1. 함수의 극한

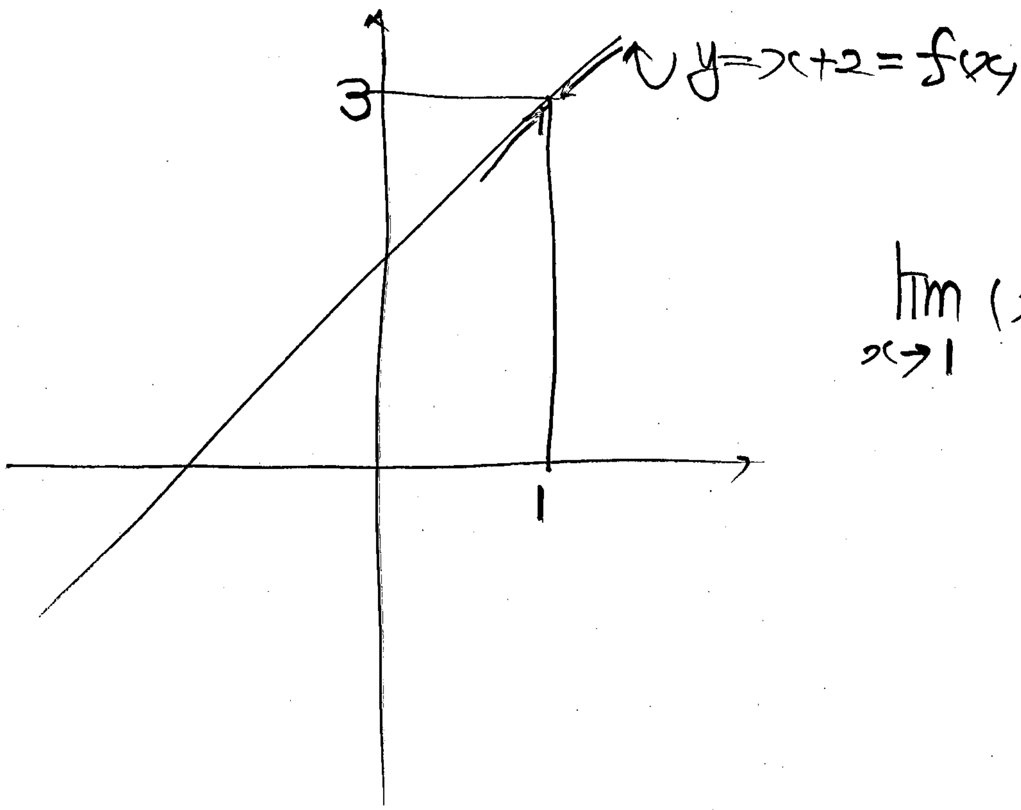
limit ⟹ lim (리미트)

∞ : 무한대

$$\lim_{x \to a} f(x) = L \quad \text{(L로 수렴)}$$

$$\lim_{x \to a-} f(x) = \lim_{x \to a+} f(x) = L$$

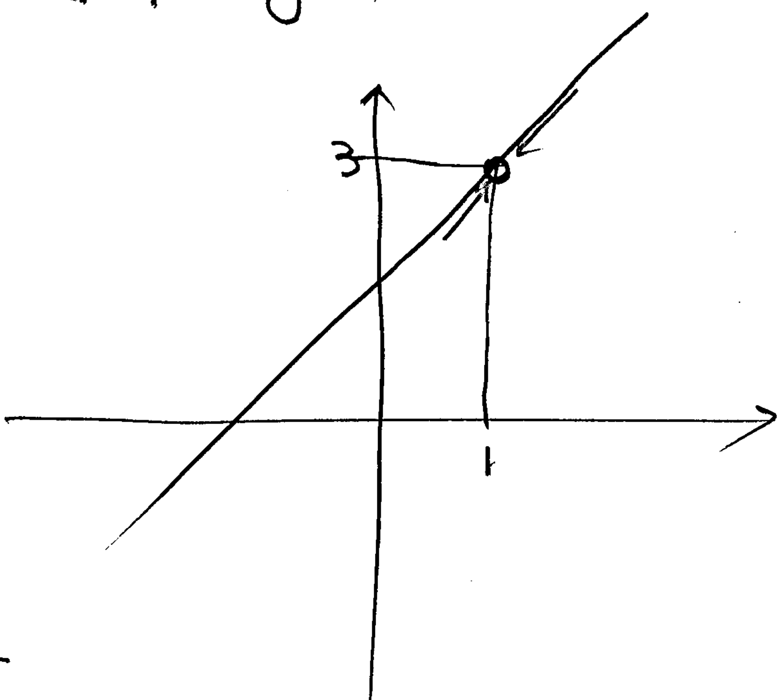
a의 좌극한　　a의 우극한



$$\lim_{x \to 1} (x+2) = 3$$

$$g(x) = \frac{x^2+x-2}{x-1} = \frac{(x-1)(x+2)}{x-1}$$

$x \neq 1 \quad g(x) = x+2$



$$\lim_{x \to 1} \frac{x^2+x-2}{x-1}$$

$$= \lim_{x \to 1} \frac{(x-1)(x+2)}{(x-1)}$$

$$= 3$$

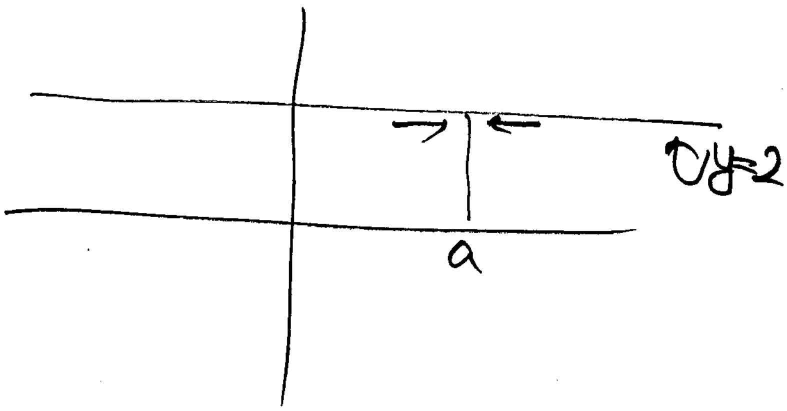
P14

EX) $\lim_{x \to 2} (x-1) = 1$

$$\boxed{f(x) : \text{다항함수} \iff \lim_{x \to a} f(x) = f(a)}$$

EX2)

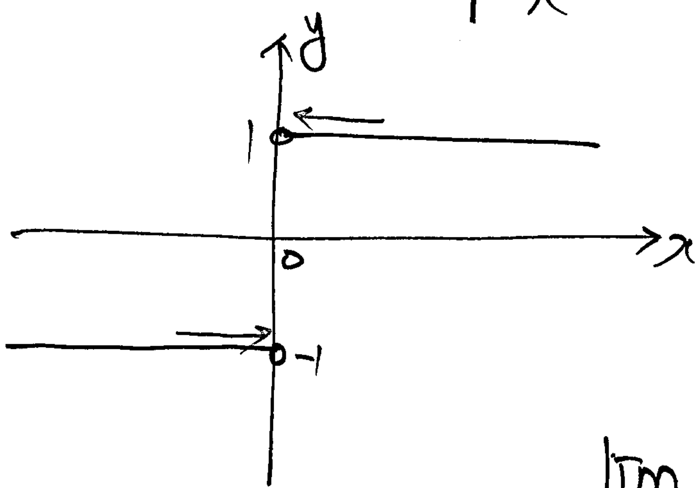
$$\lim_{x \to a} 2 = 2$$



P15

EX)

$$f(x) = \frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1 & (x > 0) \\ \frac{-x}{x} = -1 & (x < 0) \end{cases}$$



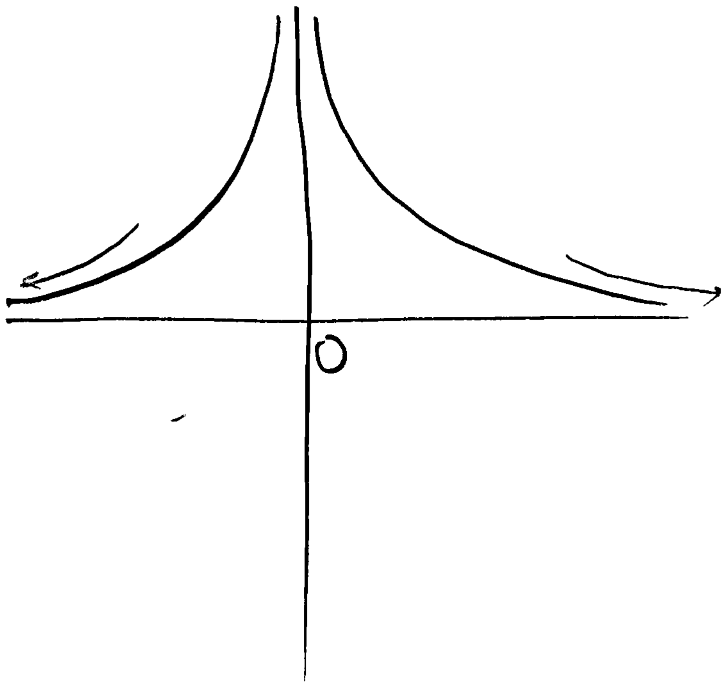
$$\lim_{x \to 0-} \frac{|x|}{x} = -1$$

$$\lim_{x \to 0+} \frac{|x|}{x} = 1$$

$$\lim_{x \to 0} \frac{|x|}{x} = \text{극한값 X}$$

P16

EX) $f(x) = \frac{1}{x^2}$



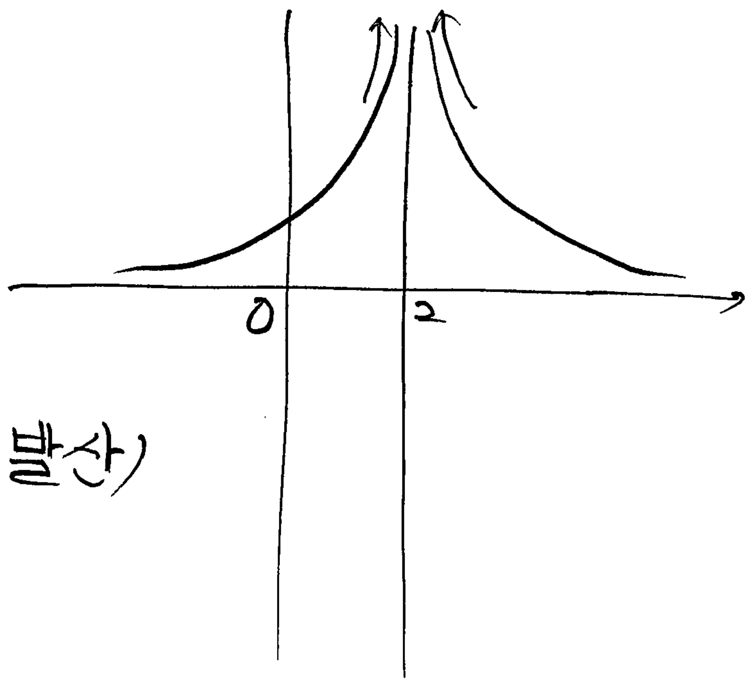
$$\lim_{x \to \infty} \frac{1}{x^2} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x^2} = 0$$

