}=(n-1)^2$

$a_n=2n-1 \quad (n\geq 2)$

$\therefore a_n=2n-1$

$B_n=b_1+b_2+\cdots+b_n=n^2+1 \quad b_1=B_1=2$

$-)\ B_{n-1}=b_1+b_2+\cdots+b_{n-1}=(n-1)^2+1$

$b_n=2n-1 \quad (n\geq 2)$

$$b_n=\begin{cases}2 & (n=1)\\ 2n-1 & (n\geq 2)\end{cases}$$

P361

11

(1) $a_n = 2 + (n-1)\cdot 3 = 3n-1$

$3n-1 = 20 \qquad n = 7$

$S_7 = \frac{7(2+20)}{2} = 77$

(2) $a_n = 33 + (n-1)\cdot(-5) = -5n+38$

$-5n+38 = -12$

$5n = 50 \qquad n = 10$

$S_{10} = \frac{10\cdot(33+(-12))}{2} = 105$

21

(1) $a_n = 1 + (n-1)\cdot 2 = 2n-1 \qquad a_1 = 1,\ a_{15} = 29$

$S_{15} = \frac{15(1+29)}{2} = 225$

(2) $a_n = -2 + (n-1)(-2) = -2n \qquad a_1 = -2,\ a_{15} = -30$

$S_{15} = \frac{15((-2)+(-30))}{2} = -240$

31

(1) $S_n = 4n \qquad a_1 = S_1 = 4$

$-) \quad S_{n-1} = 4(n-1)$

$a_n = 4 \quad (n \geq 2)$

$\therefore a_n = 4$

(2) $S_n = 2n^2$ $\quad a_1 = S_1 = 2$

$-) \; S_{n-1} = 2(n-1)^2$

$a_n = 4n - 2 \quad (n \geq 2)$

$\therefore a_n = 4n - 2$

4-11

$a_n = a + (n-1)d$

$a_4 = a + 3d = 14 \quad d = 3, \; a = 5$

$-) \; a_7 = a + 6d = 23$

$-3d = -9$

$a_n = 5 + (n-1) \cdot 3 = 3n + 2 \quad a_1 = 5, \; a_{20} = 62$

$S_{20} = \frac{20(5+62)}{2} = 670$

4-21 ④

$a_n = 50 + (n-1) \cdot d \quad a_1 = 50, \; a_n = -10$

$S_n = \frac{n(50 + (-10))}{2} = 20n = 420 \quad n = 21$

$a_{21} = 50 + 20d = -10 \quad d = -3$

$a_{30} = 50 + 29d = 50 - 87 = -37$

4-3|

(1) $S = 1+4+7+\cdots+100$

$= \frac{34(1+100)}{2}$

$= 17 \cdot 101 = 1717$

$a_n = 1+(n-1)\cdot 3$

$= 3n-2 = 100$

$3n = 102$

$n = 34$

(2) $S = 104+112+120+128+136+144+152$

$+160+168+176+184+192$

$= \frac{12(104+192)}{2} = 6 \cdot 296 = 1776$

5-11

(1) $S_n = 3n^2+2n \qquad a_1 = S_1 = 5$

$-) \; S_{n-1} = 3(n-1)^2+2(n-1)$

$a_n = 6n-3+2 = 6n-1 \quad (n \geq 2)$

$a_n = 6n-1$

(2) $S_n = 2n^2-n-1 \qquad a_1 = S_1 = 2-1-1 = 0$

$-) \; S_{n-1} = 2(n-1)^2-(n-1)-1$

$a_n = 4n-2-1 = 4n-3 \quad (n \geq 2)$

$$a_n = \begin{cases} 0 & (n=1) \\ 4n-3 & (n \geq 2) \end{cases}$$

5-2 ②

$S_n = n^2 + 2n$ $\quad a_1 = S_1 = 3$

$-) \; S_{n-1} = (n-1)^2 + 2(n-1)$

$a_n = 2n - 1 + 2 = 2n + 1 \quad (n \geq 2)$

$a_n = 2n + 1$

$S = \underbrace{3 + 7 + 11 + \cdots + 199}_{50\text{개}} = \frac{50(3+199)}{2} = 50 \cdot 101$

