10. 등비수열

P382

EX1)

(1) $a_n = 12 \times \left(\frac{1}{2}\right)^{n-1}$

(2) $a_n = 1 \times (-2)^{n-1}$

EX2)

$x^2 = 36 \quad x = \pm 6$

P383

1]

(1) $a_n = \frac{1}{4} \times (4)^{n-1}$

(2) $a_n = 27 \times \left(-\frac{1}{3}\right)^{n-1}$

2]

(1) $a_n = 64 \times r^{n-1}$

$a_3 = 64 \times r^2 = 16 \quad r^2 = \frac{1}{4} \quad r = \pm\frac{1}{2}$

$r = \frac{1}{2}$; $a = 32$, $b = 8$

$r = -\frac{1}{2}$; $a = -32$, $b = -8$

(2) $a_n = \frac{1}{2} \times r^{n-1}$

$a_4 = \frac{1}{2} \times r^3 = 500 \quad r^3 = 1000 = 10^3 \quad r = 10$

$a = 5$, $b = 50$

1-11

(1) $a_n = 4\times(-3)^{n-1} = -972$

$(-3)^{n-1} = -243 = (-3)^5$ $\quad n=6$

(2) $a_n = a\times r^{n-1}$ $\quad (a>0,\ r>0)$

$a_4 = ar^3 = 54,\quad a_6 = 486 = ar^5$

$r^2 = \frac{486}{54} = 9 \quad r=3,\ a=2$

$\therefore 2+3=5$

(3) $a_n = a\times r^{n-1}$

$a_2 = ar = 6 \qquad r^3 = 8$

$a_5 = ar^4 = 48 \qquad r=2 \quad a=3$

$a_n = 3\cdot 2^{n-1} = 1536$

$2^{n-1} = 512 = 2^9 \qquad n=10$

(4) $a_n = ar^{n-1}$

$a+ar+ar^2+ar^3=4$

$ar^4+ar^5+ar^6+ar^7=32$ $\quad r^4=8$

(준식) $= \frac{ar^4+ar^7}{a+ar^3} = r^4 = 8$

1-2 | ③

$$a_m = ar^{m-1} = n \quad \cdots ㄱ \qquad \frac{ㄱ}{ㄴ} ; \ r^{m-n} = \frac{n}{m}$$

$$a_m = ar^{n-1} = m \quad \cdots ㄴ$$

$$a_{2m-n} = ar^{2m-n-1} = \frac{n^2}{m}$$

1-3 | ④

$$a_m = ar^{n-1} \qquad a_9 b_{12} = ar^8 \cdot bs^{11} = 10 \quad \cdots ㄱ$$

$$b_n = b \cdot s^{n-1} \qquad a_{16} b_{20} = ar^{15} \cdot bs^{19} = 20 \quad \cdots ㄴ$$

$$ㄱ^2 \div ㄴ ; \ a_2 b_4 = ar \cdot bs^3 = 5$$

2-11

(1) $$a_n = p \cdot r^{n-1}$$

$$a_5 = p \cdot r^4 = q \qquad r^4 = \frac{q}{p} \qquad r = \frac{q^{\frac{1}{4}}}{p^{\frac{1}{4}}}$$

$$a_2 = x = pr = p^{\frac{3}{4}} q^{\frac{1}{4}}$$

$$a_3 = y = pr^2 = p^{\frac{2}{4}} q^{\frac{2}{4}}$$

$$a_4 = z = pr^3 = p^{\frac{3}{4}} q^{\frac{1}{4}}$$

(2) $$r^4 = \frac{81}{16} \qquad r = \frac{3}{2}$$

$$x = 16 \times \frac{3}{2} = 24$$

$$y = 24 \times \frac{3}{2} = 36$$

$$z = 36 \times \frac{3}{2} = 54$$

2-21

(1) $2x=6$ $x=3$, $y^2=5$ $x^2+y^2=14$

(2) $a_n=10+(n-1)d$

$a_5=10+4d=90$ $d=20$

$b=a_3=10+2d=50$

$b_n=10\cdot r^{n-1}$

$b_5=10r^4=90$ $r^4=9$ $r^2=3$

$e=b_3=10\cdot r^2=30$ $b+e=80$

2-31-2

㉠ $4, p, q$: 등차수열 $2p=4+q$

㉡ $p, q, 4$: 등비수열 $q^2=4p$

$q^2=8+2q$

$q^2-2q-8=0$

$(q-4)(q+2)=0$

$q=-2$, $p=1$

$\boxed{pq=-2}$

3-1121

$x^3-px^2+105x-125=0$ 세근 a, ar, ar^2

$a+ar+ar^2=p$

$a^2r+a^2r^2+a^2r^3=105$

$a^3r^3=125=5^3$ $ar=5$

$a+5r=16$

$5a+25+25r=105$

$5a+25r=80$

$r=\frac{5}{a}$; $a+\frac{25}{a}=16$

$a^2-16a+25=0$

$\underline{ar}(a+ar+ar^2)=105$

$5p=105$ $\boxed{p=21}$

3-2) ⑤

$x^3+8=kx^2+6x$

$x^3-kx^2-6x+8=0$ 세근 a, ar, ar^2

$a+ar+ar^2=k$

$a^2r+a^2r^3+a^2r^2=-6$

$a^3r^3=-8$ $ar=-2$

$\underline{ar}(\underline{a+ar^2+ar})=-6$ $\boxed{k=3}$

3-3) 21

a, ar, ar^2

$a+ar+ar^2=7$

$a^3r^3=8$ $ar=2$

$\frac{2}{r}+2+2r=7$

$2r^2-5r+2=0$

$(2r-1)(r-2)=0$

1, 2, 4

$1+4+16=21$

P390

EX) (1) $a_n = 2\times\left(\frac{1}{2}\right)^{n-1}$

$$S_{10} = \frac{2\left\{1-\left(\frac{1}{2}\right)^{10}\right\}}{1-\frac{1}{2}} = 4\times\frac{1023}{1024} = \frac{1023}{256}$$

(2) $a_n = 4\times 2^{n-1}$

$$S_5 = 4+8+16+32+64 = 124$$

복리법

원금 : A , 연이율 : r (%) 기간 : n (년)

$$A \xrightarrow{1\text{년후}} A + Ar \xrightarrow{2\text{년후}} A(1+r) + A(1+r)r$$

$$= A(1+r)$$

$$= A(1+2r+r^2)$$

$$= A(1+r)^2$$

$$\boxed{n\text{년후} : A(1+r)^n}$$

P392

EX)

$$100 \xrightarrow{10년후} 100+4\times 10=140$$

P393

EX) $$100 \xrightarrow{10년후} 100(1.04)^{10} \fallingdotseq 148$$

P395

11

(1) $S=64+32+16+8+4+2+1$

$=127$

(2) $S=1+3+9+27+81+243=364$

21

(1) $a_n=1\times(-2)^{n-1}$

$$S_n=\frac{1\cdot(1-(-2)^n)}{1-(-2)}=\frac{1-(-2)^n}{3}$$

(2) $a_n=3\times\left(\frac{1}{3}\right)^{n-1}$

$$S_n=\frac{3\cdot\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}=\frac{9}{2}\left(1-\left(\frac{1}{3}\right)^n\right)$$