P18

EX)

$$f(x) = \frac{1}{|x-2|}$$



$$\lim_{x \to 2} \frac{1}{|x-2|} = \infty \quad (\text{발산})$$

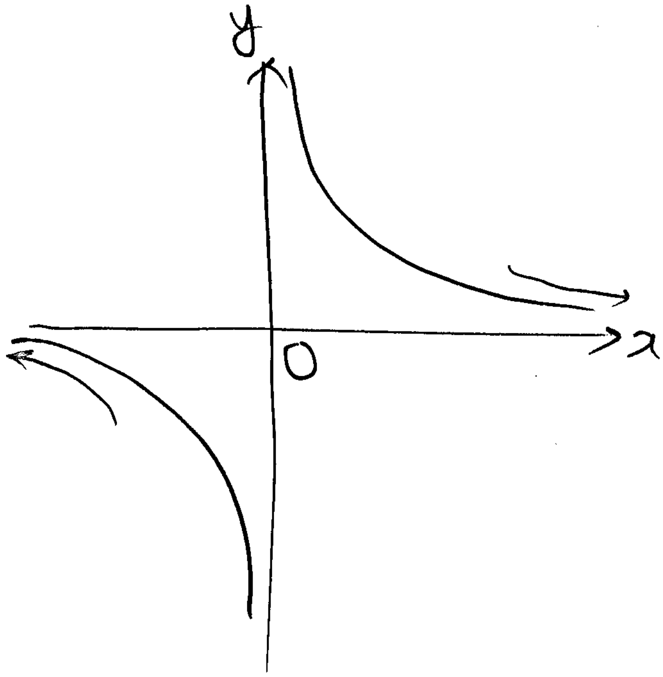
P19

1)

(1) $\lim_{x \to 1} (x^2+1) = 2$

(2) $\lim_{x \to 1} \frac{(x-1)(x+1)}{(x-1)} = 2$

2)