$= 5050$

5-3 16

$a_{10} = (100 + 10k) - (81 + 9k) = 19 + k$

$b_{10} = (200 - 30) - (162 - 27)$

$= 170 - 135 = 35$

$19 + k = 35 \quad k = 16$

6-11 -36

$a_n = -11 + (n-1) \cdot 2 = 2n - 13 < 0$

$2n < 13 \quad n = 1, 2, \cdots, 6$

$S_6 = -11 - 9 - 7 - 5 - 3 - 1 = -36$

6-21 ④

$a_n = a + (n-1)d$

$a_7 = a + 6d = 2$ $\quad d=-3,\ a=20$

$-\ a_{10} = a + 9d = -7$

$-3d = 9$

$a_n = 20 + (n-1)(-3) = -3n + 23 > 0$

$3n < 23$ $\quad a_1 = 20$

$n = 1, 2, \ldots, 7$ $\quad a_7 = 2$

$S_7 = \frac{7 \cdot (20+2)}{2} = 77$

6-31

$S_n = 2n^2 - 39n$ $\quad a_1 = S_1 = 2 - 39 = -37$

$-\ S_{n-1} = 2(n-1)^2 - 39(n-1)$

$a_n = 4n - 2 - 39 = 4n - 41 \quad (n \geq 2)$

$a_{10} < 0,\ a_{11} > 0, \ldots$

$S = -a_1 - a_2 - \cdots - a_{10} + a_{11} + a_{12} + \cdots + a_{20}$

$= 40 \cdot 10$

$= 400$

$S = -S_{10} + (S_{20} - S_{10})$

$= S_{20} - 2S_{10}$

7-11 1020

$a_n = a + (n-1)d$

$S_n = \frac{n(2a + (n-1)d)}{2}$

$S_5 = a + (a+d) + \cdots + (a+4d) = 140$

$S_{10} = \underbrace{a + (a+d) + \cdots + (a+4d)}_{=140} + \underbrace{(a+5d) + (a+6d) + \cdots + (a+9d)}_{=340}$

$= 480$

$S_{15} = 480 + (a+10d) + (a+11d) + \cdots + (a+14d)$

$= 480 + 540 = 1020$

7-214

$a_n = a+(n-1)d$, $S_n = \frac{n(2a+(n-1)d)}{2}$

$S_5 = \frac{5(2a+4d)}{2} = 70$ $\quad 2a+4d=28$

$S_{15} = \frac{15(2a+14d)}{2} = 360$ $\quad 2a+14d=48$

$d=2,\ a=10$

$S = S_{25} - S_{10}$

$S_{15} = \frac{25(20+48)}{2} = 25\times 34$

$S_{10} = \frac{10(20+18)}{2} = 10\times 19$ $\quad 17$ 5

$S = 10\times(85-19) = 660$

7-3125

$a_n = a+(n-1)d$, $S_n = \frac{n(2a+(n-1)d)}{2}$

$(a\neq 0)$

$S_{3k} = \frac{3k(2a+(3k-1)d)}{2}$

$S_k = \frac{k(2a+(k-1)d)}{2}$

$\frac{3k(2a+(3k-1)d)}{2} = 9\times\frac{k(2a+(k-1)d)}{2}$ (3)

