* 연속 *

[1] 함수의 연속

$y=f(x)$가 $x=a$ 에서 연속

㉠ $x=a$ 에서 정의 $\Leftrightarrow$ $f(a)$가 존재

㉡ $\lim_{x\to a} f(x)$: 존재 $\Leftrightarrow$ $\lim_{x\to a-} f(x) = \lim_{x\to a+} f(x) = \alpha$

㉢ $f(a) = \lim_{x\to a} f(x)$

[2] 구간

$(a, b) \Leftrightarrow a<x<b$

$[a, b] \Leftrightarrow a\le x\le b$

$(a, b] \Leftrightarrow a<x\le b$

$[a, b) \Leftrightarrow a\le x<b$

[3] 최대·최소 정리

함수 $f(x)$가 닫힌구간 $[a, b]$ 에서 연속이면

이 구간에서 반드시 최댓값과 최솟값을 가진다.

[3] 사잇값의 정리

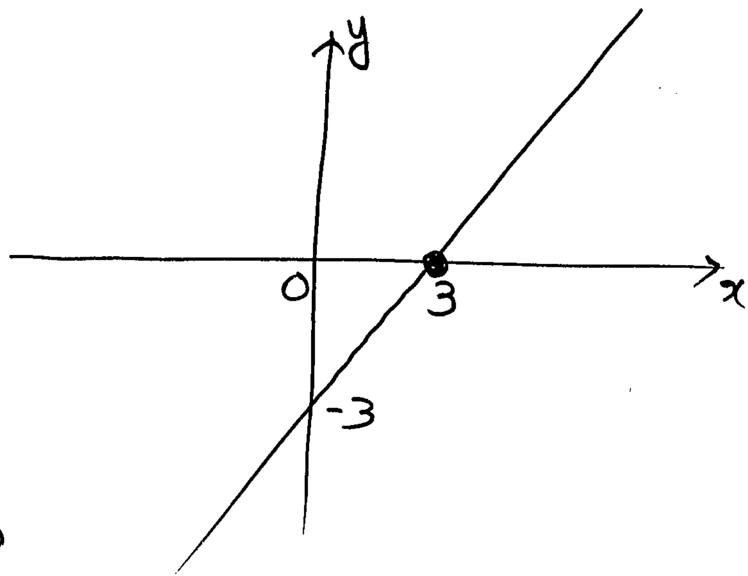
함수 $f(x)$가 닫힌구간 $[a, b]$ 에서 연속이고 $f(a)\ne f(b)$

일때 $f(a)$와 $f(b)$ 사이의 임의의 값 k에 대하여

$f(c)=k$ 인 c가 a와 b 사이에 적어도 하나 존재한다.

P58

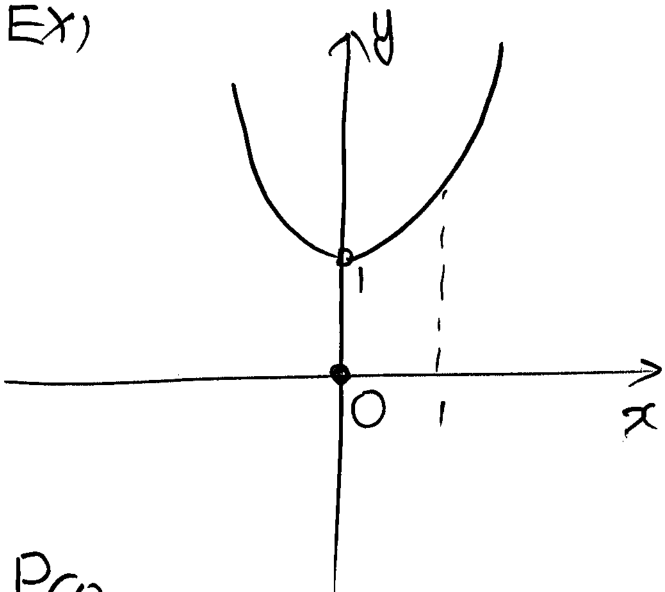
EX) $f(x) = \begin{cases} \dfrac{x^2-6x+9}{x-3} = x-3 & (x \neq 3) \\ 0 & (x = 3) \end{cases}$



$x=3$ 에서 연속

P59

EX)



$x=0$ 에서 불연속

$x=1$ 에서 연속

P60

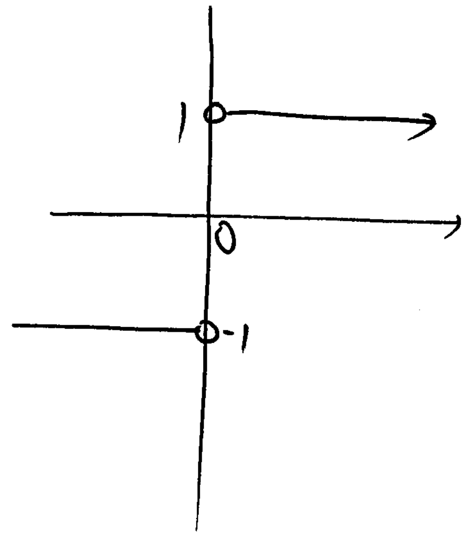
EX) (1) $(-3, 4] \iff -3 < x \leq 4$

(2) $(-2, \infty) \iff x > -2$

P61

EX1)

$$f(x) = \begin{cases} 1 & (x>0) \\ 0 & (x=0) \\ -1 & (x<0) \end{cases}$$



$(-\infty, 0)$, $(0, \infty)$ 에서 연속

EX2) $f(x) = \sqrt{x-1}$

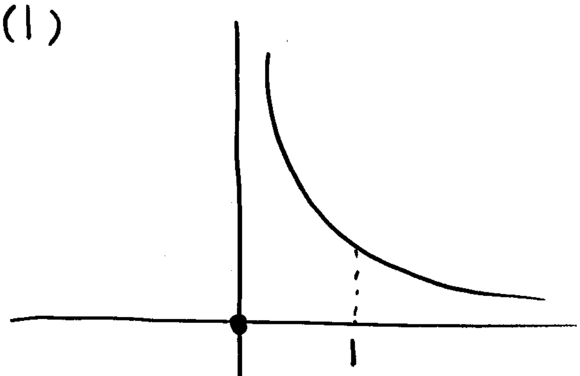
$x-1 \geq 0$

$x \geq 1$

$[1, \infty)$ 에서 연속

P62 EX)

(1)

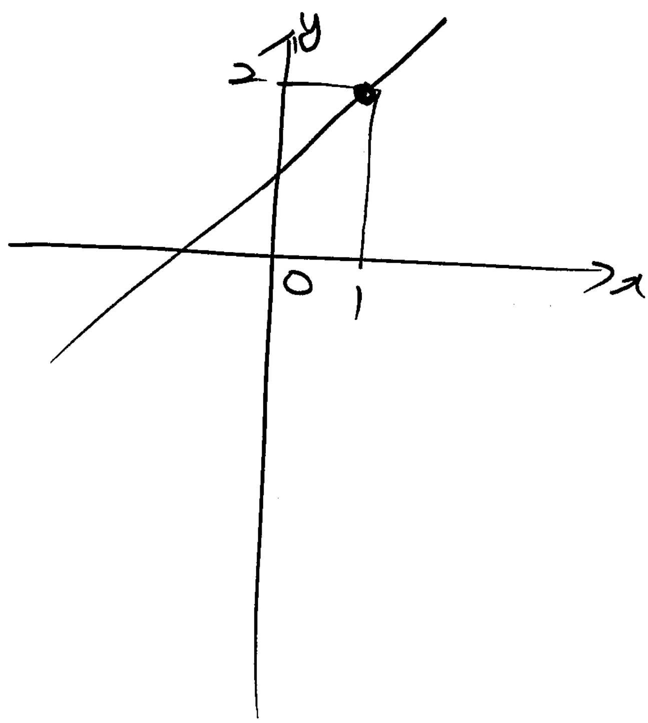


$(0, 1)$ 연속

$(0, 1]$ 연속

$$y = f(x) = \begin{cases} \frac{1}{x} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$$