P395

31

(1) $S=100(1+0.1\times5)=150$ (만원)

(2) $S=100(1+0.1)^5=160$ (만원)

4-11

$a_n=a\times r^{n-1}\quad(r>0)$

$\left.\begin{array}{l} ar+ar^3=30 \\ ar^3+ar^5=270 \end{array}\right\}\times r^2$

$r^2=9\quad r=3$

$3a+27a=30a=30$

$a=1$

$a_n=1\times3^{n-1}$

$$S_{10}=\frac{1\cdot(3^{10}-1)}{3-1}=\frac{1}{2}(3^{10}-1)$$

4-21

(1) $a_n=3\times2^{n-1}$

$$S_n=\frac{3\cdot(2^n-1)}{2-1}=3(2^n-1)=189$$

$2^n-1=63$

$2^n=64\qquad n=6$

(2) $a_n=(\sqrt{3}-1)\times(\sqrt{3})^{n-1}$

$$S_8=\frac{(\sqrt{3}-1)\{(\sqrt{3})^8-1\}}{\sqrt{3}-1}=80$$

(3) $a_n = a \times r^{n-1}$ (r: 자연수)

$S_2 = a + ar = 10$

$S_4 = \underline{a + ar} + \underline{\underline{ar^2 + ar^3}} = 2570$

$r^2(\underline{a+ar}) = 2560$

$r^2 = 256 \qquad r = 16$

4-31

$a_n = a \times r^{n-1}$

$a + ar^3 = 27 \qquad \dfrac{1+r^3}{r+r^2} = \dfrac{3}{2}$

$ar + ar^2 = 18$

$2r^3 - 3r^2 - 3r + 2 = 0$

$$\begin{array}{r|rrrr} -1 & 2 & -3 & -3 & 2 \\ & & -2 & 5 & -2 \\ \hline & 2 & -5 & 2 & \boxed{0} \end{array}$$

$(r+1)(2r^2 - 5r + 2) = 0$

$\underbrace{(r+1)}_{\neq 0}(2r-1)(\underline{r-2}) = 0$

$r = 2$; $a + 8a = 27 \qquad a = 3$

$a_1 + a_2 = 3 + 6 = 9$

$r = \frac{1}{2}$; $a + \dfrac{a}{8} = \dfrac{9a}{8} = 27 \qquad a = 24$

$a_1 + a_2 = 24 + 12 = 36$

5-1 200

$$a_n = a \cdot r^{n-1}$$

$$S_5 = \frac{a(r^5-1)}{r-1} = 5$$

$$S_{10} = \frac{a(r^{10}-1)}{r-1} = \frac{a(r^5-1)(r^5+1)}{r-1} = 20$$

$$r^5 = 3$$

$$S_{20} = \frac{a(r^{20}-1)}{r-1} = \frac{a(r^{10}-1)(r^{10}+1)}{r-1}$$

$$= 20 \cdot 10 = 200$$

5-2 576

$$a_n = a r^{n-1}$$

$$S_n = \frac{a(r^n-1)}{r-1} = 36$$

$$S_{2n} = \frac{a(r^{2n}-1)}{r-1} = \frac{a(r^n-1)(r^n+1)}{r-1} = 180$$

$$r^n+1 = 5 \qquad r^n = 4$$

$$S = S_{3n} - S_{2n} = 36 \cdot 21 - 36 \cdot 5 = 36 \cdot 16 = 576$$

$$\left(S_{3n} = \frac{a(r^{3n}-1)}{r-1} = \frac{a(r^n-1)(r^{2n}+r^n+1)}{r-1} = 36 \cdot (16+4+1) = 36 \cdot 21 \right)$$

5-3140

$a_n = a \cdot r^{n-1}$, $S_n = \dfrac{a(r^n-1)}{r-1}$

$S_{2k} = 4S_k$

$\dfrac{a(r^{2k}-1)}{r-1} = 4 \times \dfrac{a(r^k-1)}{r-1}$ $r^k+1=4$

$r^k = 3$

$S_{4k} = \dfrac{a(r^{4k}-1)}{r-1}$

$= \dfrac{a(r^k-1)(r^k+1)(r^{2k}+1)}{r-1} = 40 \cdot S_k$

6-111

$S_n = 3^{n+k} - 3$ $a_1 = S_1 = 3^{1+k} - 3$

$-\underline{|\ S_{n-1} = 3^{n-1+k} - 3}$

$a_n = \underline{3^{n-1+k}(3-1)}$ $(n \geq 2)$

$2 \cdot 3^k = 3 \cdot 3^k - 3$

$3^k = 3$ $k = 1$

6-2) ⑤

$$S_n + 1 = 10^n$$

$S_n = 10^n - 1$ $\qquad a_1 = S_1 = 10 - 1 = 9$

$-)\ S_{n-1} = 10^{n-1} - 1$

$a_n = 10^{n-1}(10-1) = 9 \cdot 10^{n-1} \quad (n \geq 2)$

$a_n = 9 \times 10^{n-1}$ $\qquad p = 9,\ q = 10 \qquad p + q = 19$

6-3) 12

$$a_n = ar^{n-1}$$

$n = 1;\quad a_1 = a = 2^3 \qquad a = 8$

$n = 2;\quad a_1 \cdot a_2 = a \cdot ar = 2^8$

$r = 2^2 = 4 \qquad a + r = 12$

7-1)