$2a+(3k-1)d = 6a+(3k-3)d$

$2d = 4a$ $\quad \boxed{d=2a}$

$$S_{5k} = \frac{5k(d+(5k-1)d)}{2} = \frac{25k^2d}{2}$$

$$S_k = \frac{k(d+(k-1)d)}{2} = \frac{k^2d}{2}$$

∴ 2584

8-1) $\frac{16}{31}$

① $\{A_n\}$: $1, a_1, a_2, \cdots, a_{15}, 10$

$\{B_n\}$: $1, b_1, b_2, \cdots, b_{30}, 10$

$A_n = 1+(n-1)d_1$ $\quad A_{17} = 1+16d_1 = 10$ $\quad d_1 = \frac{9}{16}$

$B_n = 1+(n-1)d_2$ $\quad B_{32} = 1+31d_2 = 10$ $\quad d_2 = \frac{9}{31}$

② $a_{15} - a_1 = 14d_1$

$b_{30} - b_{16} = 14d_2$

(공차) $= \frac{d_2}{d_1} = \frac{9}{31} \times \frac{16}{9} = \frac{16}{31}$

8-2) 14

$$S = \frac{(n+2)(-5+20)}{2} = \frac{15(n+2)}{2} = 120$$

$n+2 = 16$

$n = 14$

8-3 ②

공차 : d

① $4, a, a_2, \cdots, a_m, 20$ $\quad 20 = 4 + (m+1)d$

$(m+1)d = 16$

② $20, b_1, b_2, \cdots, b_n, 52$ $\quad 52 = 20 + (n+1)d$

$(n+1)d = 32$

$2m+2 = n+1$

$\therefore n = 2m+1$

P372

9-11 19

$a_n = 6 + (n-1)\cdot(-2) = -2n+8$

$b_n = 8 + (n-1)(-1) = -n+9$

$a_k = 3b_k$

$-2k+8 = -3k+27 \quad k = 19$

9-21 ⑤

$a_n = a + (n-1)d_1$

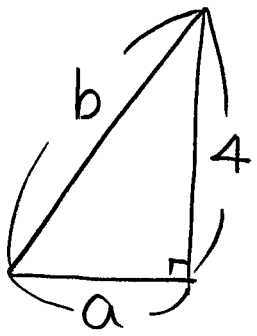
① : $\underline{a + a+d_1}, \underline{a+2d_1 + a+3d_1}, \cdots$

$d_2 = 4d_1$

② : $\underline{a + a+d_1 + a+2d_1}, \underline{a+3d_1 + a+4d_1 + a+5d_1}, \cdots$

$d_3 = 9d_1 \quad 9d_2 = 4d_3$

9-3| 5



$a, 4, b$: 등차수열 $(a<4<b)$

$8 = a+b$

$b^2 - a^2 = 16$ $\quad b=5$

$b-a=2$

9-4| 34

$109 = 4+(k+1)d$

$(k+1)d = 105 = 3 \cdot 35$ $\quad d=3 \quad k=34$

9-5| 11

① $25 = -2+(n+1)\cdot d$ $\quad (n+1)d = 27$

② $\frac{(n+2)(-2+25)}{2} = 115$

$n+2 = 10$ $\quad n=8,\ d=3$ $\quad n+d=11$

9-6| 43

$36+37+38+39+40+41+42 = 39\times 7$

$a_1 + a_1+1 + a_1+2 + a_1+3 + a_1+4 + a_1+5$

$= 6a_1 + 15$

$6a_1 + 15 = 39\times 7$

$2a_1 + 5 = 91$ $\quad a_1 = 43$

9-7 ①

$a_n = a + (n-1)d$

$S = a + (a+2d) + \cdots + (a+20d) = \frac{11 \cdot (2a+20d)}{2}$

$T = (a+d) + (a+3d) + \cdots + (a+19d) = \frac{10(2a+20d)}{2}$

$S : T = 11 : 10$

9-8 198

$a_n = a + (n-1)d$

$a + (a+2d) + \cdots + (a+98d)$

$= \frac{50(2a+98d)}{2} = 100 \qquad 2a+98d = 4$

$S = a_1 + a_2 + \cdots + a_{99}$

$= \frac{99 \cdot (a + a+98d)}{2} = 198$

9-9 400

$a_n = a + (n-1)d$

$(a+d) + (a+3d) + \cdots + (a+(2n-1)d)$

$= \frac{n(2a+2nd)}{2} = 2n^2 + n$

$a + nd = 2n + 1 \qquad a=1, d=2$

$S = \frac{20(1+39)}{2} = 400$

$a_n = 1 + (n-1) \cdot 2$
$= 2n - 1$

9-10| 5|

3	⑤	7
⑥	⑧	⑩
⑨	11	⑬

9-11| ②

$a_n = a + (n-1)\cdot 2$ $\quad \dfrac{2^{a+6} + 2^{a+10}}{2^a + 2^{a+4}} = 2^6 = N$

$$\log_4 N = \log_{2^2} 2^6 = \frac{6}{2}\log_2 2 = 3$$

9-12| ⑤

$\{a_n\}$: 1, 5, 9, 13, 17, ㉑

$$S = 21 + 41 + 61 + 81 + 101 + 121 + 141$$

$$= \frac{7 \cdot 162}{2} = 7 \cdot 81 = 567$$

9-13| 3

㉠ a, b, c : 등차수열 $\Leftrightarrow 2b = a + c$

㉡ b, c, a : 조화수열

$\frac{1}{b}, \frac{1}{c}, \frac{1}{a}$: 등차수열 $\Leftrightarrow \frac{2}{c} = \frac{1}{b} + \frac{1}{a}$