(2) $g(x) = \begin{cases} \dfrac{x^2-1}{x-1} = x+1 & (x \neq 1) \\ 2 & (x=1) \end{cases}$



P63

__11__

$x=2$ 에서 연속

$f(2) = \lim_{x\to 2} f(x)$

$a = \lim_{x\to 2} \dfrac{x^2-x-2}{x-2} = \lim_{x\to 2} \dfrac{(x-2)(x+1)}{(x-2)} = 3$

$\therefore a=3$

__21__

(1) $f(x) = \dfrac{1}{x-1}$ $\quad f(1)$: 정의 X

$x=1$ 에서 불연속

(2) $x=1$ 일때

㉠ $g(1)=1$

㉡ $\lim_{x\to1} g(x) = \lim_{x\to1} \frac{(x-1)(x+1)}{x-1} = 2$

㉢ $g(1) \neq \lim_{x\to1} g(x)$ $x=1$ 에서 불연속

31

(1) $4-x \geq 0$
$x \leq 4$
$(-\infty, 4]$ 에서 연속

(2) $2x-6 \geq 0$
$x \geq 3$
$[3, \infty)$ 에서 연속

(3) $x+1 \neq 0$
$x \neq -1$
$(-\infty, -1), (-1, \infty)$ 에서 연속

P65

1-1) 20

$x=1$ 에서 연속

$f(1) = \lim_{x\to1} f(x)$

$\frac{1}{2} = \lim_{x\to1} \frac{a\sqrt{x+3}-b}{x-1}$

$= \lim_{x\to1} \frac{a(\sqrt{x+3}-2)}{x-1}$

$= \lim_{x\to1} \frac{a(x-1)}{(x-1)(\sqrt{x+3}+2)}$

$2a-b=0$
$b=2a$

$\frac{a}{4} = \frac{1}{2}$
$a=2, b=4$
$a^2+b^2 = 4+16$
$=20$

1-2) 5

$x=1$ 에서 연속

$$f(1)=\lim_{x\to 1} f(x)$$

$$3a-b=\lim_{x\to 1}\frac{(x-1)(x+b)}{x+a}$$

$a=-1$

$$-3-b=1+b$$

$$2b=-4 \quad b=-2$$

$$\boxed{a^2+b^2=5}$$

1-3) $9x$

$x=0$ 에서 연속

$$f(0)=\lim_{x\to 0} f(x)$$

$$6=\lim_{x\to 0}\frac{g(x)-3x}{x}$$

$$\boxed{g(0)=0}$$

$$g(x)=x\cdot h(x)$$

$$=\lim_{x\to 0}\frac{h(x)-3}{1}$$

$$6=h(0)-3$$

$$\boxed{h(0)=9}$$

$$g(x)=x^2\cdot Q(x)+ax+b$$

$$g(0)=b=0$$

$$xh(x)=x^2Q(x)+ax$$

$$h(x)=xQ(x)+a$$

$$h(0)=a=9$$

$\boxed{\text{나머지}: 9x}$

2-1)

$x \neq 2$; $f(x) = \dfrac{x^2+4x+a}{x-2}$

$f(2) = \lim_{x \to 2} f(x)$

$= \lim_{x \to 2} \dfrac{x^2+4x+a}{x-2} = \lim_{x \to 2} \dfrac{(x-2)(x+6)}{x-2}$

$f(2) = 8$, $a = -12$

2-2) $\frac{2}{3}$

$x \neq 1$; $f(x) = \dfrac{x+2\sqrt{x}-3}{x^2+x-2}$

$x=1$ 에서 연속

$f(1) = \lim_{x \to 1} f(x)$

$= \lim_{x \to 1} \dfrac{(\sqrt{x}+3)(\sqrt{x}-1)(\sqrt{x}+1)}{(x+2)(x-1)(\sqrt{x}+1)}$

$= \dfrac{4}{3 \cdot 2} = \dfrac{2}{3}$

2-3)

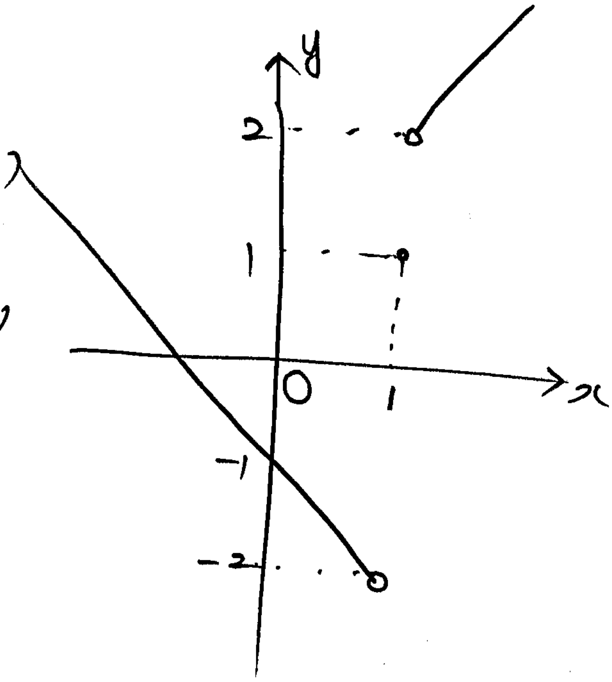
$x \neq 2$; $f(x) = \dfrac{x^3+ax+b}{(x-2)^2}$ $x=2$ 에서 연속

$f(2) = \lim_{x \to 2} f(x)$

$= \lim_{x \to 2} \dfrac{(x^2-4x+4)(x+4)}{(x^2-4x+4)} = 6$

3-1

(1) $f(x) = \begin{cases} -(x+1) & (x<1) \\ 1 & (x=1) \\ x+1 & (x>1) \end{cases}$



$x=1$ 에서 불연속

(2)