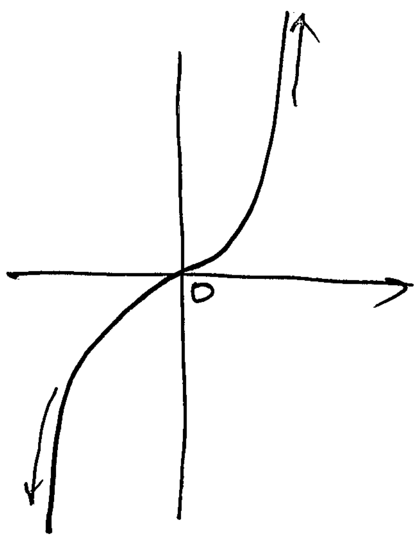
$y = \frac{1}{x}$

점근선 : $x=0$, $y=0$

(1) $\lim_{x \to \infty} \frac{1}{x} = 0$

(2) $\lim_{x \to -\infty} \frac{1}{x} = 0$

(3) $y = x^3$



$\lim_{x \to \infty} x^3 = \infty$

(4) $\lim_{x \to -\infty} x^3 = -\infty$

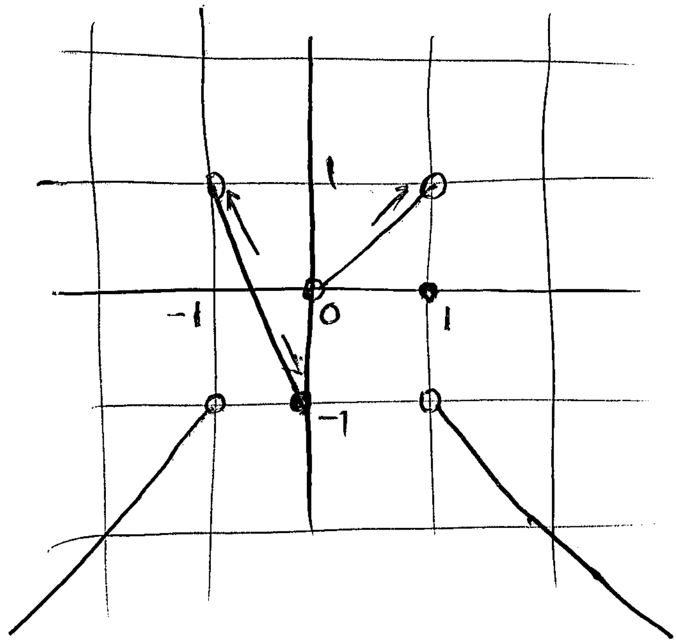
P21

1-11

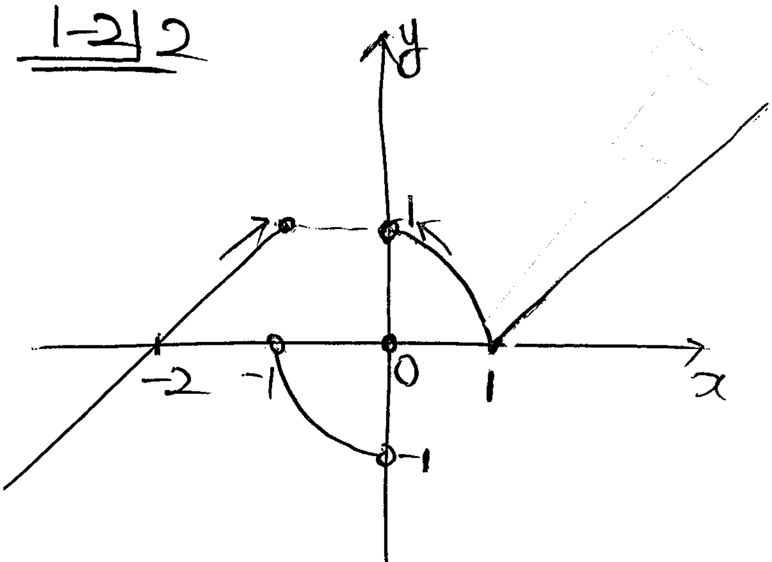
(1) $\lim_{x \to -1+} f(x) = 1$

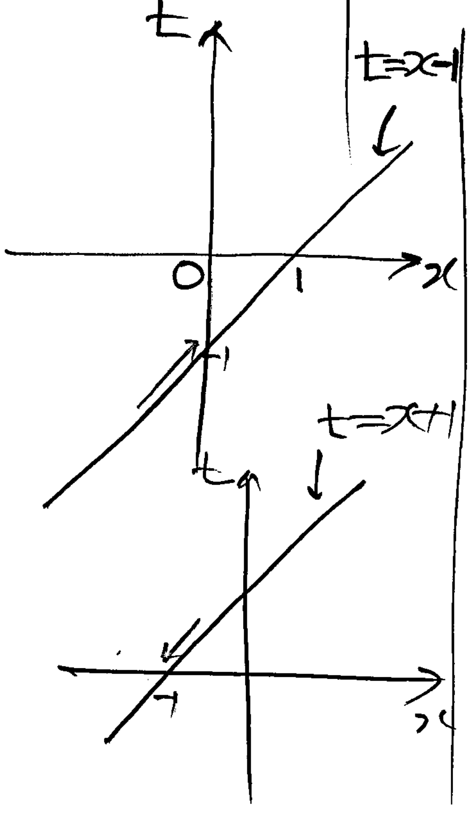
(2) $\lim_{x \to 0-} f(x) = -1$

(3) $\lim_{x \to 1-} f(x) = 1$



1-212





ア $\lim_{x \to 0-} f(x-1)$

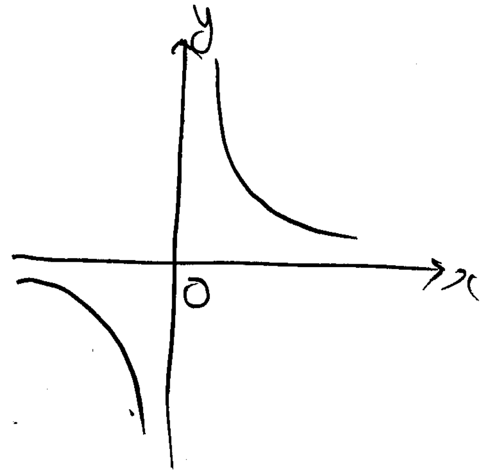
$= \lim_{t \to -1-} f(t) = 1$

イ $\lim_{x \to -1+} f(x+1)$

$= \lim_{t \to 0+} f(t) = 1$

$\therefore\ 1+1=2$

$y = \frac{k}{x}$ $(k > 0)$



정의역 : $\{x \mid x \neq 0$ 인 실수$\}$

치역 : $\{y \mid y \neq 0$ 인 실수$\}$

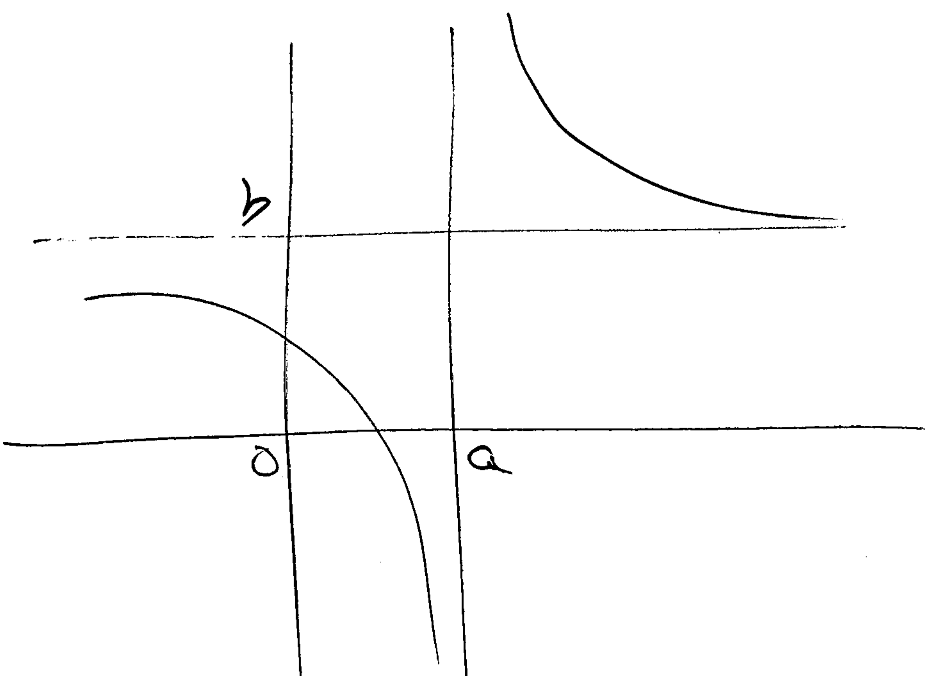
점근선 : $x = 0$, $y = 0$

x축 : a
평행이동
y축 : b

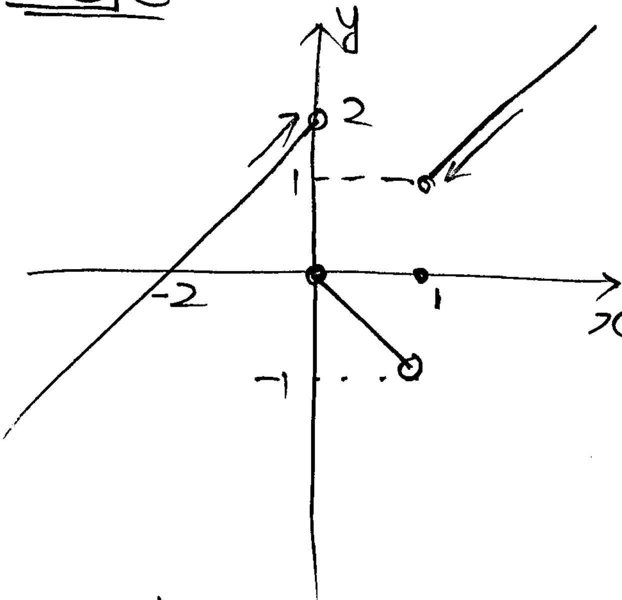
$\longrightarrow$ $y - b = \frac{k}{x - a}$

$y = \frac{k}{x-a} + b$

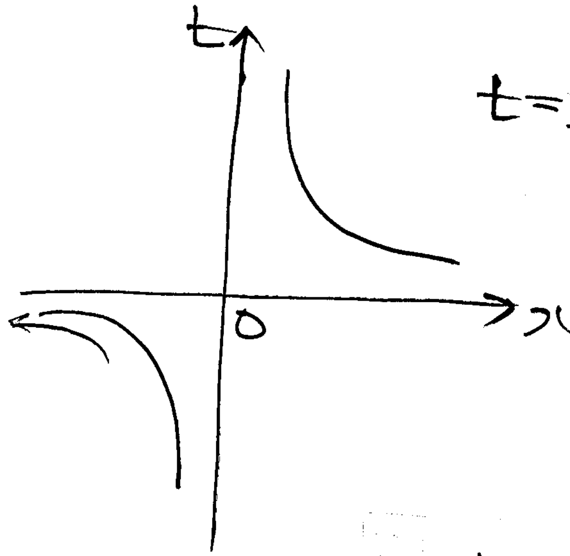
점근선 : $x = a$, $y = b$



1-3) 3

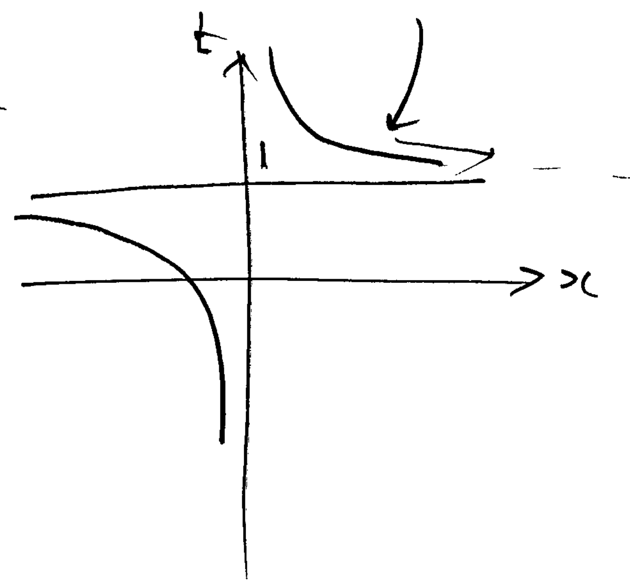


$t=\frac{1}{x}$



$$\lim_{x\to-\infty} f\left(\frac{1}{x}\right) = \lim_{t\to 0-} f(t) = 2$$

$t = 1+\frac{1}{x}$



$$\lim_{x\to\infty} f\left(1+\frac{1}{x}\right)$$

$$= \lim_{t\to 1+} f(t) = 1$$

$$2+1=3$$

2-11-3

$$\lim_{x\to -1-} f(x) = \lim_{x\to -1+} f(x)$$

$$-a+7 = 1+a^2$$

$$a^2+a-6=0$$

$$(a+3)(a-2)=0$$

$a<0$ 이므로 $\therefore a=-3$

2-21

$$\lim_{x\to 2} f(x) = 3$$

$$\lim_{x\to 2-} f(x) = \lim_{x\to 2+} f(x) = 3$$

$$\underline{4-2+b} = \underline{4+a=3}$$ $\boxed{a=-1,\ b=1}$

2-312

$$\lim_{x\to 1-} f(x) = \lim_{x\to 1+} f(x)$$

$$b = 1+a+1 \qquad b=a+2$$

$$a^2+b^2 = a^2+(a+2)^2$$
$$= 2a^2+4a+4$$
$$= 2(a^2+2a+1)+2 = 2(a+1)^2+2$$

$a=-1$ 일때 $\boxed{\text{최솟값} : 2}$

P25

EX)

(1) $\lim_{x\to 2}(2x^2+3) = 8+3 = 11$

(2) $\lim_{x\to 1}(x\sqrt{x}) = 1$

(3) $\lim_{x\to -1}\dfrac{x^2+4}{x+2} = \dfrac{5}{1} = 5$

P26

EX) (1) $\lim_{x\to -1} \frac{x^3-1}{x+2} = \frac{-1-1}{1} = -2$

(2) $\lim_{x\to\sqrt{2}} \frac{2-x^2}{x} = \frac{0}{\sqrt{2}} = 0$

P27

EX1) $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = 4$

EX2) $\lim_{x\to 0} \frac{1-\sqrt{1+x}}{x}$

$= \lim_{x\to 0} \frac{1-(1+x)}{x(1+\sqrt{1+x})} = \lim_{x\to 0} \frac{-1}{1+\sqrt{1+x}} = \frac{-1}{2}$

EX3)

$\lim_{x\to\infty} \frac{3x^2+5x+4}{-x^2+5} = -3$

EX4)

(1) $\lim_{x\to\infty} \frac{\sqrt{6x^2+1}}{2x+3} = \sqrt{6}$

$\sqrt{6x^2} = \sqrt{6}|x|$

$= \sqrt{6}x \quad (x\to\infty)$

(2) $\lim_{x \to -\infty} \frac{2x+1}{3x+\sqrt{x^2+2}}$ | $-x=t$

$= \lim_{t \to \infty} \frac{-2t+1}{-3t+\sqrt{t^2+2}}$

$= 1$

P28

EX1) (1) $\lim_{x \to \infty}(2x^2-3x+1) = \infty$

(2) $\lim_{x \to \infty}(-3x^2+2x+1) = -\infty$

EX2) $\lim_{x \to \infty}(\sqrt{x^2-x}-x) = \lim_{x \to \infty} \frac{(\sqrt{x^2-x}-x)(\sqrt{x^2-x}+x)}{\sqrt{x^2-x}+x}$

$= \lim_{x \to \infty} \frac{-x}{\sqrt{x^2-x}+x} = -\frac{1}{2}$

EX3) $\lim_{x \to 0} \frac{1}{x}\left(\frac{x+1-1}{x+1}\right) = \lim_{x \to 0} \frac{1}{x+1} = 1$

EX4)

(준식) $= \lim_{x \to \infty}(x-\sqrt{x^2+x})$

$= \lim_{x \to \infty} \frac{-x}{x+\sqrt{x^2+x}} = -\frac{1}{2}$

P30

EX) $\lim_{x\to 1} \frac{ax+b}{x-1} = 2$

$\lim_{x\to 1}(x-1)=0$ 이므로 $\lim_{x\to 1}(ax+b)=a+b=0$

$b=-a$; $ax+b = ax-a = a(x-1)$

$\lim_{x\to 1} \frac{a(x-1)}{x-1} = a = 2$ $\boxed{a=2,\ b=-2}$

P31

EX1) $\lim_{x\to -1} \frac{x+1}{ax+b} = 3$

$\lim_{x\to -1}(x+1)=0$ 이므로 $\lim_{x\to -1}(ax+b)=-a+b=0$

$b=a$ $ax+b = ax+a = a(x+1)$

$\lim_{x\to -1} \frac{(x+1)}{a(x+1)} = \frac{1}{a} = 3$

$a=\frac{1}{3}$, $b=\frac{1}{3}$

EX2)

(1) $\lim_{x\to \infty} \frac{2x^2+1}{3x^2+5x+1} = \frac{2}{3}$

(2) $\lim_{x\to \infty} \frac{x-2x^2}{x^3-5x^2} = 0$

P32

EX1) $$\lim_{x\to\infty} \frac{\overset{0}{a}x^2+bx+1}{2x-3}=2$$

$\boxed{a=0}$ $\frac{b}{2}=2$ $\boxed{b=4}$

EX2) $$\lim_{x\to 1} f(x)=\infty$$

$$\lim_{x\to 1}\{f(x)-g(x)\}=\lim_{x\to 1} f(x)\cdot\left\{1-\frac{g(x)}{f(x)}\right\}=2$$

$$\boxed{\lim_{x\to 1}\frac{g(x)}{f(x)}=1}$$

P33

1)

(1) $\lim_{x\to 2}(x^2+3x-1)=4+6-1=9$

(2) $\lim_{x\to 1}\frac{x^2-x+1}{2x-1}=1-1+1=1$

2)

(1) $\lim_{x\to 1}\frac{(x+3)(x-1)}{(x-1)}=4$

(2) $\lim_{x\to 4}\frac{(x-4)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)}=\lim_{x\to 4}(\sqrt{x}+2)=2+2=4$

(3) $\lim_{x\to 1}\frac{(\sqrt{x^2+3}-2)(\sqrt{x^2+3}+2)}{(x-1)(\sqrt{x^2+3}+2)}$

$=\lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)(\sqrt{x^2+3}+2)}=\frac{2}{2+2}=\frac{1}{2}$

3)