(1) $a_n = 1 + (n-1)(-2) = -2n + 3$

$$b_n = 3^{-2n+3}$$

$\{b_n\}: 3, 3^{-1}, 3^{-3}, \ldots$

첫째항 3 이고 공비가 $\frac{1}{9}$ 인 등비수열

(2) $a_n = 1 \times \left(\frac{1}{2}\right)^{n-1}$

$b_n = \log\left(\frac{1}{2}\right)^{n-1} = 0 + (n-1)\log\frac{1}{2}$

첫째항: 0, 공차가 $\log\frac{1}{2}$인 등차수열

<u>7-21</u>

$a_n = a \times r^{n-1} \quad (r \neq 1)$

㉠ $\{b_n\}$: $ar^4, ar^9, ar^{14}, \cdots$

㉡ $c_n = ar^n - ar^{n-1} = ar^{n-1}(r-1)$

$= (ar-a)\cdot r^{n-1}$

㉢ $S_n = \dfrac{a(r^n-1)}{r-1}$

$$d_n = \frac{a(r^{5n+5}-1)}{r-1} - \frac{a(r^{5n}-1)}{r-1}$$

$$= \frac{ar^{5n+5} - ar^{5n}}{r-1}$$

$$= \frac{ar^{5n}(r^5-1)}{r-1}$$

7-3) 800

$$a_n = 1\cdot 3^{n-1}$$

$$\{a_n\} : 1,\ 3,\ 9,\ 27,\ 81,\ 243,\ 729,\ \cdots$$

$$\{b_n\} : 3^2,\ 3^6,\ 3^{10},\ \cdots$$

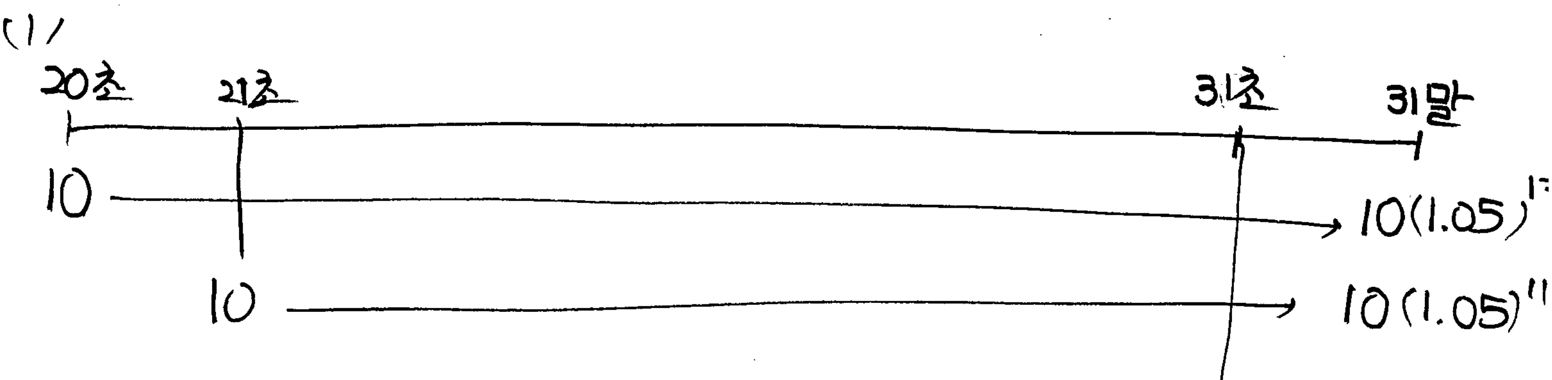
$$\{\log_3 b_n\} : 2,\ 6,\ 10,\ \cdots$$

$$\log_3 b_n = 2 + (n-1)\cdot 4$$

$$S = \frac{20(2+78)}{2} = 800$$

8-1)

(1)



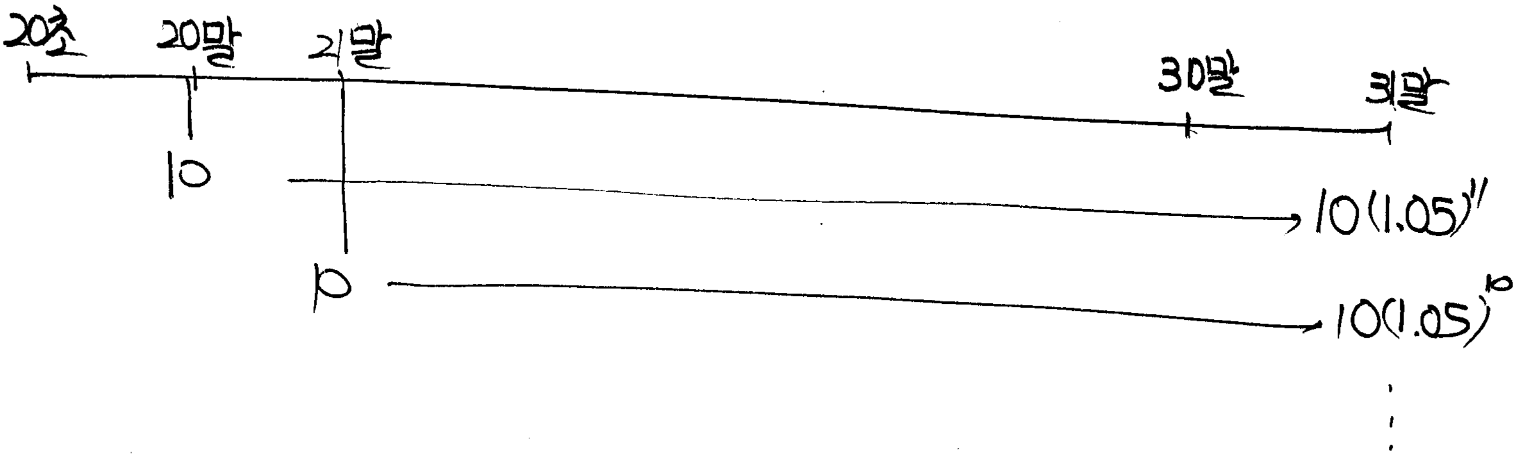
$$S = 10(1.05) + 10(1.05)^2 + \cdots + 10(1.05)^{12}$$

$10 \rightarrow 10(1.05)$

$$= \frac{10(1.05)\left((1.05)^{12}-1\right)}{1.05-1}$$

$$= 10\cdot 21\cdot 0.8 = 168$$

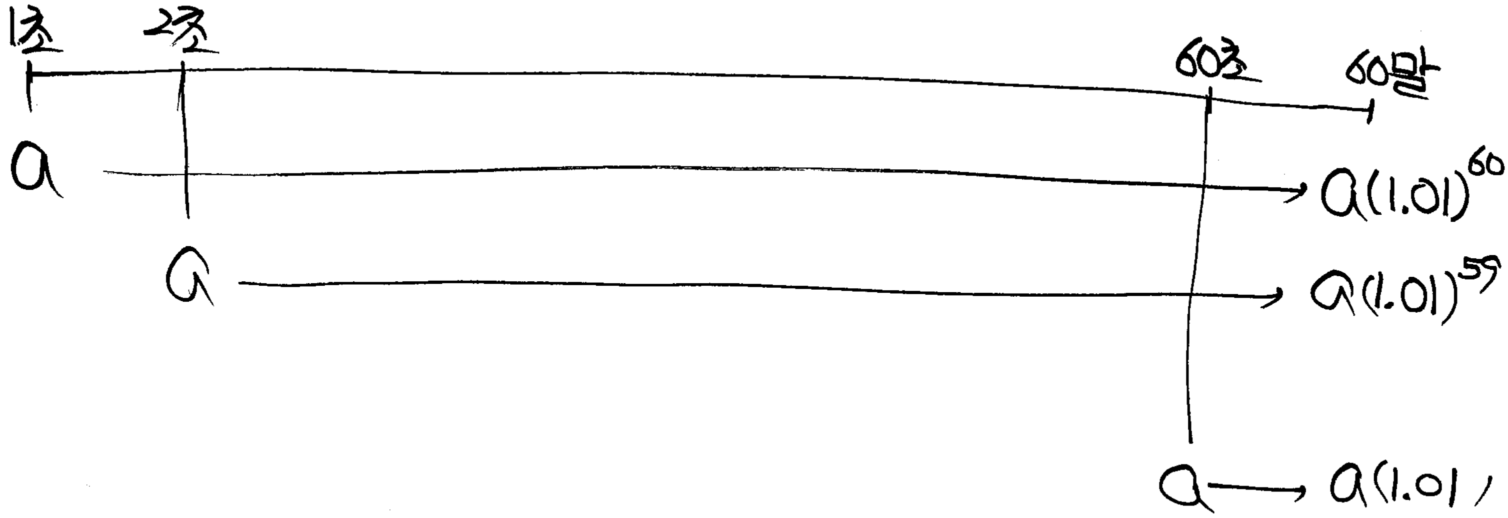
(2)