$2ab = ca + bc$

$a=2b-c$; $2(2b-c)b=c(2b-c)+bc$

$4b^2-2bc=2bc-c^2+bc$

$4b^2-5bc+c^2=0$

$(4b-c)(b-c)=0$

$c=4b$ $(a\neq b\neq c\neq 0)$

$a=-2b$

$(\text{주식})=\frac{b}{(-2b)}+\frac{4b}{b}+\frac{(-2b)}{4b}$

$=-\frac{1}{2}+4+(-\frac{1}{2})=3$

9-14 ⑤

$a_1=2+4+6+\cdots+20=\frac{10\cdot 22}{2}=110$

$a_n=110+(n-1)\cdot 2=2n+108$

$a_{50}=208$

9-15 ②

$a_n=a+(n-1)d_1$, $b_n=b+(n-1)d_2$

$A_n=\frac{n(2a+(n-1)d_1)}{2}$ $B_n=\frac{n(2b+(n-1)d_2)}{2}$

$2a+(n-1)d_1=3n+6$ $d_1=3$ $a=\frac{9}{2}$

$2b+(n-1)d_2=7n+2$ $d_2=7$ $b=\frac{9}{2}$

$a_7:b_7=\frac{9}{2}+18:\frac{9}{2}+42=45:93=15:31$

9-16번 8

$a_n = a + (n-1)d_1$, $b_n = b + (n-1)d_2$

$S_n = \frac{n(2a+(n-1)d_1)}{2}$ $T_n = \frac{n(2b+(n-1)d_2)}{2}$

$S_n T_n = \frac{d_1 d_2 n^4 + \cdots}{4}$

$\frac{d_1 d_2}{4} = 2$ $d_1 d_2 = 8$

9-17번 6

$a_n = 3 + (n-1)d$

$3(a_1 + a_2 + \cdots + a_n) = (a_{n+1} + \cdots + a_{2n})$

$4(a_1 + a_2 + \cdots + a_n) = (a_1 + a_2 + \cdots + a_{2n})$

$\cancel{4}^{2} \cdot \frac{n(6+(n-1)d)}{\cancel{2}} = \frac{\cancel{2n}(6+(2n-1)d)}{\cancel{2}}$