1) $\lim_{x\to\infty}\frac{2x^2-3x-1}{x^2+1}=2$

(2) $\lim_{x\to\infty}\frac{x}{\sqrt{x^2+1}+x}=\frac{1}{2}$

$\underset{x}{\shortparallel}$

(3) $\lim_{x\to\infty}(\sqrt{4x^2-x}-2x)$

$= \lim_{x\to\infty}\dfrac{(\sqrt{4x^2-x}-2x)(\sqrt{4x^2-x}+2x)}{\sqrt{4x^2-x}+2x}$

$= \lim_{x\to\infty}\dfrac{-x}{\underbrace{\sqrt{4x^2}}_{=2x}-x+2x} = -\dfrac{1}{4}$

41

$\lim_{x\to 1}\dfrac{(x-1)(x+2)}{(x-1)}=3$

(분자) $= x^2+x-2$

$\boxed{a=1,\ b=-2}$

P35

3-11

(1) $(x-1)^3+1 = x^3-3x^2+3x$

(준식) $= \lim_{x\to 0}(x^2-3x+3)=3$

(2) $(\sqrt{x+3}-(x+1))(\sqrt{x+3}+(x+1))$

$= x+3-(x+1)^2 = -x^2-x+2$

$= -(x^2+x-2) = -(x-1)(x+2)$

(준식) $= \lim_{x\to 1}\dfrac{-(x-1)(x+2)}{(x-1)(\sqrt{x+3}+(x+1))} = \dfrac{-3}{4}$

(3) $(\sqrt{x+1}-2)(\sqrt{x+1}+2)=x+1-4=x-3$

$$(준식)=\lim_{x\to 3}\frac{x-3}{(x-3)(x-1)(\sqrt{x+1}+2)}$$

$$=\frac{1}{2\cdot 4}=\frac{1}{8}$$

3-21

(1) $\underline{x^3-ax^2}+\underline{a^2x-a^3}$

$=x^2(x-a)+a^2(x-a)=(x-a)(x^2+a^2)$

$x^3-a^3=(x-a)(x^2+ax+a^2)$

$$(준식)=\lim_{x\to a}\frac{x^2+a^2}{x^2+ax+a^2}=\frac{2a^2}{3a^2}=\frac{2}{3}$$

(2) $(x\sqrt{a+1}-a\sqrt{x+1})(x\sqrt{a+1}+a\sqrt{x+1})$

$=x^2(a+1)-a^2(x+1)$

$=(a+1)x^2-a^2x-a^2$

$=(x-a)((a+1)x+a)$

$$(준식)=\lim_{x\to a}\frac{(x-a)((a+1)x+a)}{(x-a)(x\sqrt{a+1}+a\sqrt{x+1})}$$

$$=\frac{a^2+a+a}{a\sqrt{a+1}+a\sqrt{a+1}}=\frac{a(a+2)}{2a\sqrt{a+1}}$$

$$=\frac{a+2}{2\sqrt{a+1}}$$

3-31

$f(x)=ax(x-5) \quad (a>0)$

$f(x-1)=a(x-1)(x-6)$

$f(x+4)=a(x+4)(x-1)$

$f(x+3)=a(x+3)(x-2)$

$$\lim_{x\to 1}\frac{a(x-1)(x-6)}{x-1}=-5a=-10 \quad \boxed{a=2}$$

(1) $$\lim_{x\to 1}\frac{2(x+4)(x-1)}{(x-1)(x+1)}=\frac{2\cdot 5}{2}=5$$

(2) $(\sqrt{x+2}-2)(\sqrt{x+2}+2)=x+2-4=x-2$

$$\lim_{x\to 2}\frac{2(x+3)(x-2)(\sqrt{x+2}+2)}{x-2}=2\cdot 5\cdot 4=40$$

4-11

1) $$\lim_{x\to\infty}\frac{6x^2-5x+1}{3x^2+2x-1}=2$$

(2) $$\lim_{x\to\infty}\frac{\sqrt{x^2+3}-2x}{4x}=\frac{-1}{4}$$

(3) $-x=t$ $$\lim_{t\to\infty}\frac{\sqrt{2t^2-1}}{-t+1}=\frac{\sqrt{2}}{-1}=-\sqrt{2}$$

4-21

(1) $\lim_{x\to\infty} \frac{2x+3}{\sqrt{4x^2+x}+\sqrt{x^2-2x}} = \frac{2}{3}$

$2x \quad\quad x$

(2) $-x=t \quad \lim_{t\to\infty} \frac{-t+1}{\sqrt{t^2-t}+t} = \frac{-1}{2}$

$\sqrt{t^2-t} = t$

(3) $\lim_{x\to\infty} \frac{\sqrt{1+x}-\sqrt{1+x^2}}{\sqrt{1+x^2}+x} = \frac{-1}{2}$

$x \quad x$

4-31

$x^2-x-1=0 \quad \alpha, \beta$

$\alpha+\beta=1, \quad \alpha\beta=-1$

㉠ $(\sqrt{x+\alpha^2}-\sqrt{x+\beta^2})(\sqrt{x+\alpha^2}+\sqrt{x+\beta^2})$

$=(x+\alpha^2)-(x+\beta^2)=\alpha^2-\beta^2=(\alpha-\beta)(\alpha+\beta)$

$=\alpha-\beta$

㉡ $(\sqrt{4x+\alpha}-\sqrt{4x+\beta})(\sqrt{4x+\alpha}+\sqrt{4x+\beta})$

$=(4x+\alpha)-(4x+\beta)=\alpha-\beta$

$$(\text{준식}) = \lim_{x\to\infty} \frac{(\alpha-\beta)(\overbrace{\sqrt{4x+\alpha}}^{2\sqrt{x}}+\overbrace{\sqrt{4x+\beta}}^{2\sqrt{x}})}{(\alpha-\beta)(\underbrace{\sqrt{x+\alpha^2}}_{\sqrt{x}}+\underbrace{\sqrt{x+\beta^2}}_{\sqrt{x}})} = 2$$

5-11

(1) $\lim_{x\to\infty}\left(\overset{2x}{\sqrt{4x^2+3x}}-2x\right)$

$= \lim_{x\to\infty}\dfrac{3x}{\underset{2x}{\sqrt{4x^2+3x}}+2x}$

$= \dfrac{3}{4}$

$(\sqrt{4x^2+3x}-2x)(\sqrt{4x^2+3x}+2x)$

$= 4x^2+3x-4x^2$

$= 3x$

(2) $(\sqrt{x^2+4}-\sqrt{x^2+1})(\sqrt{x^2+4}+\sqrt{x^2+1})$

$= (x^2+4)-(x^2+1) = 3$

(준식) $= \lim_{x\to\infty}\dfrac{3x}{\underset{x}{\sqrt{x^2+4}}+\underset{x}{\sqrt{x^2+1}}} = \dfrac{3}{2}$

(3) $\dfrac{1}{x+1}-\dfrac{1}{4} = \dfrac{4-(x+1)}{4(x+1)} = \dfrac{-(x-3)}{4(x+1)}$

(준식) $= \lim_{x\to 3}\dfrac{-1}{4(x+1)} = \dfrac{-1}{16}$

$n \le x < n+1 \iff [x]=n$ (n은 정수)

$\boxed{[x] = x-\alpha \quad (0 \le \alpha < 1)}$

5-21

(1) $(\sqrt{4x^2+[x]+3}-2x)(\sqrt{4x^2+[x]+3}+2x)$

$= (4x^2+[x]+3)-4x^2 = [x]+3$

$= x-\alpha+3 \qquad (0 \le \alpha < 1)$

$(\text{준식}) = \lim_{x\to\infty} \dfrac{x-\alpha+3}{\sqrt{\underbrace{4x^2}_{2x}+x-\alpha+3}+2x} = \dfrac{1}{4}$

(2) $-x=t$

$(\text{준식}) = \lim_{t\to\infty}(-3t+1+\sqrt{9t^2-4t+1})$

$= \lim_{t\to\infty}(\sqrt{9t^2-4t+1}-(3t-1))$

(이) $\Big((\sqrt{9t^2-4t+1}-(3t-1))(\sqrt{9t^2-4t+1}+(3t-1))$

$= (9t^2-4t+1)-(9t^2-6t+1)$

$= 2t\Big)$

$= \lim_{t\to\infty} \dfrac{2t}{\sqrt{\underbrace{9t^2}_{3t}-4t+1}+(3t-1)}$

$= \dfrac{1}{3}$

5-3 | 1

$\boxed{a=1}$

$$(\sqrt{x^2+2x+3}-(x+b))(\sqrt{x^2+2x+3}+(x+b))$$

$$=(x^2+2x+3)-(x^2+2bx+b^2)$$

$$=(2-2b)x+(3-b^2)$$

$$\lim_{x\to\infty}\frac{(2-2b)x+(3-b^2)}{\underbrace{\sqrt{x^2}}_{=x}+2x+3+(x+b)}=\frac{2-2b}{2}=0$$

$\boxed{b=1}$

$$(\text{준식})=\lim_{x\to\infty}\frac{2x}{\underbrace{\sqrt{x^2}}_{=x}+2x+3+(x+1)}=1$$

6-1 |

(1) $\lim_{x\to 2}(x-2)=0$ 이므로

$$\lim_{x\to 2}(\sqrt{x^2+a}-b)=\sqrt{4+a}-b=0 \quad \therefore b=\sqrt{4+a}$$

$$(\sqrt{x^2+a}-\sqrt{4+a})(\sqrt{x^2+a}+\sqrt{4+a})$$

$$=(x^2+a)-(4+a)=(x^2-4)=(x-2)(x+2)$$

$$\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)(\sqrt{x^2+a}+\sqrt{4+a})}=\frac{4}{2\sqrt{4+a}}=\frac{2}{\sqrt{4+a}}=\frac{2}{5}$$