$$S = 10 + 10(1.05) + \cdots + 10(1.05)^{11}$$

$$= \frac{10\{(1.05)^{12} - 1\}}{1.05 - 1} = 10 \times 20 \times 0.8 = 160$$

8-21 5

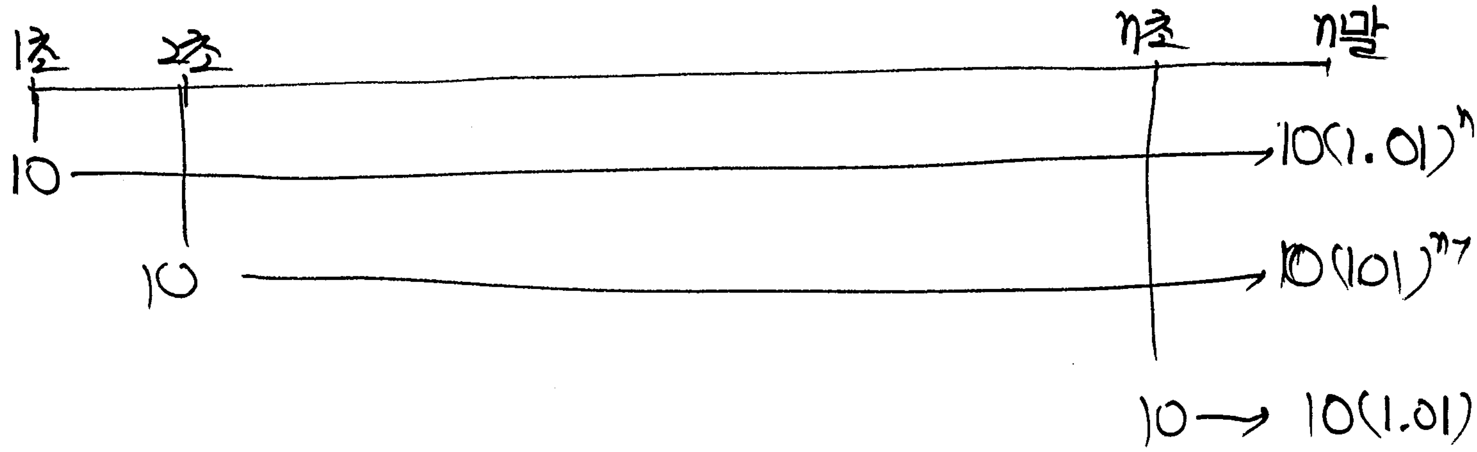


$$a(1.01) + a(1.01)^2 + \cdots + a(1.01)^{60} = 404$$

$$\frac{a(1.01)\{(1.01)^{60} - 1\}}{1.01 - 1} = a \times 101 \times (0.8) = 404$$

$$a \times \frac{8}{10} = 4 \qquad a = 4 \times \frac{10}{8} = 5$$

8-3) 70



$$10(1.01)+10(1.01)^2+\cdots+10(1.01)^n=1010$$

$$\frac{10(1.01)((1.01)^n-1)}{1.01-1}=1010$$

$$(1.01)^n-1=1$$

$$(1.01)^n=2$$

$$\log(1.01)^n=\log 2$$

$$n\log 1.01=\log 2$$

$$n=70$$

$$\begin{array}{r} 70 \\ 43\overline{)3010} \\ 301 \\ \hline 0 \end{array}$$

10-1| ②

$a_1 = 1 \quad \left(\frac{a_{n+1}}{a_n}\right)^3 = 3^2$

$a_n = 1 \times r^{n-1} \qquad r^3 = 9 = 3^2$

$a_{10} = r^9 = r^3 \times r^3 \times r^3 = 3^2 \times 3^2 \times 3^2 = 3^6$

10-2| ④

$a_n = 1 \cdot r^{n-1} \qquad a_6 = r^5 = 4$

$x_1 \cdot x_2 \cdot x_3 \cdot x_4 = r \cdot r^2 \cdot r^3 \cdot r^4 = r^{10} = 16$

10-3| 64

$a_n = ar^{n-1}$

$a_3 \cdot a_4 \cdot a_5 = ar^2 \cdot ar^3 \cdot ar^4 = a^3 r^9 = 2$

$a_8 \cdot a_9 \cdot a_{10} = ar^7 \cdot ar^8 \cdot ar^9 = a^3 r^{24} = 4$

$r^{15} = 2 \rightarrow r = 2^{\frac{1}{15}}$

$a_1 \cdot a_2 \cdots a_{11} \cdot a_{12}$

$= a^{12} \cdot r^{66}$

$= (a^3 \cdot r^9)^4 \cdot r^{30}$

$= 2^4 \cdot 2^2 = 64$

10-4 | 48

$a_n = a + (n-1)d$

$b_n = -2 \cdot r^{n-1}$

$a = -2r^2$, $a + d = -2$, $a + 2d = -2r$

$-4 = -2r^2 - 2r$

$r^2 + r - 2 = 0$

$(r-1)(r+2) = 0$

$r = -2$

$a = -8$, $d = 6$

$a_n = -8 + (n-1) \cdot 6$, $b_n = -2 \cdot (-2)^{n-1}$

$a_5 = -8 + 24 = 16$

$b_5 = -2 \cdot 16 = -32$

$a_5 - b_5 = 48$

10-5 | 20

$a_n = 1 \cdot 2^{n-1}$ $a_{2n} = 2^{2n-1}$

$b_n = (2^{2n-1})^2 = 2^{4n-2}$

$b_1 = b = 2^2 = 4$

$b_2 = 2^6$

$r = 16$

$b + r = 20$

10-6| 2

$a_n = ar^{n-1}$

$3a_1 - a_2 = 3a - ar = 10$

$3a_2 - a_3 = 3ar - ar^2 = 10 \times (-2)$

$r = -2$; $\boxed{a = 2}$

10-7| -8

$x^2 - 2x + k = 0$ 두근 α, β

$\alpha + \beta = 2$

$\alpha\beta = k$

$k = -8$

$\frac{\alpha}{\beta}, \alpha+\beta, \alpha\beta$: 등비수열

$(\alpha+\beta)^2 = \alpha^2$

$4 = \alpha^2$

$\alpha = -2, \beta = 4$

10-8| 40

$(\cancel{\sqrt{3}}-1) + (\cancel{3}-\cancel{\sqrt{3}}) + (\cancel{3\sqrt{3}}-\cancel{3}) + (9-\cancel{3\sqrt{3}}) = 8$