$12 + 2nd - 2d = 6 + 2nd - d$

$d = 6$

9-18번 3

a, b, c : 등차수열 $\Leftrightarrow$ $2b = a + c$ $c = 2b - a$

b^2, c^2, a^2 : 등차수열 $\Leftrightarrow$ $2c^2 = a^2 + b^2$ ←

$2(4b^2 - 4ab + a^2) = a^2 + b^2$ $a = 7b$

$a^2 - 8ab + 7b^2 = 0$ $b = 1, a = 7, c = -5$

$(a-b)(a-7b) = 0$ $a + b + c = 3$

9-19 | 150

$a_n = a + (n-1)\cdot 2$

$S_{50} = \frac{50(a + a+98)}{2} \leq 10000$

$\therefore \; a+49 < 200$

$a < 151 \qquad \therefore 150$

9-20 | 216

$a_n = a + (n-1)d \qquad (d: 정수)$

㈎ $(a+d) + (a+3d) + (a+5d) = 102$

$3a + 9d = 102$

$a + 3d = 34 \qquad a = 34 - 3d$

$a_{13} = a + 12d = 34 - 3d + 12d$

$= 9d + 34 \leq 100$

$a_{14} = a + 13d = 10d + 34 > 100$

$\frac{66}{10} < d \leq \frac{22}{3} \qquad d = 7$

$a = 34 - 21 = 13$

$a_{30} = a + 29d = 13 + 29 \times 7$

$= 13 + 203 = 216$

9-21 | 30

$a_n = a + (n-1)d \qquad a_1 = a > 0$

$S_n = \frac{n(2a+(n-1)d)}{2}$

$S_{40} = S_{20}$

$\frac{40(2a+39d)}{2} = \frac{20(2a+19d)}{2}$

$4a + 78d = 2a + 19d$

$2a = -59d \qquad a = \frac{-59d}{2} \quad (d<0)$

$a_n = -\frac{59d}{2} + (n-1)d > 0$

$-59 + 2n - 2 < 0$

$2n < 61 \qquad n = 30$

9-22 | 15

$a_n = 1 + (n-1)d \qquad (d: 자연수)$

$S_n = \frac{n(2+(n-1)d)}{2} = 94$

$n(2+(n-1)d) = 94 \times 2 = 2 \times 2 \times 47$

$n = 4 ; \quad 4(2+3d) = 4 \times 47$

$d = 15$

9-23| 100

$$\underbrace{a+(a+2)+(a+4)+\cdots\cdots+(a+38)}_{20개}=n^3$$