$\sqrt{4+a}=5 \qquad a=21,\ b=5$

(2) $x^2-(a+2)x+2a=(x-2)(x-a)$

$\lim_{x\to 2}(x^2-(a+2)x+2a)=0$ 이므로

$\lim_{x\to 2}(x^2-b)=4-b=0 \qquad \boxed{b=4}$

$\lim_{x\to 2}\frac{(x-2)(x-a)}{(x+2)(x-2)}=\frac{2-a}{4}=3$

$2-a=12 \qquad \boxed{a=-10}$

6-21 ③

$\lim_{x\to 0}x^2=0$ 이므로

$\lim_{x\to 0}(\sqrt{1+x}-ax-1)=0$

$(\sqrt{1+x}-(ax+1))(\sqrt{1+x}+(ax+1))$

$=(1+x)-(a^2x^2+2ax+1)$

$=(1-2a)x-a^2x^2$

$\lim_{x\to 0}\frac{(1-2a)x-a^2x^2}{x^2(\sqrt{1+x}+(ax+1))}$

$=\lim_{x\to 0}\frac{\cancel{(1-2a)}-a^2x}{x(\sqrt{1+x}+(ax+1))}=b$

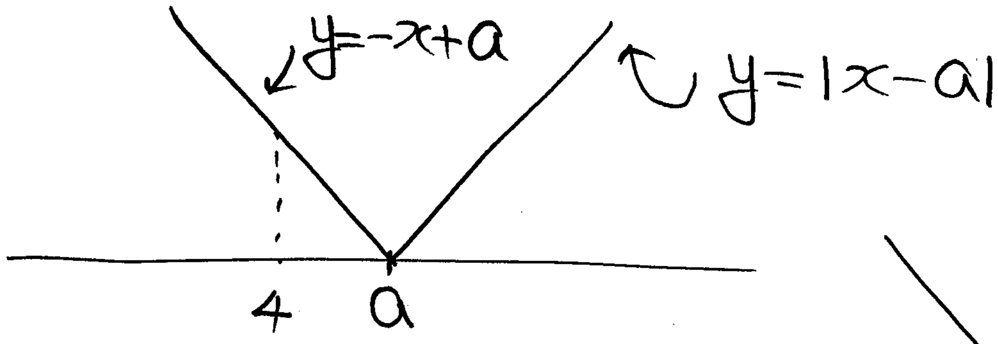
$a=\frac{1}{2}$

$b=\frac{-\frac{1}{4}}{2}=\frac{-1}{8}$

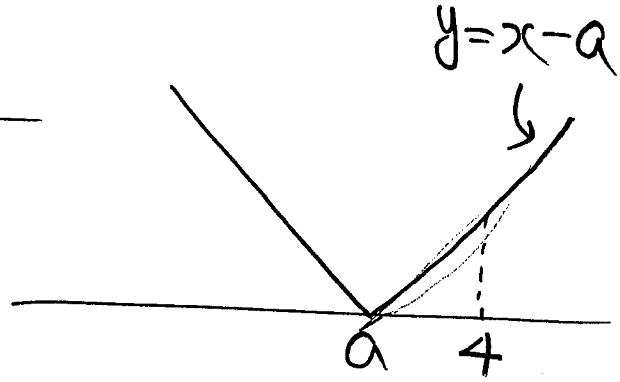
$a+b=\frac{3}{8}$

6-31

$$|x-a| = \begin{cases} x-a & (x \geq a) \\ -x+a & (x<a) \end{cases}$$



㉡ $a \geq 4$



㉠ $a<4$

㉠ $a<4$; $\lim_{x \to 4} \frac{(x-a)+(a-4)}{x-4} = \lim_{x\to 4} \frac{x-4}{x-4} = 1$

$\boxed{a=1,2,3 \quad b=1}$

㉡ $a \geq 4$; $\lim_{x \to 4} \frac{-(x-a)-(a-4)}{x-4}$

$= \lim_{x \to 4} \frac{-(x-4)}{x-4} = -1 \quad (\times)$

$a^2+b^2 = 9+1 = 10$

7-1| 4

㉠ $f(x)=2x^2+ax+b$

㉡ $\lim_{x\to 2}\frac{(x-2)(2x-2)}{x(x-2)}=1$ $\therefore f(x)=(x-2)(2x-2)$

$f(3)=1\cdot 4=4$

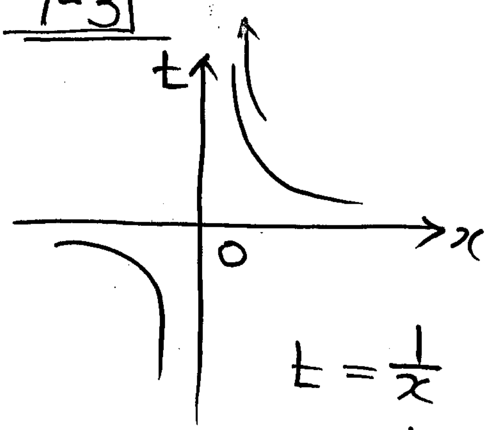
7-2| ②

(가) $f(x)=x^3-2x^2+ax+b$

(나) $\lim_{x\to 1}\frac{(x-1)(x^2-x+2)}{(x-1)}=2$

$f(x)=(x-1)(x^2-x+2)$ $f(-1)=(-2)(1+1+2)$
$=-8$

7-3|



$t=\frac{1}{x}$

$x=\frac{1}{t}$

$\lim_{t\to\infty}\frac{\frac{1}{t^3}f(t)-1}{\frac{1}{t^3}+\frac{1}{t}}$

$=\lim_{t\to\infty}\frac{f(t)-t^3}{1+t^2}=2$

$f(t)=t^3+2t^2+at+b$

$f(x)=x^3+2x^2+ax+b$

$$\lim_{x\to1}\frac{(x-1)(x^2+3x-1)}{(x+1)(x-1)} \doteq \frac{3}{2}$$

$$f(x)=(x-1)(x^2+3x-1)$$

$$\lim_{x\to2}(x-1)(x^2+3x-1) = 4+6-1=9$$

<u>8-11</u>

$$\lim_{x\to-1}\frac{f(x)}{x+1}=3$$

(1) $$\lim_{x\to-1}\frac{f(x)}{(x+1)(x-1)}=\frac{3}{-2}=-\frac{3}{2}$$

(2) $$\lim_{x\to-1}\frac{(x+1)(x^2-x+1)}{f(x)}=\frac{1}{3}\cdot(1+1+1)=1$$

(3) $$\lim_{x\to-1-}\frac{f(x)}{x+1}\cdot\frac{x+1}{\sqrt{x^2-1}}$$

$$\boxed{x+1=-\sqrt{(x+1)^2}}$$

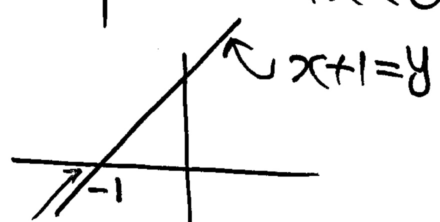
$$=\lim_{x\to-1-}\frac{f(x)}{x+1}\cdot\frac{-\sqrt{(x+1)(x+1)}}{\sqrt{(x-1)(x+1)}}$$

$$=0$$

$$\sqrt{x^2}$$

$$=|x|$$

$$=\begin{cases} x & (x\geq0)\\ -x & (x<0)\end{cases}$$



8-21

$$\lim_{x\to 1}\frac{f(x)}{x-1}=3$$

(1) $$\lim_{x\to 1}\frac{(x-1)+f(x)}{(x^2-1)-f(x)}=\lim_{x\to 1}\frac{1+\frac{f(x)}{x-1}}{(x+1)-\frac{f(x)}{x-1}}$$

$$=\frac{1+3}{2-3}=-4$$

(2) $(\sqrt{x}-1)(\sqrt{x}+1)=x-1$

$$\lim_{x\to 1}\frac{(\sqrt{x}-1)+f(x)}{(x-1)(x^2+x+1)}$$

$$=\lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(x^2+x+1)(\sqrt{x}+1)}+\lim_{x\to 1}\frac{f(x)}{(x-1)(x^2+x+1)}$$

$$=\frac{1}{6}+3\times\frac{1}{3}=\frac{1}{6}+1=\frac{7}{6}$$

8-3|4

$$\lim_{x\to 0}\frac{f(x)}{x}=3,\quad \lim_{x\to 1}\frac{f(x)}{x-1}=4$$

$$\lim_{x\to 0} f(x)=0$$

$$\lim_{x\to 1} f(x)=0$$

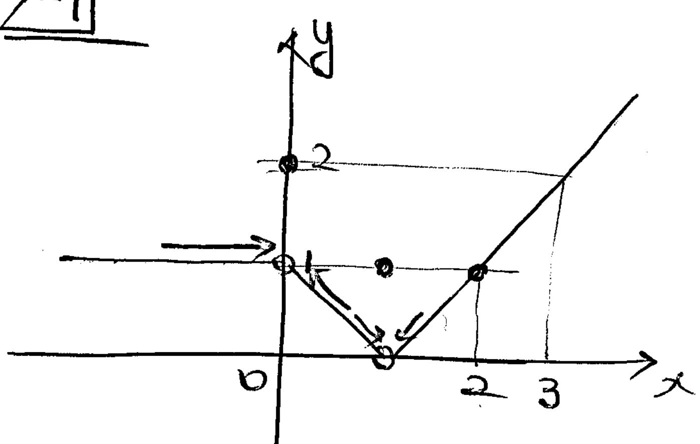
$$\lim_{x\to 1}\frac{f(f(x))\cdot f(x)}{f(x)\cdot(x-1)(x^2+x+1)}$$

$$=\boxed{\lim_{x\to 1}\frac{f(f(x))}{f(x)}}\cdot\frac{4}{3}$$

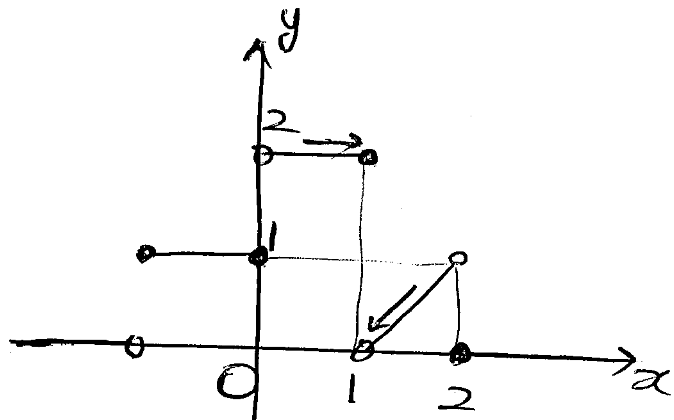
$f(x)=t$

$$=\lim_{t\to 0}\frac{f(t)}{t}\cdot\frac{4}{3}=4$$

9-1|



$y=f(x)$



$y=g(x)$

(1) $\lim_{x\to 0-} g(f(x)) = g(1)=2$

$f(x)=t$

$\lim_{x\to 0+} g(f(x)) = \lim_{t\to 1-} g(t)=2$

$\therefore 2$

(2) $\lim_{x\to 1-} f(g(x)) = f(2) = 1$

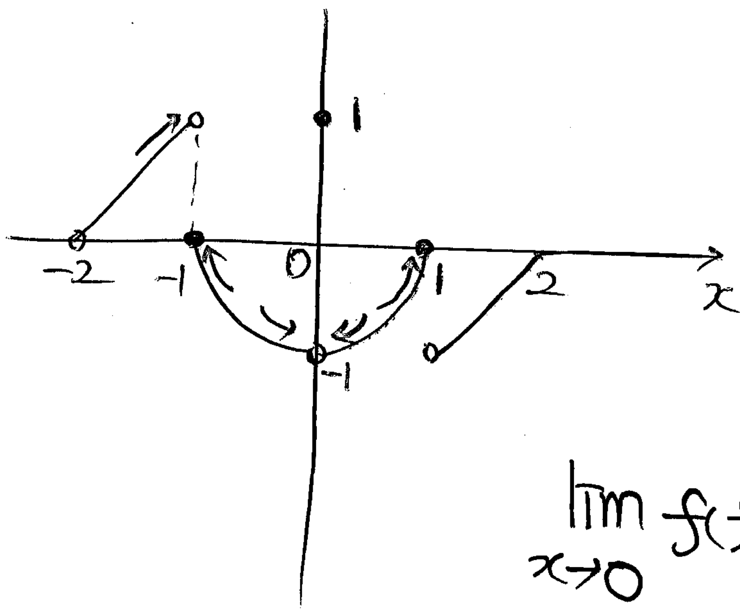
$\lim_{x\to 1+} f(g(x)) = \lim_{t\to 0+} f(t) = 1$