$x^2 - 3x - k = 0$ 두근 $8, -5$

$-k = -40$

$k = 40$

10-9)

$a_n = 1 \cdot 3^{n-1}$

$S_n = \frac{1 \cdot (3^n - 1)}{3-1} = \frac{1}{2} \cdot 3^n - \frac{1}{2}$

$p = \frac{1}{2}$

10-10)

1) $a_n = 10^n - 1$

$S_{10} = \frac{10(10^{10}-1)}{10-1} - 10 = \frac{1}{9}(10^{11} - 100)$

2) $a_n = \frac{5}{9}(10^n - 1)$

$S_n = \frac{5}{9}\left(\frac{10(10^n-1)}{10-1} - n\right)$

10-11) ③

$a_n = ar^{n-1}$

$a + ar + ar^2 + ar^3 + ar^4 = \frac{31}{2}$

$a^5 r^{10} = 32$

$(ar^2)^5 = 2^5 \qquad ar^2 = 2$

$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4}$

$= \frac{a(r^4 + r^3 + r^2 + r + 1)}{a^2 r^4} = \frac{\frac{31}{2}}{4} = \frac{31}{8}$

10-121 ⑤

$$a_n = a\cdot(\sqrt{2})^{n-1}$$

$$\frac{a_{10}-a_9}{a_{10}+a_9}+\frac{a_5+a_4}{a_5-a_4}$$

$$=\frac{a(\sqrt{2})^9-a(\sqrt{2})^8}{a\cdot(\sqrt{2})^9+a(\sqrt{2})^8}+\frac{a(\sqrt{2})^4+a(\sqrt{2})^3}{a(\sqrt{2})^4-a(\sqrt{2})^3}$$

$$=\frac{\sqrt{2}-1}{\sqrt{2}+1}+\frac{\sqrt{2}+1}{\sqrt{2}-1}=(\sqrt{2}-1)^2+(\sqrt{2}+1)^2$$

$$=6$$

10-131 ①

$$f(x)=x^3+2x^2+ax+1$$

$$f(0)=1$$

$$f(1)=1+2+a+1=a+4$$

$$f(-2)=-8+8-2a+1=-2a+1$$

$$(a+4)^2=-2a+1$$

$$a^2+10a+15=0$$

$\therefore$ (두 근의 합) $=-10$

10-141 ⑤

$$x^{10}+x^{9}+\cdots+x+1=(x-1)f(x)+11$$

$x=2$; $2^{10}+2^{9}+\cdots+2+1=f(2)+11$

$$\frac{1\cdot(2^{11}-1)}{2-1}=f(2)+11$$

$$f(2)=2^{11}-12$$

㉠ $N=2^{8}$

$(1, 2, 2^{2}, \cdots, 2^{8})$

개수 : $(8+1)$

합 : $(1+2+2^{2}+\cdots+2^{8})$

㉡ $N=2^{3}\cdot 3^{4}$

$(1, 2, 2^{2}, 2^{3})$ $(1, 3, 3^{2}, 3^{3}, 3^{4})$

개수 : $(3+1)(4+1)$

$\boxed{\text{합 : } (1+2+2^{2}+2^{3})(1+3+3^{2}+3^{3}+3^{4})}$

10-15 ④

$$6^{10} = 2^{10} \times 3^{10}$$

$$S = (1+2+2^2+\cdots+2^{10})(1+3+3^2+\cdots+3^{10})$$

$$= \frac{1\cdot(2^{11}-1)}{2-1} \times \frac{1\cdot(3^{11}-1)}{3-1} = \frac{1}{2}(2A-1)(3B-1)$$

10-16 ⑤

$$10^{10} = 2^{10} \times 5^{10}$$

$$S = (\cancel{1}+\cancel{2}+\cancel{2^2}+\underbrace{2^3+2^4}+\cancel{2^5}+\cdots+\cancel{2^{10}})(1+5+5^2+\cdots+5^{10})$$

$$= 24 \times \frac{1\cdot(5^{11}-1)}{5-1} = 6\times(5^{11}-1)$$

10-17 $\frac{21}{4}$

a, b, c : 등비수열

$b^2 = ac$ (4) $b^3 = 1$ $b = 1$

$a + c = \frac{5}{2}$, $ac = 1$

$$a^2+b^2+c^2$$

$$= 1 + (a+c)^2 - 2ac$$

$$= 1 + \frac{25}{4} - 2 = \frac{21}{4}$$

3-11

(1) $b_n = \dfrac{a_1+a_2+\cdots+a_n}{n}$

$$b_1+b_2+b_3+\cdots+b_n = n^2 \qquad b_1 = 1^2 = 1$$

$$-\underline{\left| b_1+b_2+b_3+\cdots+b_{n-1} = (n-1)^2\right.}$$

$$b_n = 2n-1 \quad (n \geq 2)$$

$$\therefore\ b_n = \frac{a_1+a_2+\cdots+a_n}{n} = 2n-1$$

$$a_1+a_2+\cdots+a_n = 2n^2-n$$

$$\sum_{k=1}^{20} a_k = a_1+a_2+\cdots+a_{20} = 2\times 20^2-20 = 800-20$$

$$= 780$$

(2) $n=1$; $a_1 = 1\cdot 2\cdot 3 = 6$

$$na_1+(n-1)a_2+(n-2)a_3+\cdots+2a_{n-1}+a_n = \underline{n(n+1)}(n+2)$$

$$-\underline{\left| (n-1)a_1+(n-2)a_2+(n-3)a_3+\cdots+a_{n-1} = (n-1)\cdot\underline{n(n+1)}\right.}$$

$$a_1+a_2+a_3+\cdots+a_n = 3n(n+1) \quad (n \geq 2)$$

$$a_1+a_2+a_3+\cdots+a_n = 3n(n+1)$$

$$\sum_{k=1}^{20} a_k = a_1+a_2+\cdots+a_{20} = 3\cdot 20\cdot 21 = 1260$$

3-21

(1) $a_1+a_2+a_3+\cdots\cdots+a_n=2n^2 \qquad a_1=2$

$-\underline{)\ a_1+a_2+a_3+\cdots+a_{n-1} \quad =2(n-1)^2}$

$$a_n=4n-2 \quad (n\geq 2)$$

$$a_n=4n-2. \qquad a_{2k}=8k-2$$

$$\sum_{k=1}^{10} a_{2k}=\sum_{k=1}^{10}(8k-2)=8\sum_{k=1}^{10}k-\sum_{k=1}^{10}2$$