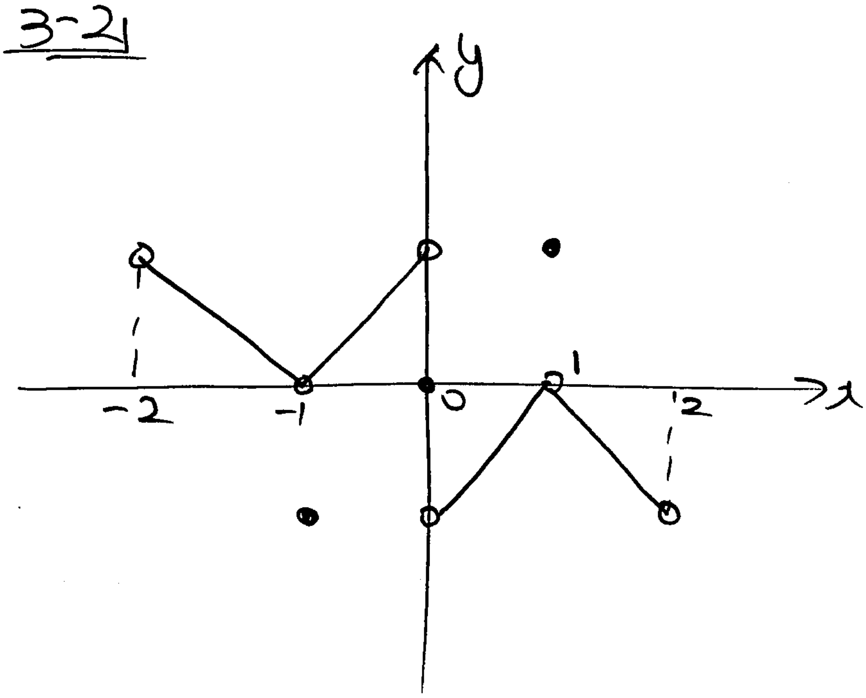
$g(1) = [2] - [1] = 2-1 = 1$

$\lim_{x \to 1-} g(x) = \lim_{x \to 1-} ([2x] - [x]) = 1-0 = 1$

$\lim_{x \to 1+} g(x) = \lim_{x \to 1+} ([2x] - [x]) = 2-1 = 1$

$x=1$ 에서 연속

3-2



$-2<x<2$

㉠ $x=0$ 에서 극한 ×

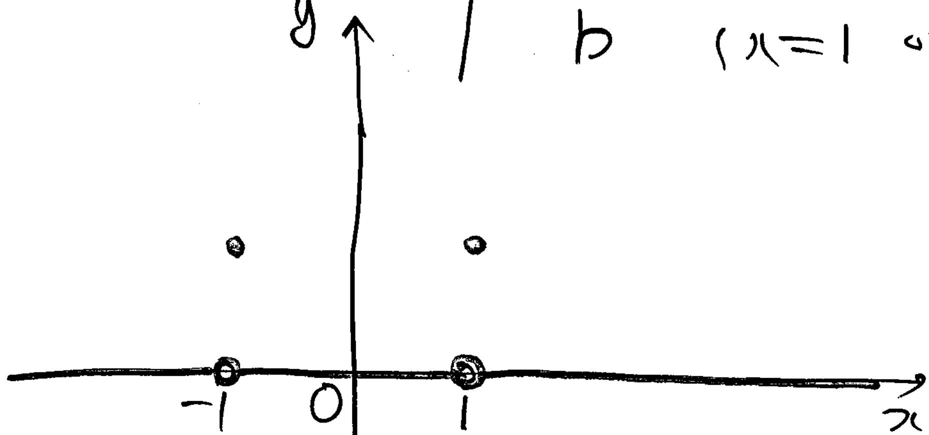
$m=1$

㉡ $n=3$

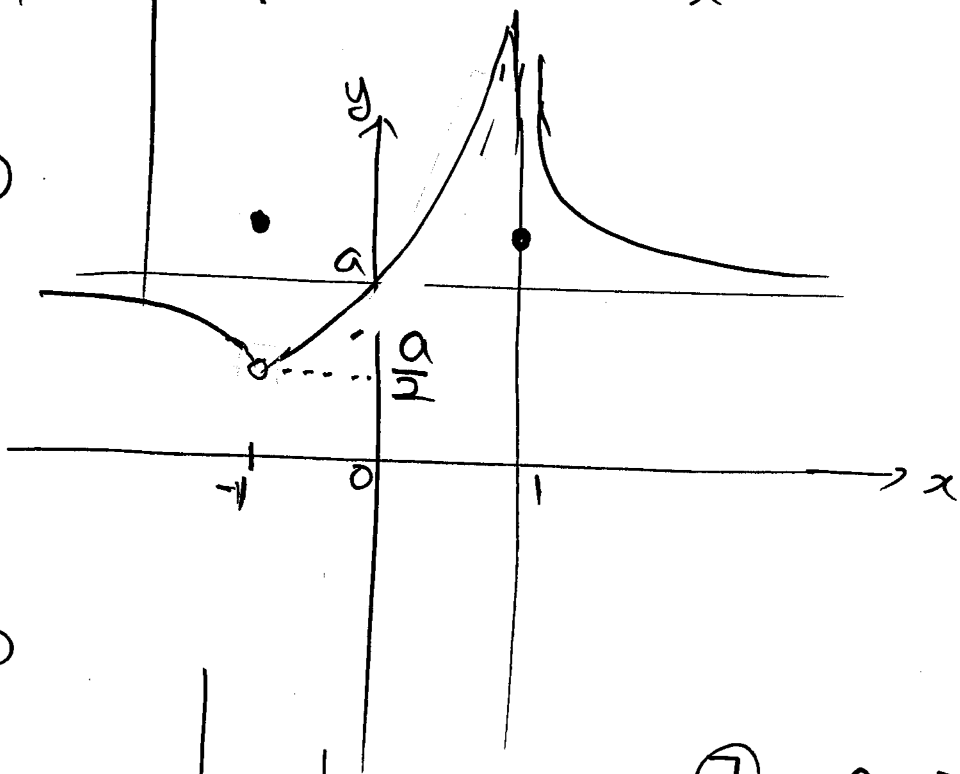
$10m+n=13$

3-31

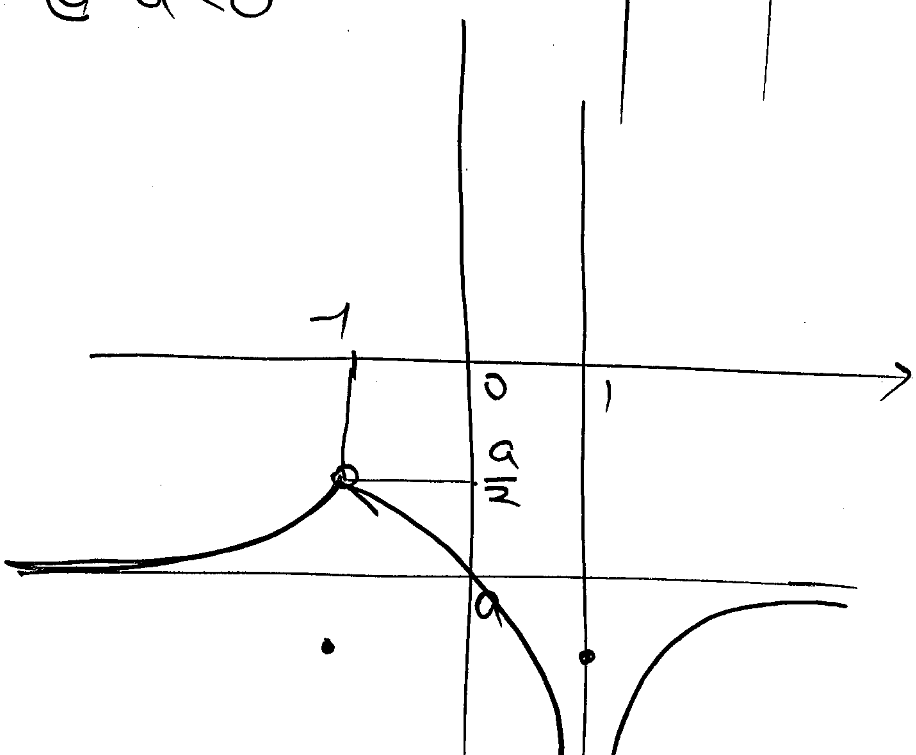
① $a=0$ $f(x)=\begin{cases} 0 & (x<-1,\ x>1) \\ 0 & (-1<x<1) \\ b & (x=1 \text{ or } x=-1) \end{cases}$