$g(x) = t$

$\therefore\ \underline{\underline{1}}$

(3) $g(\lim_{x\to 1} f(x)) = g(0) = 1$

9-210



$f(x) = t$

$\lim_{x\to -1-} f(f(x))$

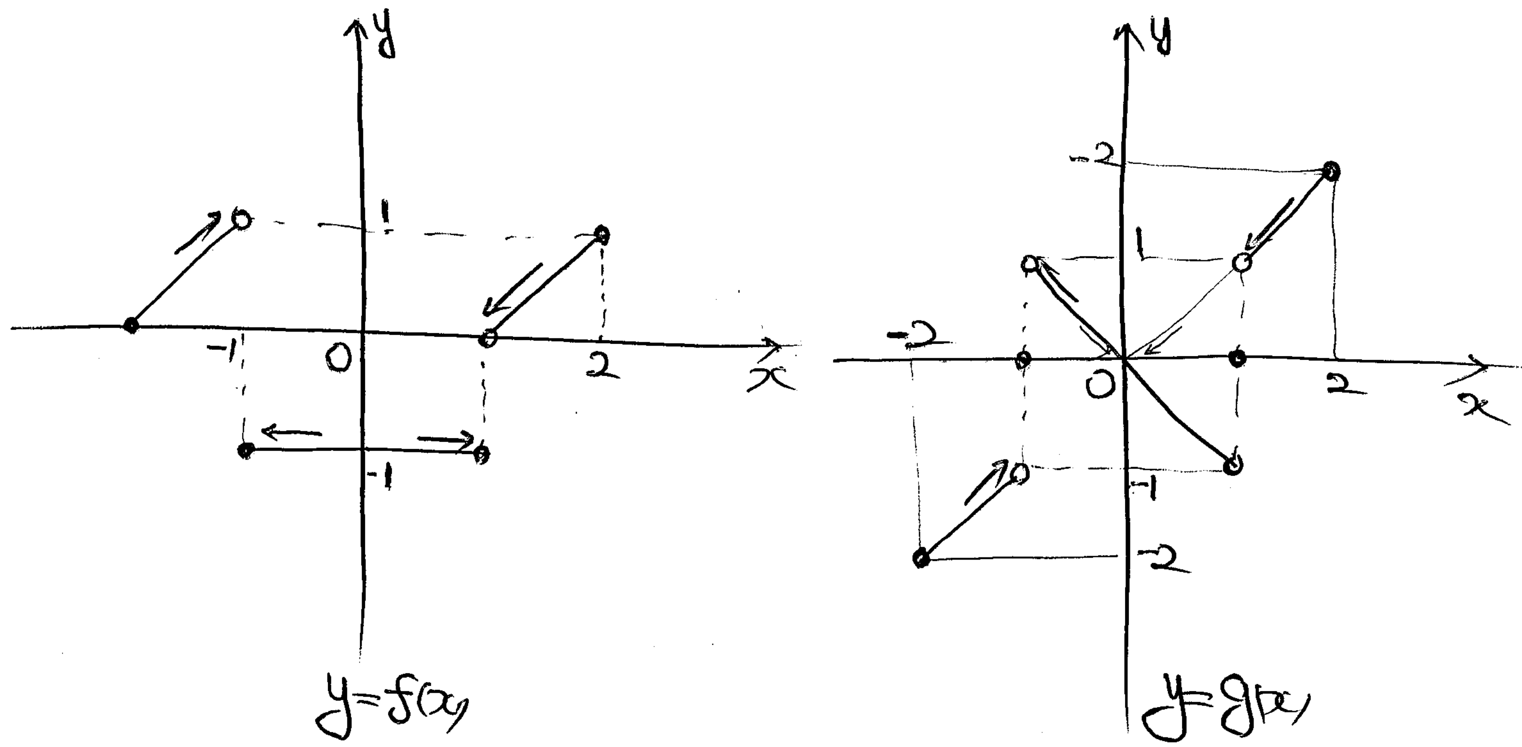
$= \lim_{t\to 1-} f(t) = 0$

$\lim_{x\to 0} f(f(x))$

$= \lim_{t\to -1+} f(t) = 0$

$\therefore\ 0 + 0 = 0$

9-31 ⑤



㉠. $\lim_{x\to 1-} g(f(x))$ $\quad \lim_{x\to 1+} f(g(x))$ $\quad g(x)=t$

$= g(-1)=0$ $\quad \lim_{t\to 1+} f(t)=0$

㉡. $\lim_{x\to 0} f(|g(x)|+1)$ $\quad |g(x)|+1=t$

$=\lim_{t\to 1+} f(t)=0$

㉢ $\lim_{x\to -1-} \{f(x)+g(x)\}=1+(-1)$ $\lim_{x\to -1+} \{f(x)+g(x)\}=-1+1$

$=0$ $\quad =0$

(좌변) $=f(0)=-1$

10-11 ㄷ

~~ㄱ~~ (반례) $f(x)=x-a$, $g(x)=\frac{1}{x-a}$ $f(x)g(x)=1$

~~ㄴ~~

ㄷ) $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)\cdot\frac{f(x)}{g(x)}$ = 수렴

10-21

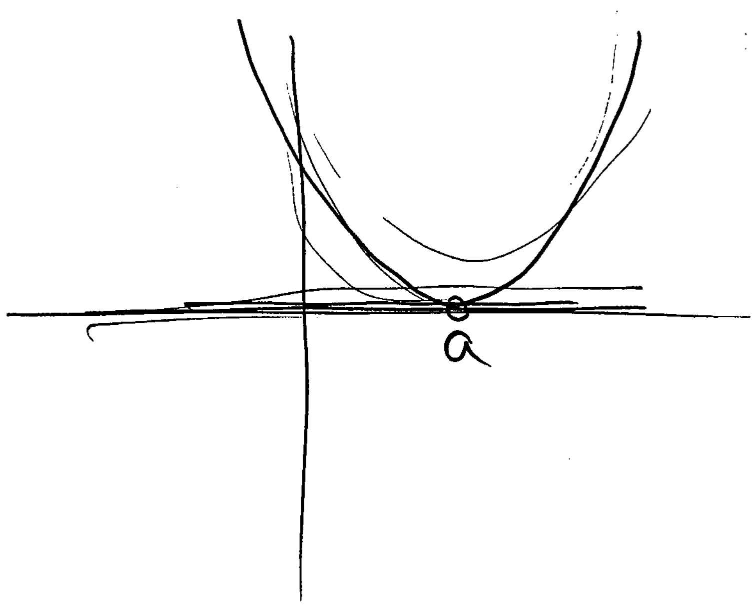
ㄱ. $\lim_{x\to a}\{f(x)+g(x)\}=\alpha$, $\lim_{x\to a}\{f(x)-g(x)\}=\beta$

$\lim_{x\to a} f(x)=\frac{\alpha+\beta}{2}$, $\lim_{x\to a} g(x)=\frac{\alpha-\beta}{2}$

~~ㄴ~~ (반례) $f(x)=\frac{1}{x-a}$, $g(x)=\frac{1}{x-a}$

$f(x)-g(x)=0$

~~ㄷ~~ (반례) $f(x)=0$, $g(x)=(x-a)^2$ $(x\neq a)$



$\lim_{x\to a} f(x)=0$

$\lim_{x\to a} g(x)=0$

10-31

ㄱ. $\lim_{x\to a} f(x) = \alpha$ $\lim_{x\to a} (f(x))^2 = \lim_{x\to a} f(x)\cdot f(x) = \alpha^2$

ㄴ. $\lim_{x\to a} f(x) = \beta$, $\lim_{x\to a} |f(x)| = |\beta|$ (대우)

ㄷ. (ㄴ 대우)

$\lim_{x\to a} f(x)$: 존재 하면 $\lim_{x\to a} f(f(x))$: 존재

$\lim_{x\to a} f(x) = b$

P50

1-1|

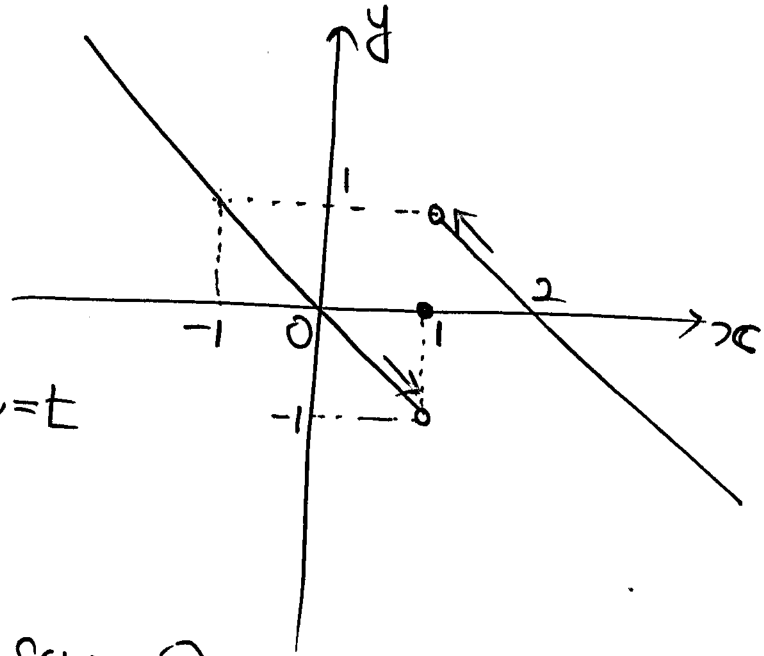
(1) $\lim_{x\to -1} f(x) = 1$

(2) $\lim_{x\to 1-} f(x) = -1$

(3) $\lim_{x\to 1+} f(f(x))$ $\quad f(x)=t$

$= \lim_{t\to 1-} f(t) = -1$

(4) $f(\lim_{x\to -1} f(x)) = f(1) = 0$



1-2|

(1) $\lim_{x\to 0} \dfrac{x - \frac{f(x)}{x}}{x + \frac{f(x)}{x}} = \dfrac{-1}{1} = -1$