$$=\cancel{8}^{4}\cdot\frac{10\cdot 11}{\cancel{2}} \;-20 \;=420$$

(2) $\sum_{n=1}^{100} na_n=a_1+2a_2+3a_3+\cdots+100a_{100}=500$

$-\underline{)\ \sum_{n=1}^{99} na_{n+1}= \quad a_2+2a_3+\cdots\cdots+99a_{100}=200}$

$$a_1+a_2+a_3+\cdots\cdots+a_{100}=300$$

3-31

$P_n=a_1\times a_2\times a_3\times\cdots\times a_n=3^{n(n-1)} \qquad P_1=a_1=1$

$\div\underline{)\ P_{n-1}=a_1\times a_2\times a_3\times\cdots\cdots\times a_{n-1}=3^{(n-1)\cdot(n-2)}}$

$$a_n=3^{2(n-1)} \quad (n\geq 2)$$

$$a_n=3^{2(n-1)}$$

$$a_{100}=3^{198}=3^m \qquad m=198$$

4-11

(1) $1,\ 2,\ 4,\ 7,\ 11,\ \cdots\cdots$

$1 \quad 2 \quad 3 \quad 4 \longrightarrow b_n = n$

$$a_n = 1 + (1+2+3+\cdots\cdots+(n-1))$$

$$= 1 + \frac{(n-1)\cdot n}{2} = \frac{n^2-n}{2} + 1$$

$$S_n = \sum_{k=1}^{n} a_k = \frac{1}{2}\sum_{k=1}^{n} k^2 - \frac{1}{2}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= \frac{1}{2}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{1}{2}\cdot\frac{n(n+1)}{2} + n$$

(2) $2,\ 3,\ 5,\ 9,\ 17,\ \cdots\cdots$

$1 \quad 2 \quad 4 \quad 8 \longrightarrow b_n = 1\cdot 2^{n-1}$

$$a_n = 2 + (\underbrace{1+2+2^2+\cdots\cdots+2^{n-2}}_{n-1\text{개}})$$

$$= 2 + \frac{1\cdot(2^{n-1}-1)}{2-1} = 2^{n-1}+1$$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} 2^{k-1} + \sum_{k=1}^{n} 1$$

$$= \frac{1\cdot(2^n-1)}{2-1} + n = 2^n + n - 1$$

4-2) 91

$$\cancel{f(1)} = 1$$
$$\cancel{f(2)} - \cancel{f(1)} = 2$$
$$\cancel{f(3)} - \cancel{f(2)} = 4$$
$$\vdots$$
$$+\ \ f(10) - \cancel{f(9)} = 18$$

$$f(10) = 1 + \frac{9 \cdot 20}{2} = 91$$

4-3) 605

n번째 흰돌 : n^2

검은돌 : $4n$

$$a_n = n^2 + 4n$$

$$S = \sum_{k=1}^{10} a_k = \sum_{k=1}^{10} k^2 + 4\sum_{k=1}^{10} k$$

$$= \frac{10 \cdot 11 \cdot 21}{6} + 4 \cdot \frac{10 \cdot 11}{2} = 385 + 220 = 605$$

$$\frac{1}{A} - \frac{1}{B} = \frac{B-A}{AB}$$

$$\Rightarrow \frac{1}{AB} = \frac{1}{B-A}\left(\frac{1}{A} - \frac{1}{B}\right)$$

P435

EX) $a_n = \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$

$$\sum_{k=1}^{n} a_k = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{2k-1} - \frac{1}{2k+1}\right)$$

$$= \frac{1}{2}\left\{\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \cdots + \left(\cancel{\frac{1}{2n-1}} - \frac{1}{2n+1}\right)\right\}$$

$$= \frac{1}{2}\cdot\frac{2n}{2n+1} = \frac{n}{2n+1}$$

P436

EX) $a_n = \frac{1}{\sqrt{3n+2}+\sqrt{3n-1}} = \frac{\sqrt{3n+2}-\sqrt{3n-1}}{3}$

$$\sum_{k=1}^{n} a_k = \frac{1}{3}\sum_{k=1}^{n}\left(\sqrt{3k+2}-\sqrt{3k-1}\right)$$

$$= \frac{1}{3}\left[\left(\cancel{\sqrt{5}}-\sqrt{2}\right) + \left(\cancel{\sqrt{8}}-\cancel{\sqrt{5}}\right) + \left(\cancel{\sqrt{11}}-\cancel{\sqrt{8}}\right) + \cdots + \left(\sqrt{3n+2}-\cancel{\sqrt{3n-1}}\right)\right]$$

$$= \frac{1}{3}\left(\sqrt{3n+2}-\sqrt{2}\right)$$

P438

EX)

$$S = 1\times1 + 3\times3 + 5\times3^2 + \cdots + 19\times3^9$$

$$-\underline{\;3S = \qquad 1\times3 + 3\times3^2 + 5\times3^3 + 17\times3^9 + 19\times3^{10}}$$

$$-2S = 1 + \underbrace{2\times3 + 2\times3^2 + \cdots + 2\times3^9}\; - 19\times3^{10}$$

$$-2S = 1 + \frac{\cancel{2}\times 3(3^9-1)}{\cancel{3-1}} - 19\times 3^{10}$$

$$-2S = 1 + \underline{3^{10}} - 3 - \underline{19}\cdot 3^{10} = -18\cdot 3^{10} - 2$$

$$S = 9\cdot 3^{10} + 1$$

P439

EX)

$$\frac{1}{1} / \frac{1}{2}, \frac{2}{1} / \frac{1}{3}, \frac{2}{2}, \frac{3}{1} / \frac{1}{4}, \frac{2}{3}, \frac{3}{2}, \frac{4}{1} / \cdots$$

n군

$$\frac{1}{n}, \frac{2}{n-1}, \frac{3}{n-2}, \cdots\cdots, \frac{n}{1}$$

$\frac{y}{x} \Rightarrow x+y = n+1$ (일정)

항수 : n

$$1+2+3+\cdots\cdots+m = \frac{m(m+1)}{2} \le 100$$

$$m=13\ ;\ \frac{13\cdot 14}{2} = 91$$

14군 9번째 $\Rightarrow \frac{9}{6}$

P441 EX) $a_{2020} = 7$

n	1	2	3	4	5	6	7	8
2^n의 일의자리수	2	4	8	6	2	4	8	6
3^n의 일의자리수	3	9	7	1	3	9	7	1
a_n	5	3	5	7	5	3	5	7