② $a>0$



③ $a<0$

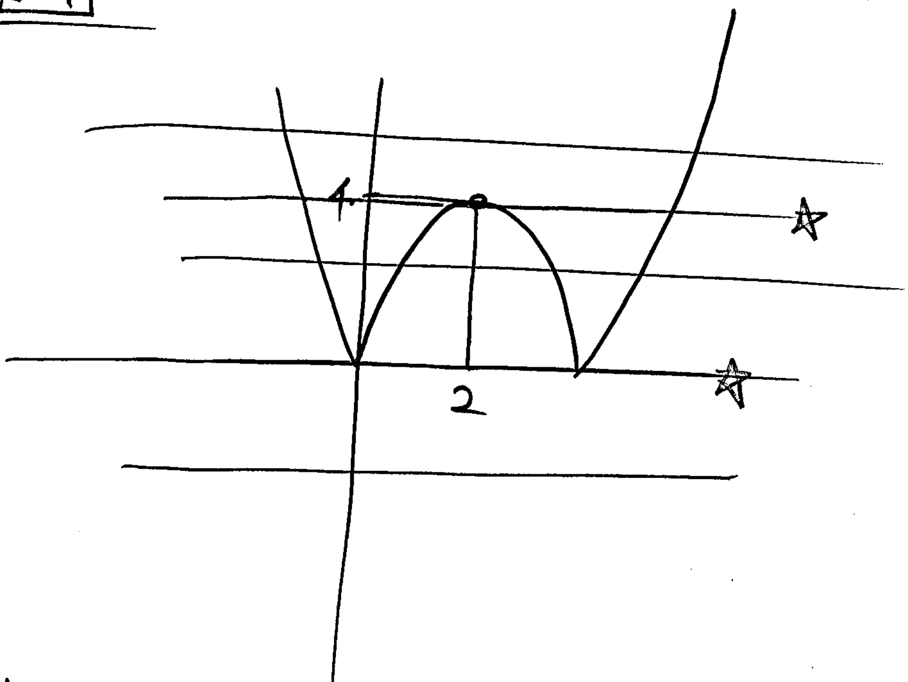


ㄱ $a=2b$

ㄴ

ㄷ

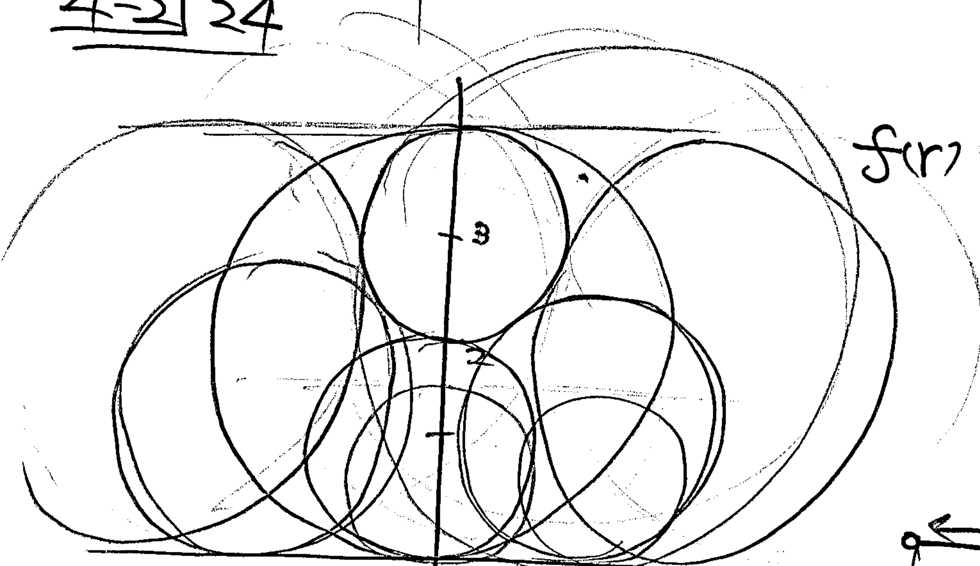
4-1)



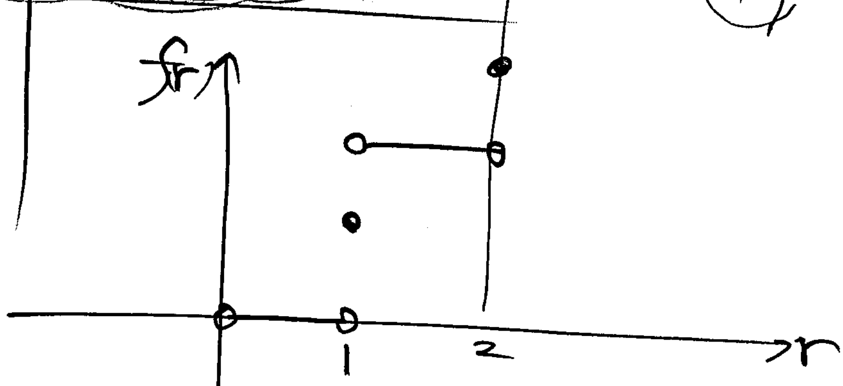
$t-1=0$ 또는 4

$\boxed{t=1 \text{ 또는 } t=5}$

4-2) 24



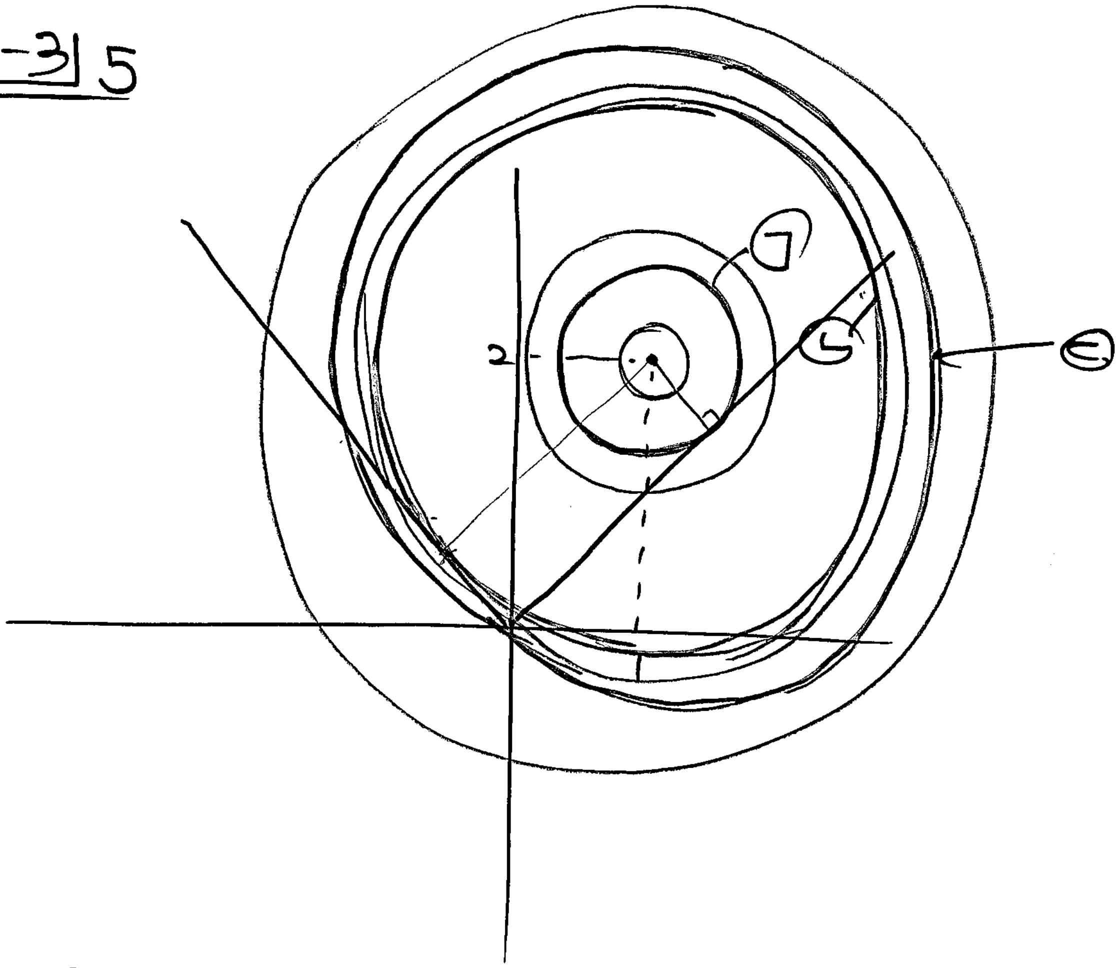
$$f(r)=\begin{cases} 0 & (0<r<1) \\ 1 & (r=1) \\ 2 & (1<r<2) \\ 3 & (r=2) \\ 4 & (2<r<4) \end{cases}$$