(2) $\lim_{x\to \infty} \dfrac{x - \frac{f(x)}{x}}{x + \frac{f(x)}{x}} = 1$

1-3|

(1) $\lim_{x\to -3} \dfrac{(x+3)(x^2-3x+9)}{(x+3)(x-4)} = \dfrac{27}{-7}$

(2) $\lim_{x\to 1} \dfrac{1}{(x-1)} \cdot \dfrac{3(x-1)}{x} = 3$

(3) $\lim_{x\to 0} \dfrac{(x^2+1)-(x-1)^2}{x(\sqrt{x^2+1}-(x-1))} = \lim_{x\to 0} \dfrac{2x}{x(\sqrt{x^2+1}-(x-1))} = 1$

(4) $\lim_{x \to 3} \frac{(x-3)(x+\sqrt{18-x^2})}{x^2-(18-x^2)}$

$= \lim_{x \to 3} \frac{(x-3)(x+\sqrt{18-x^2})}{2(x+3)(x-3)} = \frac{6}{12} = \frac{1}{2}$

1-41

(1) $(준식) = \lim_{x \to 0} \frac{x(1-4x)(\sqrt{1-x^2}+\sqrt{1-4x})}{x(-x+4)(\sqrt{1+x}+\sqrt{1+4x^2})}$

$= \frac{(1+1)}{4(1+1)} = \frac{1}{4}$

(2) $\lim_{x \to \infty} \frac{\sqrt{1+4x}-\sqrt{1+x^2}}{\sqrt{1+4x^2}-\sqrt{1+x}} = -\frac{1}{2}$

1-51

$\frac{2x^2+4x+3}{x\sqrt{4x^2+1}} < (2x^2+4x+3)\cdot f(x) < \frac{2x^2+4x+3}{2x^2}$

$\therefore (준식) = 1$

1-61

(1) $\lim_{x \to 0} x = 0$ 이므로 $\lim_{x \to 0}(a\sqrt{x+1}+b) = a+b=0$

$b=-a$

$\lim_{x \to 0} \frac{a(\sqrt{x+1}-1)}{x} = \lim_{x \to 0} \frac{ax}{x(\sqrt{x+1}+1)} = \frac{a}{2} = 1$

$a=2,\ b=-2$

(2) $\lim_{x \to 3} \dfrac{b - x - a}{(x-3) \cdot b \cdot (x+a)} = -\dfrac{1}{16}$

$b - 3 - a = 0$ $\boxed{a - b = -3}$ $b = a + 3$

$\lim_{x \to 3} \dfrac{-(x-3)}{(x-3) \cdot b \cdot (x+a)} = -\dfrac{1}{16}$

$\dfrac{-1}{b(3+a)} = -\dfrac{1}{16}$ $b(3+a) = 16$

$b^2 = 16$

$\begin{cases} b = 4 \\ a = 1 \end{cases}$ (x)

$\boxed{\begin{matrix} b = -4 \\ a = -7 \end{matrix}}$

$\underline{\underline{-7,\ \dfrac{5}{2}}}$

$\lim_{x \to 0} \dfrac{(\sqrt{1+2x+4x^2} - (ax+1))(\sqrt{1+2x+4x^2} + (ax+1))}{x^2(\sqrt{1+2x+4x^2} + (ax+1))}$

$= \lim_{x \to 0} \dfrac{(1+2x+4x^2) - (a^2x^2 + 2ax + 1)}{x^2(\sqrt{1+2x+4x^2} + (ax+1))}$

$= \lim_{x \to 0} \dfrac{x\{(2-2a) + (4-a^2)x\}}{x^2(\sqrt{1+2x+4x^2} + (ax+1))}$ $2 - 2a = 0$ $a = 1$

$= \lim_{x \to 0} \dfrac{\cancel{(2-2a)} + (4-a^2)x}{x(\sqrt{1+2x+4x^2} + (ax+1))} = \dfrac{3}{2} = b$ $a + b = \dfrac{5}{2}$

1-8)

(1) (준식) $= \lim_{x\to 1} \dfrac{1 + \dfrac{2f(x)}{x-1}}{(x+1) - \dfrac{f(x)}{x-1}} = \dfrac{1+12}{2-6}$

$= -\dfrac{13}{4}$

(2) $x+2=t$

$x=t-2$

$\lim_{t\to 1} \dfrac{f(t)}{(t-2)^3+1}$

$= \lim_{t\to 1} \dfrac{f(t)}{t^3-6t^2+12t-7}$

$= \lim_{t\to 1} \dfrac{f(t)}{(t-1)(t^2-5t+7)}$

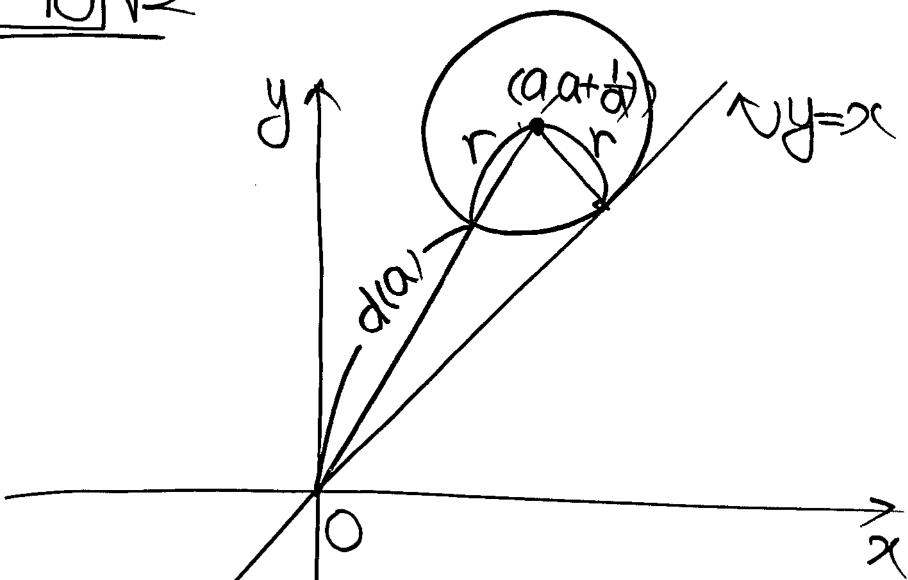
$= 6 \cdot \dfrac{1}{1-5+7} = 2$

1-9)

$\lim_{x\to a} f(x) \cdot \left\{1 - \dfrac{g(x)}{f(x)}\right\} = 2$ $\quad \therefore \lim_{x\to a} \dfrac{g(x)}{f(x)} = 1$

(준식) $= \lim_{x\to a} \dfrac{2 - \dfrac{g(x)}{f(x)}}{1 + \dfrac{2g(x)}{f(x)}} = \dfrac{2-1}{1+2} = \dfrac{1}{3}$

1-10) $\sqrt{2}$



$$\begin{cases} x-y=0 \\ (a,\ a+\frac{1}{a}) \end{cases} \qquad r=\frac{|a-(a+\frac{1}{a})|}{\sqrt{1+1}}=\frac{1}{\sqrt{2}a}$$

$$d(a)=\sqrt{a^2+a^2+2+\frac{1}{a^2}}-\frac{1}{\sqrt{2}a}$$

$$\lim_{a\to\infty}\frac{d(a)}{a}=\lim_{a\to\infty}\frac{\sqrt{2a^2+2+\frac{1}{a^2}}-\frac{1}{\sqrt{2}a}}{a}=\sqrt{2}$$

1-11) 32

$$f(x)=(x-1)f_1(x)+r$$

$$f(1)=r=3. \qquad f(x)-3=(x-1)f_1(x)$$

$$(\text{준식})=\lim_{x\to 1}\frac{f(x)-3}{x^2-1}\cdot\frac{(x^2-1)\cdot f_1(x)}{\sqrt{x}-1}$$

$$=2\cdot\lim_{x\to 1}\frac{(x+1)\cdot(f(x)-3)}{x^2-1}\cdot\frac{x^2-1}{\sqrt{x}-1}$$

$$=8\lim_{x\to 1}\frac{(x-1)(x+1)(\sqrt{x}+1)}{(x-1)}=32$$

1-12번

$\lim_{x\to 1}(x-1)=0$ 이므로 $\lim_{x\to 1}(g(x)-2x)=g(1)-2=0$

$\boxed{g(1)=2}$

$f(x)=(x-1)g(x)-(x-1)=(x-1)(g(x)-1)$

$(\text{준식})=\lim_{x\to 1}\dfrac{(x-1)(g(x)-1)\cdot g(x)}{(x+1)(x-1)}$

$=\dfrac{2}{2}=1$

1-13번

$f(x)=x(x-1)\cdot g(x)$

㉠ $\lim_{x\to 0}\dfrac{f(x)}{x}=\lim_{x\to 0}(x-1)g(x)=-g(0)=10$

$g(0)=-10$

㉡ $\lim_{x\to 1}\dfrac{f(x)}{x-1}=\lim_{x\to 1}xg(x)=g(1)=-4$

$g(x)=ax-10 \quad (a\neq 0)$

$g(1)=a-10=-4 \quad a=6$

$g(x)=6x-10$

$f(x)=x(x-1)(6x-10)$

1-14) 26

㉠ $f(-2)\neq 0$; $\lim_{x\to -2}\frac{f(x)-(x+2)}{f(x)+(x+2)}=\frac{f(-2)}{f(-2)}=1$

(모순)

㉡ $\therefore f(-2)=0$ $f(x)=(x+2)(x+a)$

$\lim_{x\to -2}\frac{(x+a)-1}{(x+a)+1}=\frac{a-3}{a-1}=\frac{3}{7}$

$7a-21=3a-3$

$4a=18$ $a=\frac{9}{2}$

$f(x)=(x+2)(x+\frac{9}{2})$ $f(2)=4\cdot\frac{13}{2}=26$

1-15) ㉠, ㉡, ㉢

$\lim_{x\to 0}g(x)=\lim_{x\to 0}\frac{x+f(x)}{x-f(x)}=3$

$f(0)\neq 0$ (모순) $f(0)=0$

㉠ $\lim_{x\to 0}\frac{1+\frac{f(x)}{x}}{1-\frac{f(x)}{x}}=3$

$1+\alpha=3-3\alpha$

$\boxed{\alpha=\frac{1}{2}}$

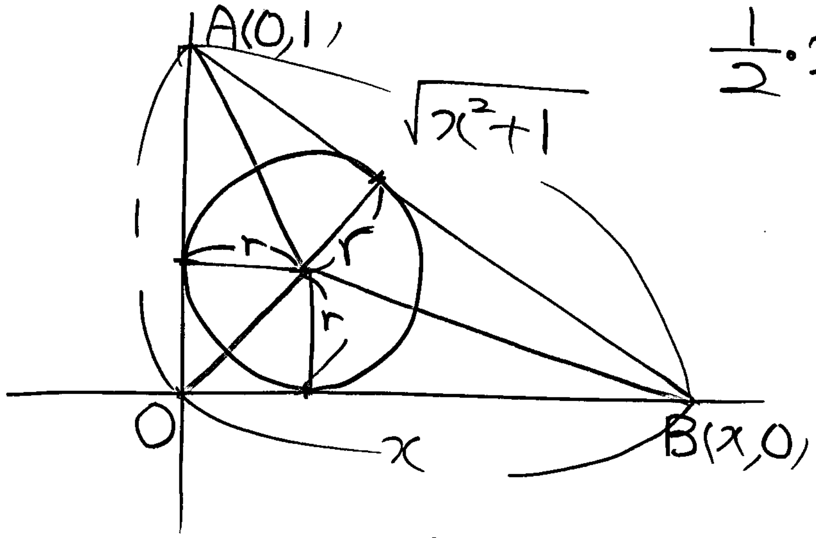
$\lim_{x\to 0}\frac{f(x)}{x}=\alpha$ $\frac{1+\alpha}{1-\alpha}=3$