P441

<u>11</u>

(1) $a_n = \dfrac{1}{(n+1)(n+2)} = \left(\dfrac{1}{n+2} - \dfrac{1}{n+3}\right)$

$$\sum_{k=1}^{n} a_k = \sum_{k=1}^{n}\left(\frac{1}{k+1} - \frac{1}{k+2}\right)$$

$$= \left(\frac{1}{2} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \cdots + \left(\cancel{\frac{1}{n+1}} - \frac{1}{n+2}\right)$$

$$= \frac{1}{2} - \frac{1}{n+2}$$

(2) $a_n = \dfrac{1}{\sqrt{n} + \sqrt{n+1}} = \dfrac{\sqrt{n+1} - \sqrt{n}}{(\sqrt{n+1} + \sqrt{n})(\sqrt{n+1} - \sqrt{n})} = \sqrt{n+1} - \sqrt{n}$

$$\sum_{k=1}^{48} a_k = \sum_{k=1}^{48}\left(\sqrt{k+1} - \sqrt{k}\right)$$

$$= (\cancel{\sqrt{2}} - \sqrt{1}) + (\cancel{\sqrt{3}} - \cancel{\sqrt{2}}) + \cdots + (\sqrt{49} - \cancel{\sqrt{48}})$$

$$= -\sqrt{1} + \sqrt{49} = -1 + 7 = 6$$

<u>5-11</u>

(1) $a_n = \dfrac{1}{2n \cdot (2n+2)} = \dfrac{1}{4n(n+1)} = \dfrac{1}{4}\left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$

$$S_n = \frac{1}{4}\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+1}\right)$$

$$= \frac{1}{4}\left[\left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \cdots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)\right]$$

$$= \frac{1}{4}\left(\frac{n+1-1}{n+1}\right) = \frac{n}{4(n+1)}$$

(2) $a_n = \dfrac{1}{(2n)^2 - 1} = \dfrac{1}{(2n-1)(2n+1)} = \dfrac{1}{2}\left(\dfrac{1}{2n-1} - \dfrac{1}{2n+1}\right)$

$$S_n = \sum_{k=1}^{n} \frac{1}{2}\left(\frac{1}{2k-1} - \frac{1}{2k+1}\right)$$

$$= \frac{1}{2}\left[\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \cdots + \left(\cancel{\frac{1}{2n-1}} - \frac{1}{2n+1}\right)\right]$$

$$= \frac{1}{2}\left(\frac{2n+\cancel{1}-\cancel{1}}{2n+1}\right) = \frac{n}{2n+1}$$

(3) $a_n = \dfrac{1}{n(n+1)(n+2)} = \dfrac{1}{2}\left(\dfrac{1}{n(n+1)} - \dfrac{1}{(n+1)(n+2)}\right)$

$$S_n = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k(k+1)} - \frac{1}{(k+1)(k+2)}\right)$$

$$= \frac{1}{2}\left[\left(\frac{1}{1\cdot 2} - \cancel{\frac{1}{2\cdot 3}}\right) + \left(\cancel{\frac{1}{2\cdot 3}} - \cancel{\frac{1}{3\cdot 4}}\right) + \cdots + \left(\cancel{\frac{1}{n(n+1)}} - \frac{1}{(n+1)(n+2)}\right)\right]$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$$

(4) $a_n = \dfrac{1}{1+2+\cdots+n} = \dfrac{2}{n(n+1)}$

$$= 2\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$S_n = 2\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+1}\right)$$

$$= 2\left[\left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \cdots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)\right]$$

$$= \frac{2n}{n+1}$$

5-21

(1) $a_1+a_2+\cdots+a_n=n^2+2n$ $\quad a_1=1+2=3$

$$-\underline{|\ a_1+a_2+\cdots+a_{n-1}=(n-1)^2+2(n-1)}$$

$$a_n=2n-1+2=2n+1 \quad (n\geq 2)$$

$\therefore a_n=2n+1$

$$\sum_{k=1}^{n}\frac{1}{(2k+1)(2k+3)}$$

$$=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{2k+1}-\frac{1}{2k+3}\right)$$

$$=\frac{1}{2}\left[\left(\frac{1}{3}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)+\cdots\cdots+\left(\cancel{\frac{1}{2n+1}}-\frac{1}{2n+3}\right)\right]$$

$$=\frac{1}{2}\cdot\frac{2n+\cancel{3}-\cancel{3}}{3(2n+3)}=\frac{n}{3(2n+3)}$$

(2) $a_1+a_2+\cdots\cdots+a_n=\frac{n(n+1)(n+2)}{3}$ $\quad a_1=\frac{1\cdot2\cdot3}{3}=2$

$$-\underline{|\ a_1+a_2+\cdots+a_{n-1}=\frac{(n-1)\cdot n(n+1)}{3}}$$

$$a_n=\frac{n(n+1)\cdot 3}{3}=n(n+1) \quad (n\geq 2)$$

$a_n=n(n+1)$

$$\sum_{k=1}^{n}\frac{1}{a_k}=\sum_{k=1}^{n}\frac{1}{k(k+1)}=\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+1}\right)$$

$$=\left(\frac{1}{1}-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)+\cdots+\left(\cancel{\frac{1}{n}}-\frac{1}{n+1}\right)$$

$$=\frac{n}{n+1}$$

5-3 ④

$x^2+4x-(2n-1)(2n+1)=0$ 두근 α_n, β_n

$$\alpha_n+\beta_n=-4$$

$$\alpha_n\beta_n=-(2n-1)(2n+1)$$

$$\frac{1}{\alpha_n}+\frac{1}{\beta_n}=\frac{\beta_n+\alpha_n}{\alpha_n\beta_n}=\frac{4}{(2n-1)(2n+1)}=2\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$$

$$(준식)=2\sum_{n=1}^{10}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$$

$$=2\left[\left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\cdots+\left(\cancel{\frac{1}{19}}-\frac{1}{21}\right)\right]$$

$$=\frac{40}{21}$$

6-11

(1) $$\frac{1}{\sqrt{k+1}+\sqrt{k-1}}=\frac{\sqrt{k+1}-\sqrt{k-1}}{2}$$

$$(준식)=\frac{1}{2}\sum_{k=2}^{99}\left(\sqrt{k+1}-\sqrt{k-1}\right)$$

$$=\frac{1}{2}\left[(\cancel{\sqrt{3}}-\sqrt{1})+(\cancel{\sqrt{4}}-\sqrt{2})+(\cancel{\sqrt{5}}-\cancel{\sqrt{3}})+(\sqrt{6}-\cancel{\sqrt{4}})+\cdots\right.$$

$$\left.\cdots+(\sqrt{99}-\cancel{\sqrt{97}})+(\sqrt{100}-\cancel{\sqrt{98}})\right]$$

$$=\frac{1}{2}(-1-\sqrt{2}+3\sqrt{11}+10)$$

$$=\frac{1}{2}(9-\sqrt{2}+3\sqrt{11})$$

(2) $$\frac{1}{\sqrt{2K+1}+\sqrt{2K-1}}=\frac{\sqrt{2K+1}-\sqrt{2K-1}}{2}$$

$$(\text{준식})=\frac{1}{2}\sum_{K=1}^{40}\left(\sqrt{2K+1}-\sqrt{2K-1}\right)$$

$$=\frac{1}{2}\left[(\cancel{\sqrt{3}}-\sqrt{1})+(\cancel{\sqrt{5}}-\cancel{\sqrt{3}})+\cdots\cdots+(\sqrt{81}-\cancel{\sqrt{79}})\right]$$