$n=2 \quad \lim_{r\to 2+} f(r) = 4 = k$

$\therefore\ 10n+k = 20+4 = 24$

4-3|5



㉠ $x-y=0$ $\quad \dfrac{|1-2|}{\sqrt{1+1}}=\dfrac{1}{\sqrt2}=\dfrac{\sqrt2}{2}$

$(1,2)$

㉡ $x+y=0$ $\quad \dfrac{|1+2|}{\sqrt{1+1}}=\dfrac{3}{\sqrt2}=\dfrac{3\sqrt2}{2}$

$(1,2)$

㉢ $(0,0)$ $(1,2)$ $\quad \sqrt5$

$$f(t)=\begin{cases} 0 & \left(0<t<\frac{\sqrt2}{2}\right) \\ 1 & \left(t=\frac{\sqrt2}{2}\right) \\ 2 & \left(\frac{\sqrt2}{2}<t<\frac{3\sqrt2}{2}\right) \\ 3 & \left(t=\frac{3\sqrt2}{2}\right) \\ 4 & \left(\frac{3\sqrt2}{2}<t<\sqrt5\right) \\ 3 & (t=\sqrt5) \\ 2 & (t>\sqrt5) \end{cases}$$

$n=3$

$f(n-1)$

$=f(2)=2$

$k=2$

$n+k=5$

P73

EX1) $f(x) = \dfrac{x^2}{x-2}$

$x \neq 2$ 에서 연속

$(-\infty, 2), (2, \infty)$ 에서 연속

EX2) $f(x) = x$, $g(x) = x^2 + 2$

$2f(x) + 3g(x)$: 연속

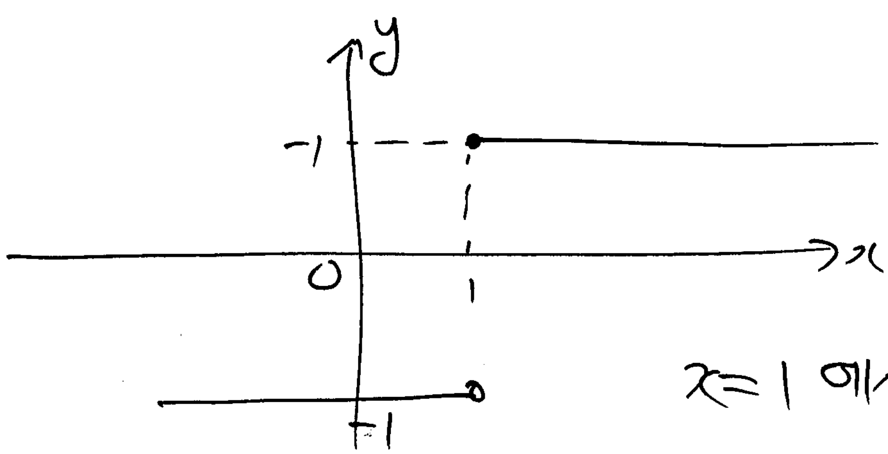
$$\frac{g(x)}{f(x)} = \frac{x^2+2}{x}$$

$x \neq 0$ 에서 연속 $(-\infty, 0), (0, \infty)$ 에서 연속

P74

EX1) $f(x) = \begin{cases} 1 & (x \geq 0) \\ -1 & (x < 0) \end{cases}$ $g(x) = x - 1$

$$f(g(x)) = f(x-1) = \begin{cases} 1 & (x-1 \geq 0) \\ -1 & (x-1 < 0) \end{cases}$$



$x = 1$ 에서 불연속

EX2)

(1) $f(x)=x^2$, $g(x)=\sqrt{x}$

$f(g(x)) = (\sqrt{x})^2 = x$ $\quad (x \ge 0)$

(2) $f(x)=\sqrt{x}$, $g(x)=1-x^2$

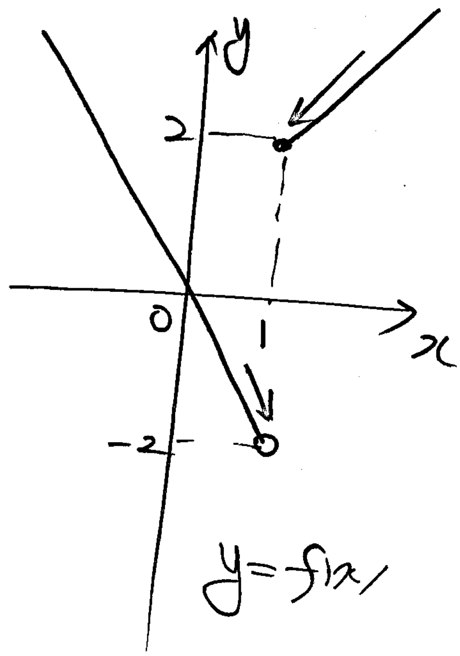
$f(g(x)) = \sqrt{1-x^2}$ $\quad 1-x^2 \ge 0$

$(-1 \le x \le 1)$ $\quad x^2-1 \le 0$

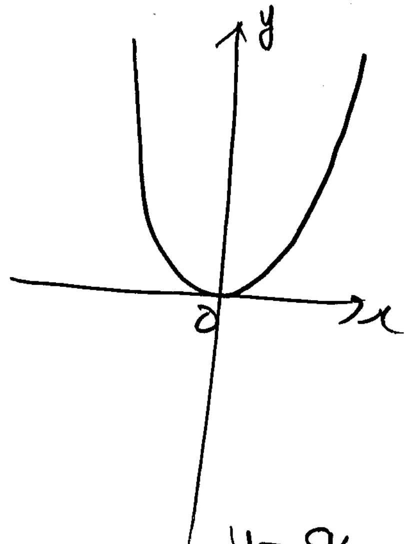
P75

EX) $y=f(x)$

$y=x^2=g(x)$



$y=f(x)$



$y=g(x)=x^2$

$y=g(f(x))$

$x=1$ 일때

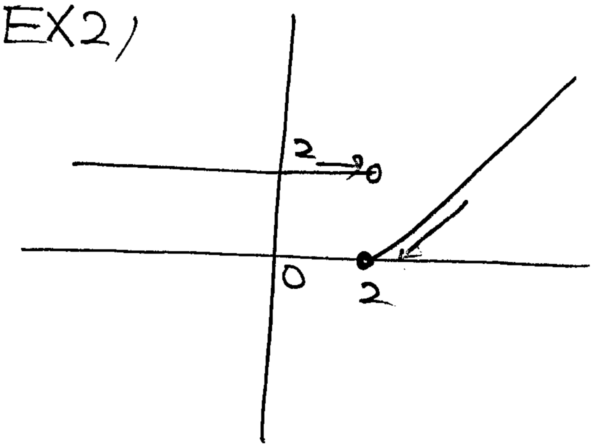
$g(f(1)) = g(2) = 4$

$\lim_{x\to 1-} g(f(x)) = \lim_{t\to -2+} g(t) = 4$ $\quad f(x)=t$

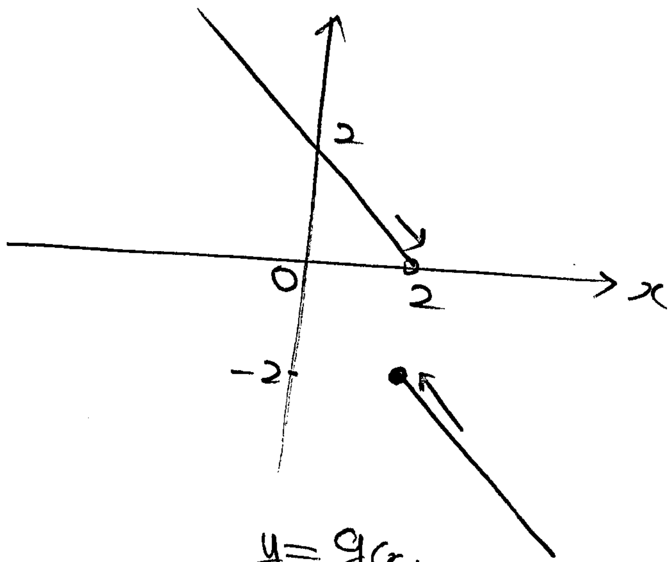
$\lim_{x\to 1+} g(f(x)) = \lim_{t\to 2+} g(t) = 4$