㉡ $\lim_{x\to 0}\frac{f(x)}{x}=\frac{1}{2}$ $\lim_{x\to 0}x=0$ 이므로

$\lim_{x\to 0}f(x)=0$

© $\lim_{x\to0}\dfrac{x+\frac{f(x)}{x}}{x-\frac{f(x)}{x}}=-1$

1-16) π



$\frac{1}{2}\cdot x\cdot 1=\frac{1}{2}\cdot r\cdot(1+x+\sqrt{x^2+1})$

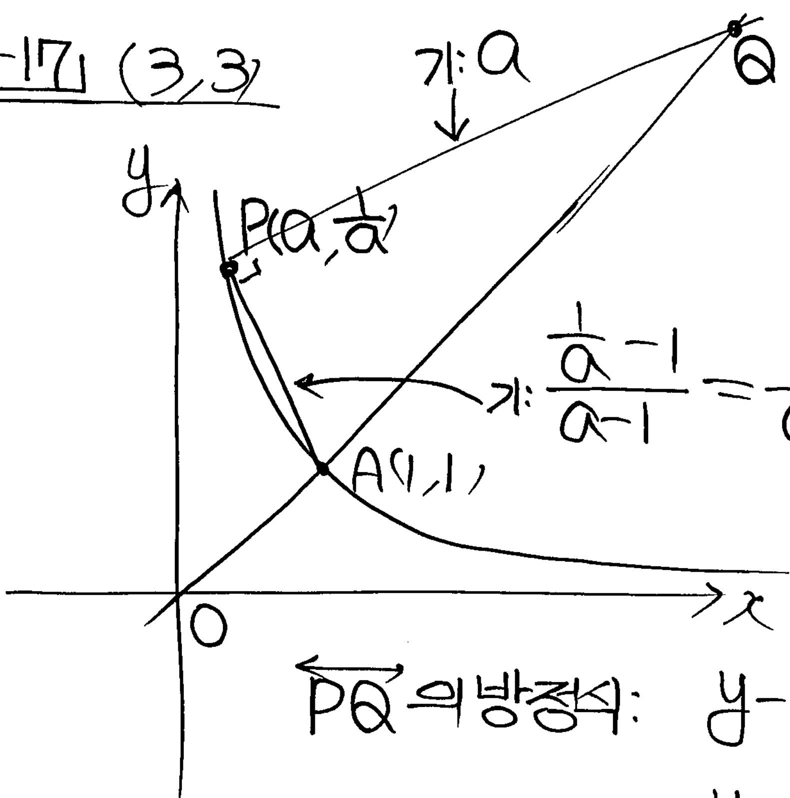
$r=\dfrac{x}{x+1+\sqrt{x^2+1}}$

$l=2\pi r$

$=\dfrac{2\pi\cdot x}{(x+1)+\sqrt{x^2+1}}$

$$\lim_{x\to0+}\frac{l}{x}=\lim_{x\to0+}\frac{2\pi}{(x+1)+\sqrt{x^2+1}}=\pi$$

1-17) (3, 3)



기: $\dfrac{\frac{1}{a}-1}{a-1}=\dfrac{1-a}{a(a-1)}=-\dfrac{1}{a}$

$\overleftrightarrow{PQ}$의 방정식: $y-\frac{1}{a}=a(x-a)$

$$\begin{cases} y=ax-a^2+\frac{1}{a} \\ y=x \end{cases}$$

$$ax - a^2 + \frac{1}{a} = x$$

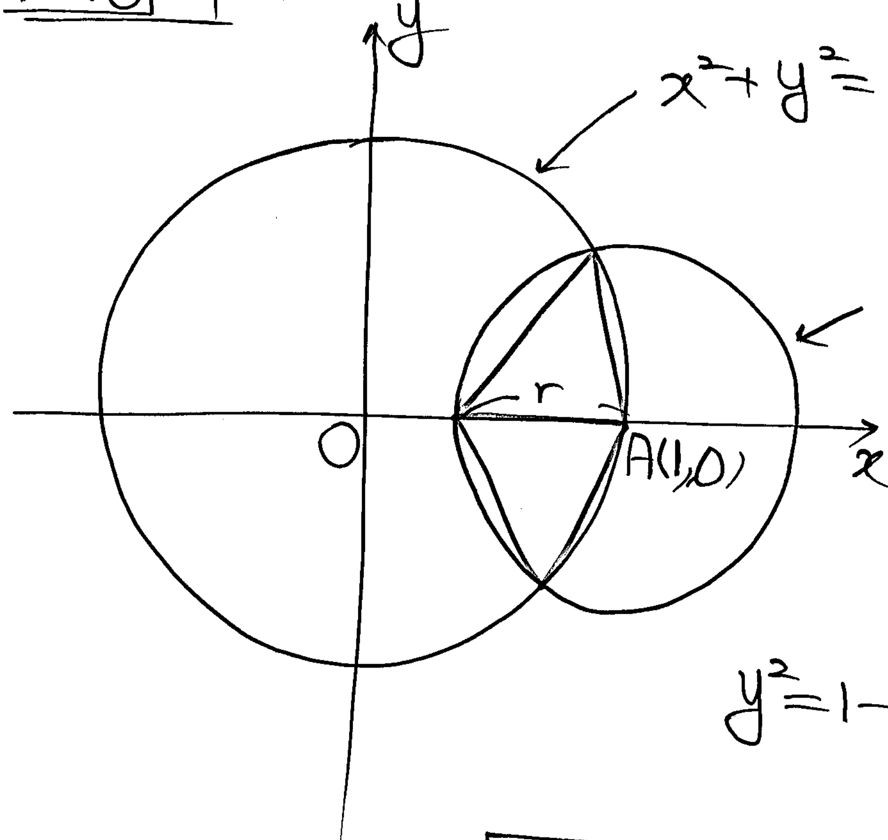
$$(a-1)x = a^2 - \frac{1}{a} = \frac{a^3-1}{a} = \frac{(a-1)(a^2+a+1)}{a}$$

$$x = \frac{a^2+a+1}{a} \quad (a \neq 1)$$

$$Q\left(\frac{a^2+a+1}{a}, \frac{a^2+a+1}{a}\right)$$

$$\lim_{a \to 1} Q = (3, 3)$$

<u>1-18| 4</u>



$$2x - 1 = 1 - r^2$$

$$x = \frac{2-r^2}{2}$$

$$y^2 = 1 - x^2 = 1 - \frac{4-4r^2+r^4}{4}$$

$$= \frac{4r^2-r^4}{4}$$

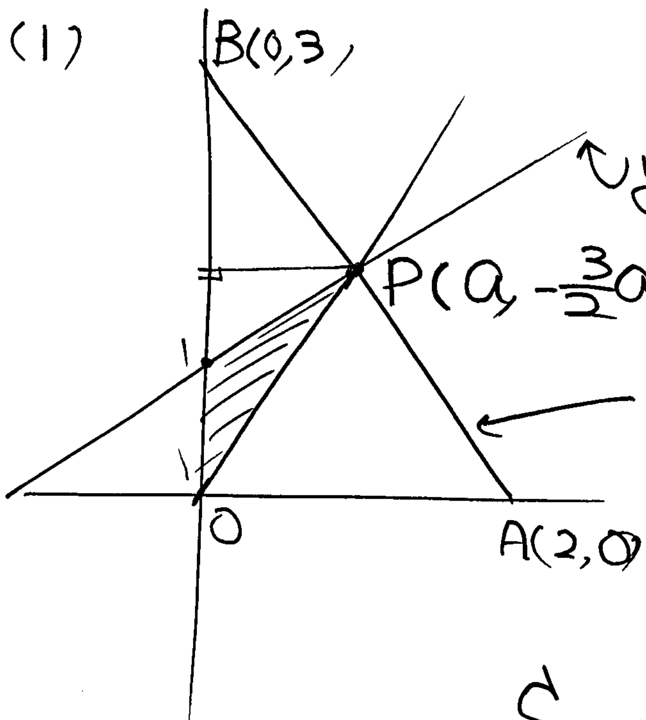
$$S(r) = \frac{1}{2} \cdot r \cdot \frac{r\sqrt{4-r^2}}{\cancel{2}} \cdot \cancel{2}$$

$$y = \frac{r\sqrt{4-r^2}}{2}$$

$$\lim_{r \to 2-} \frac{S(r)}{\sqrt{2-r}} = \lim_{r \to 2-} \frac{\frac{r^2}{2} \cdot \sqrt{2-r}\sqrt{2+r}}{\sqrt{2-r}} = 4$$

1-19

(1)



$nx+1=-\frac{3}{2}x+3$

$(n+\frac{3}{2})x=2$

$x=\frac{2}{n+\frac{3}{2}}$

$S=\frac{1}{2}\cdot 1\cdot \frac{2}{n+\frac{3}{2}}=\frac{2}{2n+3}$

$a=\frac{2}{n+\frac{3}{2}}$

$a\to 2 \qquad 2n+3=2 \qquad n=-\frac{1}{2}$

$\lim_{n\to -\frac{1}{2}} n\cdot\frac{2}{2n+3}=\frac{-1}{2}$

(2) $\begin{cases} y=-\frac{3}{2}x+3 \\ y=mx \end{cases}$

$mx=-\frac{3}{2}x+3$

$(m+\frac{3}{2})x=3$

$x=\frac{3}{m+\frac{3}{2}}$

$S=\frac{1}{2}\cdot 1\cdot\frac{3}{m+\frac{3}{2}}=\frac{3}{2m+3}$

$\lim_{m\to\infty} mS=\lim_{m\to\infty}\frac{3m}{2m+3}=\frac{3}{2}$