$$=\frac{1}{2}(-1+9)=4$$

(3) $$\frac{3}{\sqrt{3K+2}+\sqrt{3K-1}}=\frac{\cancel{3}\left(\sqrt{3K+2}-\sqrt{3K-1}\right)}{\cancel{3}}$$

$$(\text{준식})=\sum_{K=1}^{16}\left(\sqrt{3K+2}-\sqrt{3K-1}\right)$$

$$=\left[(\cancel{\sqrt{5}}-\sqrt{2})+(\cancel{\sqrt{8}}-\cancel{\sqrt{5}})+(\cancel{\sqrt{11}}-\cancel{\sqrt{8}})+\cdots+(\sqrt{50}-\cancel{\sqrt{47}})\right]$$

$$=-\sqrt{2}+5\sqrt{2}=4\sqrt{2}$$

(4) $$\frac{1}{(K+1)\sqrt{K}+K\sqrt{K+1}}$$

$$=\frac{1}{\sqrt{K}\sqrt{K+1}\left(\sqrt{K+1}+\sqrt{K}\right)}$$

$$=\frac{\left(\sqrt{K+1}-\sqrt{K}\right)}{\sqrt{K}\sqrt{K+1}}=\frac{1}{\sqrt{K}}-\frac{1}{\sqrt{K+1}}$$

$$(\text{준식})=\sum_{K=1}^{99}\left(\frac{1}{\sqrt{K}}-\frac{1}{\sqrt{K+1}}\right)=\left(\frac{1}{\sqrt{1}}-\cancel{\frac{1}{\sqrt{2}}}\right)+\left(\cancel{\frac{1}{\sqrt{2}}}-\cancel{\frac{1}{\sqrt{3}}}\right)+\cdots+\left(\cancel{\frac{1}{\sqrt{99}}}-\frac{1}{\sqrt{100}}\right)$$

$$=1-\frac{1}{10}=\frac{9}{10}$$

6-2| 48

$$\frac{1}{f(k)} = \frac{1}{\sqrt{k+1}+\sqrt{k}} = \sqrt{k+1}-\sqrt{k}$$

$$\sum_{k=1}^{n}(\sqrt{k+1}-\sqrt{k})$$

$$=(\cancel{\sqrt{2}}-\sqrt{1})+(\cancel{\sqrt{3}}-\cancel{\sqrt{2}})+(\cancel{\sqrt{4}}-\cancel{\sqrt{3}})+\cdots+(\sqrt{n+1}-\cancel{\sqrt{n}})$$

$$=\sqrt{n+1}-1=6$$

$$\sqrt{n+1}=7$$

$$n+1=49 \qquad n=48$$

6-3| ④

$$a_1^2+a_2^2+\cdots\cdots+a_n^2=n^2 \qquad a_1^2=1 \qquad a_1=1$$

$$-\underline{\left|\ a_1^2+a_2^2+\cdots+a_{n-1}^2=(n-1)^2\right.}$$

$$a_n^2=2n-1 \quad (n\geq 2)$$

$$a_n=\sqrt{2n-1}$$

$$\frac{1}{a_k+a_{k+1}}=\frac{1}{\sqrt{2k+1}+\sqrt{2k-1}}=\frac{\sqrt{2k+1}-\sqrt{2k-1}}{2}$$

$$(\text{준식})=\frac{1}{2}\sum_{k=1}^{60}(\sqrt{2k+1}-\sqrt{2k-1})$$

$$=\frac{1}{2}[(\cancel{\sqrt{3}}-\sqrt{1})+(\cancel{\sqrt{5}}-\cancel{\sqrt{3}})+\cdots+(\sqrt{121}-\cancel{\sqrt{119}})]$$

$$=\frac{1}{2}(-1+11)=5$$

7-11

(1) $S = 1 + 2\times 2 + 3\times 2^2 + 4\times 2^3 + \cdots\cdots + n\times 2^{n-1}$

$-)\ 2S = \quad 1\times 2 + 2\times 2^2 + 3\times 2^3 + \cdots \quad + (n-1)\times 2^{n-1} + n\times 2^n$

$-S = \underbrace{1 + 2 + 2^2 + \cdots\cdots + 2^{n-1}} - n\cdot 2^n$

$= \dfrac{1\cdot(2^n-1)}{2-1} - n\cdot 2^n$

$= \underline{2^n} - 1 - \underline{n\cdot 2^n}$

$S = (n-1)\cdot 2^n + 1$

(2) $S = 1 + 2x + 3x^2 + 4x^3 + \cdots\cdots + nx^{n-1}$

$-)\ xS = \quad x + 2x^2 + 3x^3 + \cdots\cdots + (n-1)\cdot x^{n-1} + nx^n$

$(1-x)S = 1 + x + x^2 + \cdots \quad + x^{n-1} - nx^n$

$(1-x)S = \dfrac{1\cdot(1-x^n)}{1-x} - nx^n$

$S = \dfrac{1-x^n}{(1-x)^2} - \dfrac{nx^n}{1-x}$

7-2 ②

$$S = 1\cdot\left(\frac{1}{2}\right)^1 + 2\cdot\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + \cdots + n\left(\frac{1}{2}\right)^n$$

$$-\,\frac{1}{2}S = \qquad 1\cdot\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^3 + \cdots + (n-1)\cdot\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^{n+1}$$

$$\frac{1}{2}S = \underbrace{\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots + \left(\frac{1}{2}\right)^n} - n\left(\frac{1}{2}\right)^{n+1}$$

$$= \frac{\cancel{\frac{1}{2}}\cdot\left(1-\left(\frac{1}{2}\right)^n\right)}{\cancel{1-\frac{1}{2}}} - n\left(\frac{1}{2}\right)^{n+1}$$

$$S = 2 - 2\left(\frac{1}{2}\right)^n - 2n\left(\frac{1}{2}\right)^{n+1}$$

$a=2,\ b=-2,\ c=-2 \qquad a+b+c=-2$

7-3 ②

$$S = 1 + 2\times3 + 3\times3^2 + 4\times3^3 + \cdots + 11\times3^{10}$$

$$-\,3S = \qquad 3 + 2\times3^2 + 3\times3^3 + \cdots + 10\times3^{10} + 11\times3^{11}$$

$$-2S = \underbrace{1 + 3 + 3^2 + \cdots + 3^{10}} - 11\cdot3^{11}$$

$$= \frac{1\cdot(3^{11}-1)}{3-1} - 11\cdot3^{11} = \frac{3^{11}-1-22\cdot3^{11}}{2}$$

$$S = \frac{21\cdot3^{11}+1}{4}$$