$x=1$에서 연속

EX2)



$y=f(x)$



$y=g(x)$

(1) $y=f(x)g(x)$

$f(2)g(2)=0\cdot(-2)=0$

$\lim_{x\to2-} f(x)g(x)=2\cdot0=0$, $\lim_{x\to2+} f(x)g(x)=0\cdot(-2)=0$

$x=2$ 에서 연속

(2) $y=f(x)+g(x)$

$f(2)+g(2)=0+(-2)=-2$

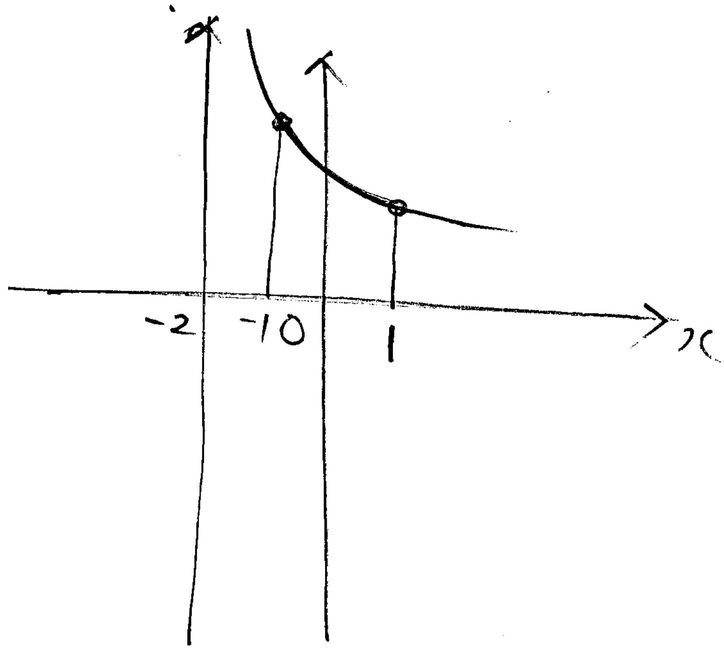
$\lim_{x\to2-}(f(x)+g(x))=2+0=2$

$\lim_{x\to2+}(f(x)+g(x))=0+(-2)=-2$

$x=2$에서 불연속

P77

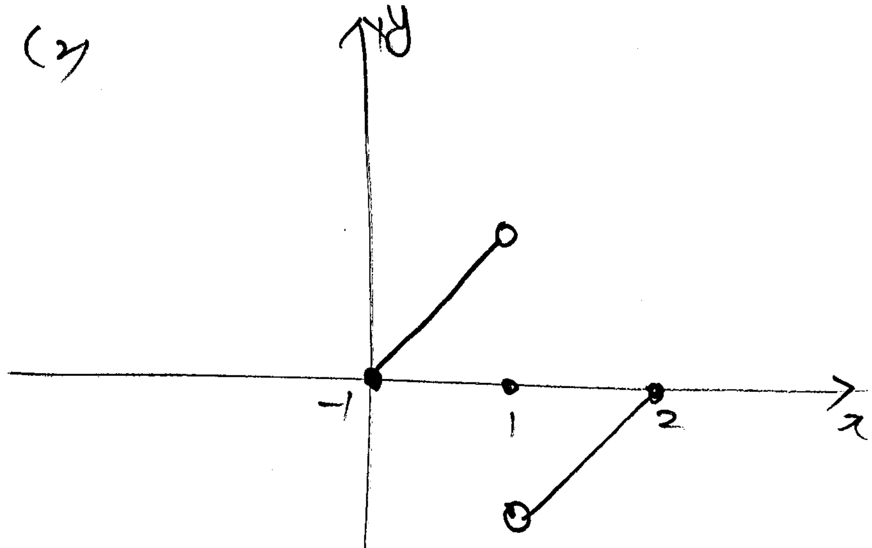
EX) (1) $f(x) = \frac{2}{x+2}$ $(-1 \leq x \leq 1)$



$M = f(-1) = 2$

$m = f(1) = \frac{2}{3}$

(2)



M: 最大

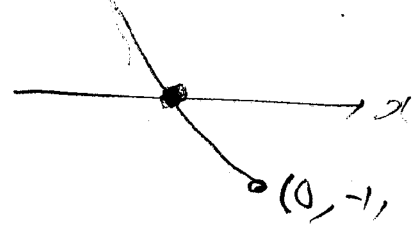
m: 最小

P78

EX) $f(x) = x^3 + 4x^2 - 3x - 1$

$f(-1) = -1 + 4 + 3 - 1 = 5 > 0$

$f(0) = -1 < 0$



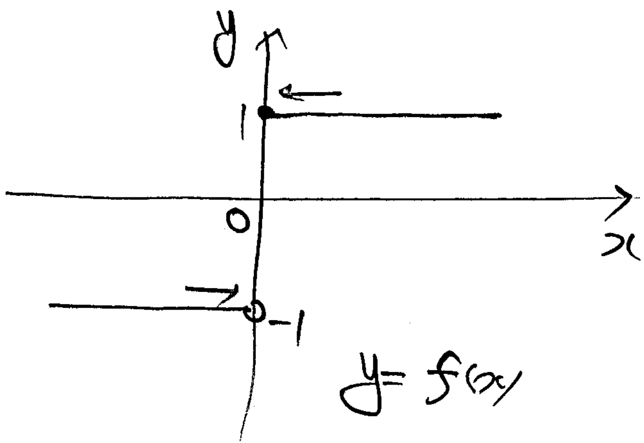
P79

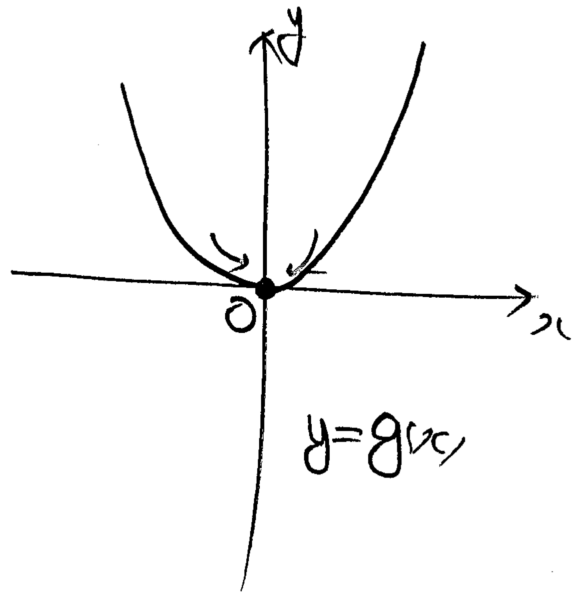
11

(1) 모든 실수 에서 연속

(2) $x \neq 2$ 인 모든 실수에서 연속

12





(1) $x=0$ 일때

$$f(0)\cdot g(0) = 1\cdot 0 = 0$$

$$\lim_{x\to 0-} f(x)g(x) = -1\cdot 0 = 0,\ \lim_{x\to 0+} f(x)g(x) = 1\cdot 0 = 0$$