a: 홀수

$$\frac{20(2a+38)}{2}=20(a+19)=n^3$$

$a+19=50$ $\quad\therefore 20\times 50=1000=10^3$

$a=31$

$\therefore 2a+38=62+38=100$

9-24| 6

$$a(m,n)=1+(n-1)(2m-1)=100001$$

$$(n-1)(2m-1)=100000=10^5=2^5\times 5^5$$

1
5
5^2
5^3
5^4 6개
5^5

9-25| 46

$a,\ a+d,\ a+2d,\ a+3d,\ a+4d \quad (d>0)$

$5a+10d=120 \qquad a+2d=24$

$a+(a+d)=\frac{1}{7}(3a+9d)$

$14a+7d=3a+9d$

$11a=2d \qquad 12a=24$

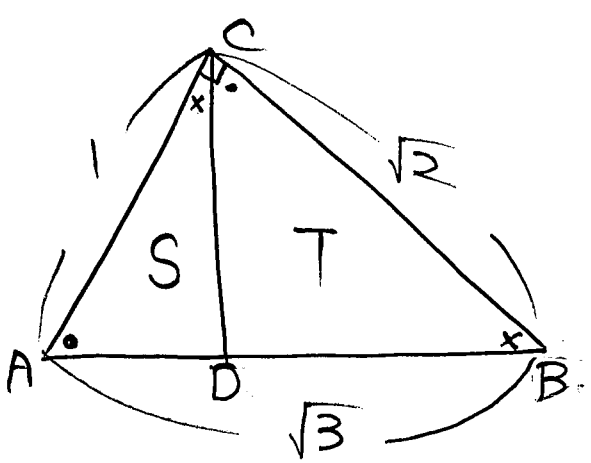
$a=2,\ d=11$

$\therefore 2+44=46$

9-26| 315

$S=\frac{14(3+42)}{2}=7\times 45=315$

9-27| $\frac{\sqrt{2}}{2}$



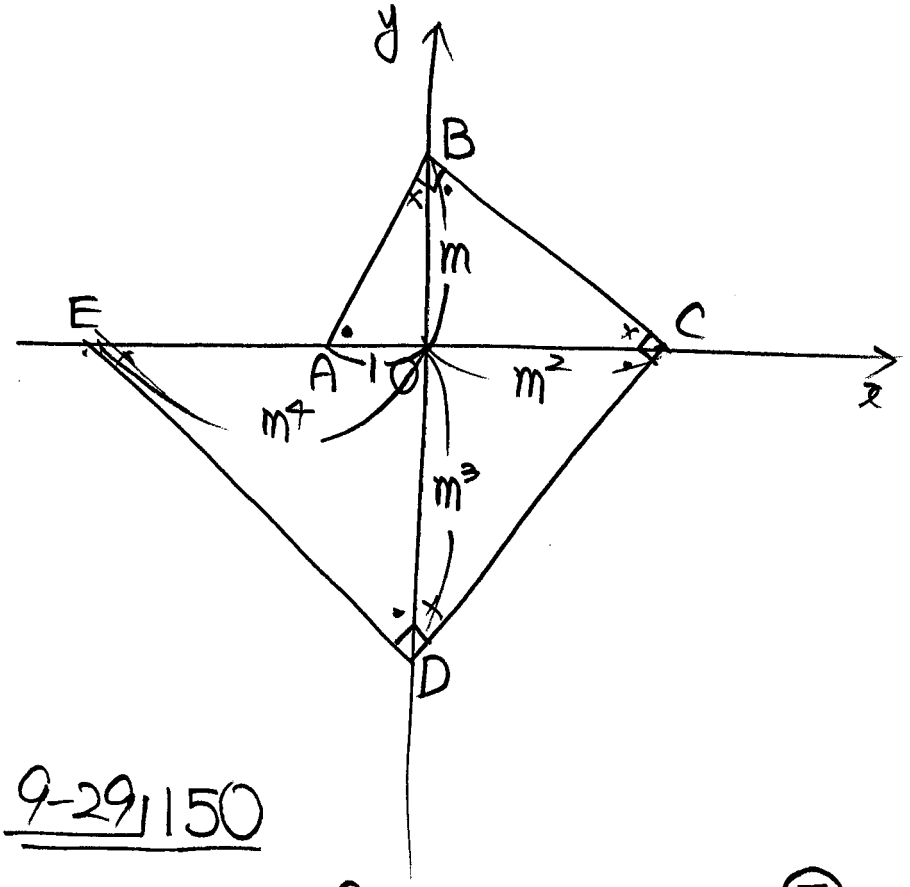
S, T, S+T: 등차수열

$2T=2S+T$

$T=2S$

$\tan B=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

9-28 | $\sqrt{2}$



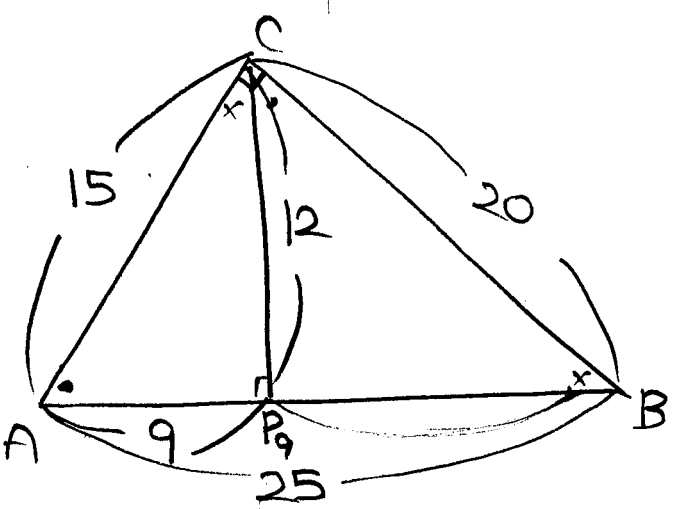
$1, m^2, m^4-1$: 등차수열

$2m^2 = m^4$ $(m>0)$

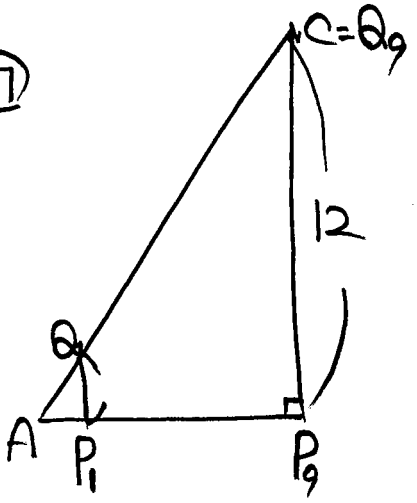
$m^2 = 2$

$m = \sqrt{2}$

9-29 | 150



①



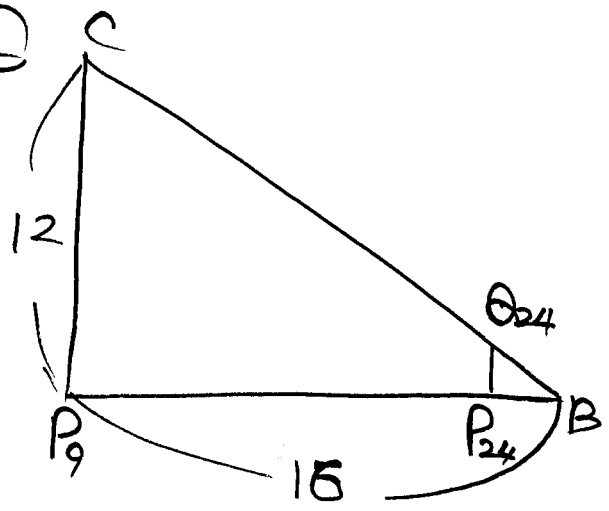
$9 \to 12$

$\overline{P_1Q_1} = \frac{4}{3}$

$\overline{P_1Q_1} + \overline{P_2Q_2} + \cdots + \overline{P_9Q_9} = \frac{4}{3}(1+2+\cdots+9)$

$= \frac{4}{3} \cdot \frac{9 \cdot 10}{2} = 60$

㉡



$\overline{P_{24}Q_{24}} + \overline{P_{23}Q_{23}} + \cdots + \overline{P_{10}Q_{10}}$

$= \frac{3}{4}(1+2+\cdots+15)$

$= \frac{3}{4} \cdot \frac{15 \cdot 16^{2}}{2} = 90$

$\therefore$ ㉠, ㉡ $60+90=150$