$x=0$ 에서 연속

(2) $f(g(0)) = f(0) = 1$

$$\lim_{x\to 0-} f(g(x)) = \lim_{t\to 0+} f(t) = 1$$

| $g(x)=t$

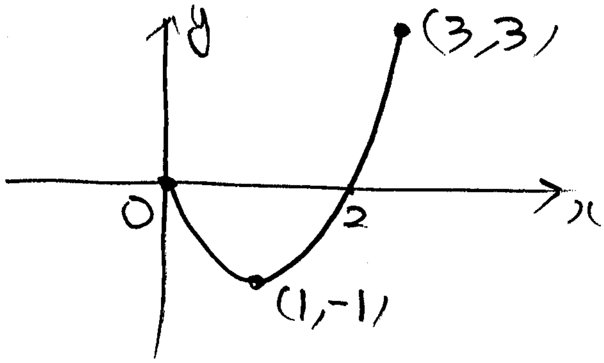
$$\lim_{x\to 0+} f(g(x)) = \lim_{t\to 0+} f(t) = 1$$

$x=0$ 에서 연속

3]

$$f(x) = x^2 - 2x = (x^2 - 2x + 1) - 1 = (x-1)^2 - 1$$

(1) $[0, 3] \iff 0 \leqq x \leqq 3$

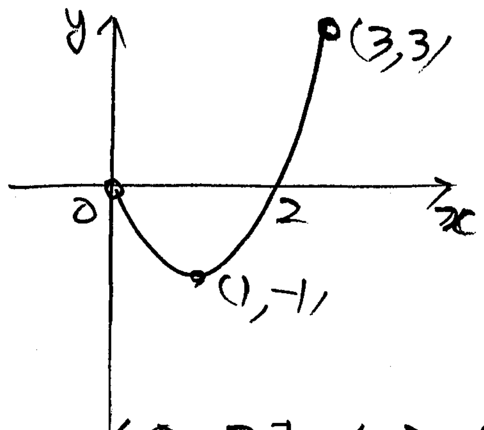


$f(3) = 9 - 6 = 3$

$M = f(3) = 3$

$m = f(1) = -1$

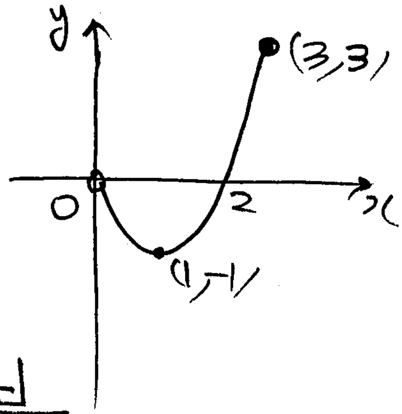
(2) $(0, 3) \iff 0 < x < 3$



M: 없다

$m = f(1) = -1$

(3) $(0, 3] \iff 0 < x \leqq 3$



$M = f(3) = 3$

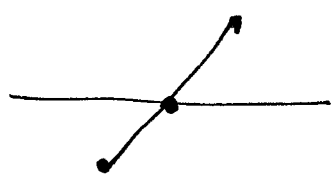
$m = f(1) = -1$

4]

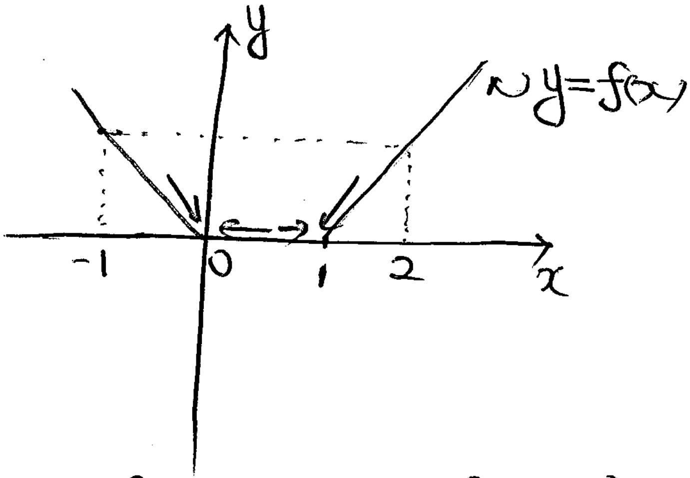
$f(x) = 2x^3 + x^2 - x - 4$

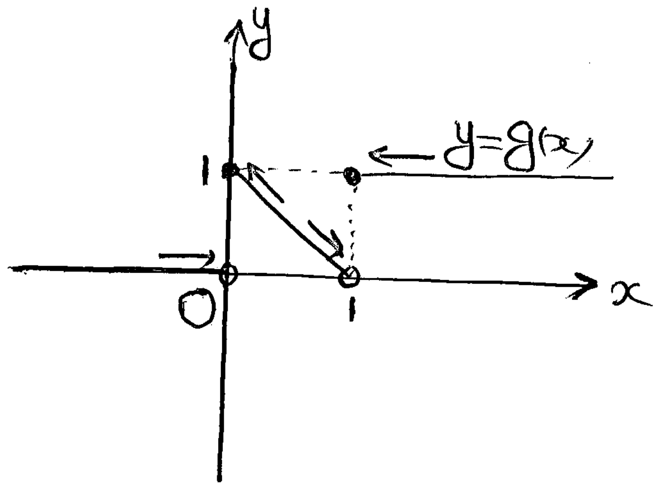
$f(1) = 2 + 1 - 1 - 4 = -2 < 0$

$f(2) = 16 + 4 - 2 - 4 = 14 > 0$



5-11





(1) $f(0)g(0) = 0 \cdot 1 = 0$

$$\lim_{x \to 0-} f(x)g(x) = 0 \cdot 0 = 0$$

$$\lim_{x \to 0+} f(x)g(x) = 0 \cdot 1 = 0$$

$x=0$ 에서 연속

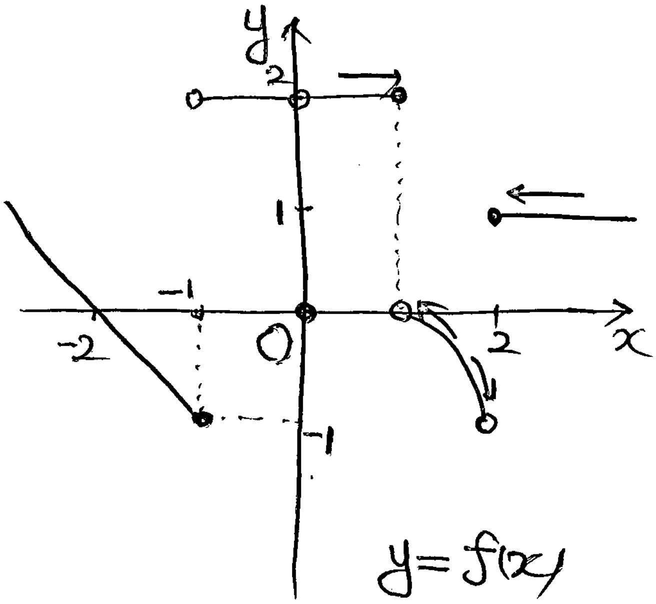
(2) $\dfrac{f(1)}{g(1)} = \dfrac{0}{1} = 0$

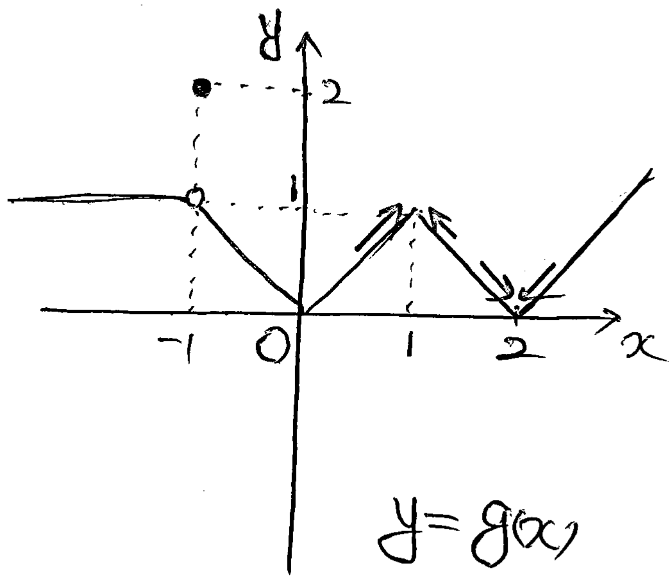
$$\lim_{x \to 1-} \frac{f(x)}{g(x)} = 0$$

$$\lim_{x \to 1+} \frac{f(x)}{g(x)} = \frac{0}{1} = 0$$

$x=0$ 에서 연속

5-21





(1) $f(2)g(2)=1\cdot 0=0$

$\lim_{x\to 2-} f(x)g(x)=-1\cdot 0=0$ x=2 에서 연속

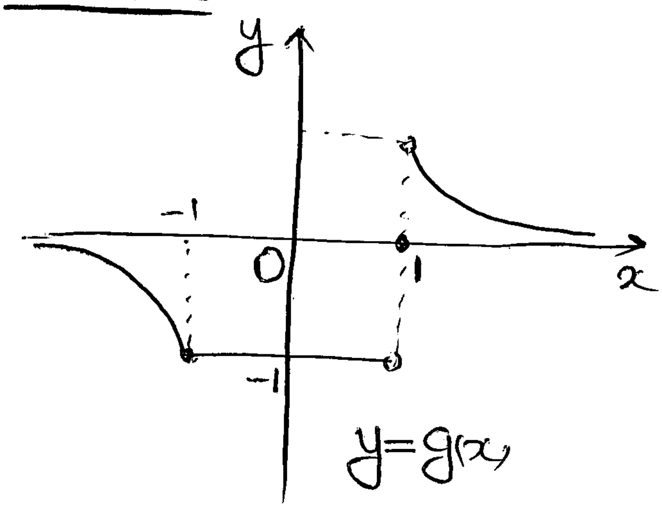
$\lim_{x\to 2+} f(x)g(x)=1\cdot 0=0$

(2) $\frac{f(1)}{g(1)}=\frac{2}{1}=2$

$\lim_{x\to 1-}\frac{f(x)}{g(x)}=\frac{2}{1}=2$ x=1 에서 불연속

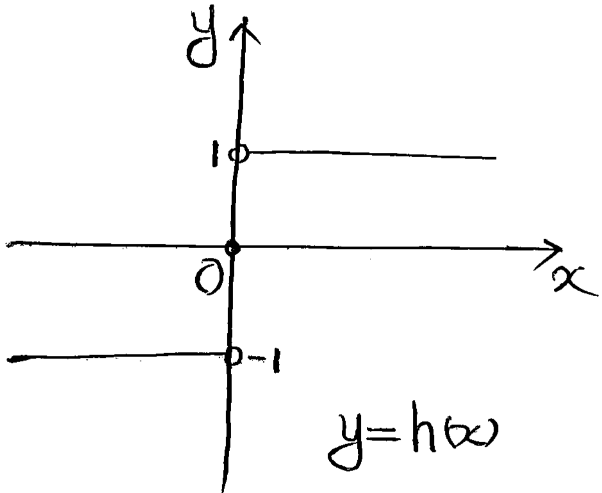
$\lim_{x\to 1+}\frac{f(x)}{g(x)}=\frac{0}{1}=0$

5-3 | 20



$y=g(x)$

$x=1$ 에서 불연속

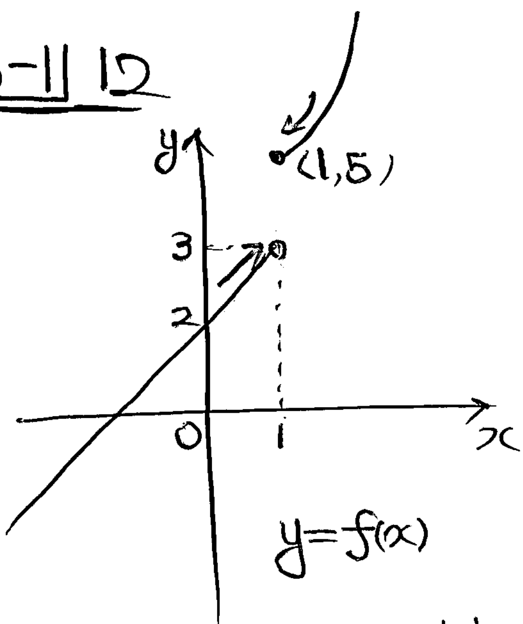


$y=h(x)$

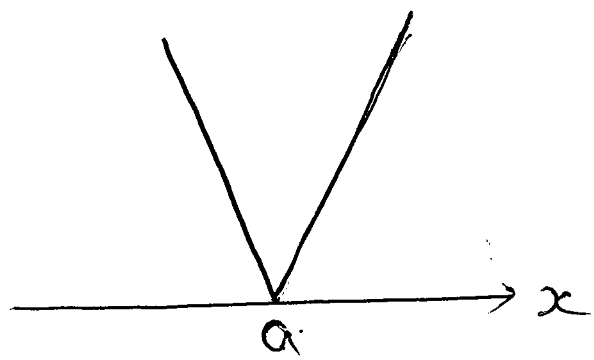
$x=0$ 에서 불연속

$f(x)=(x-1)\cdot x \qquad f(5)=4\cdot 5=20$

6-1 | 12



$y=f(x)$



$y=g(x)=|3x-a|$

$x=1$ 에서 불연속

$x=1$ 일때

$g(f(1))=g(5)=|15-a|$

$|15-a|=|9-a|$

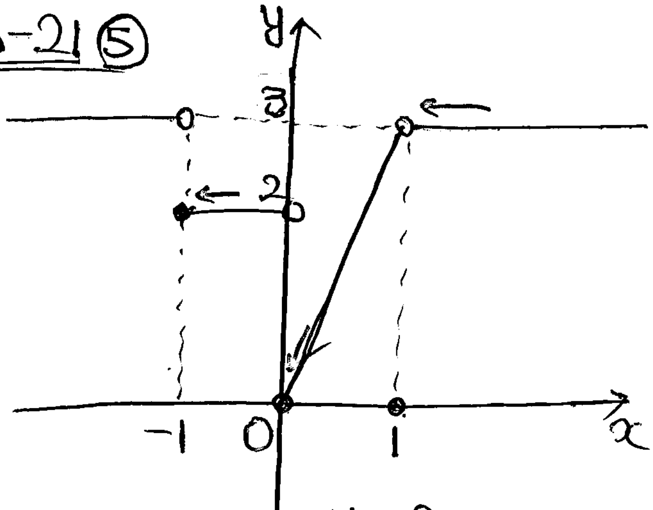
$15-a=-9+a$

$2a=24 \quad \boxed{a=12}$

$\lim_{x\to 1-} g(f(x))=\lim_{t\to 3-} g(t)=g(3)=|9-a| \quad f(x)=t$

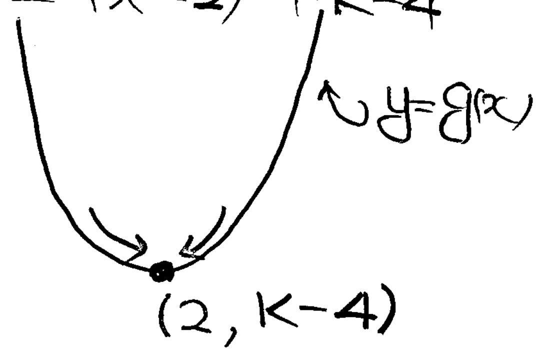
$\lim_{x\to 1+} g(f(x))=\lim_{t\to 5+} g(t)=g(5)=|15-a|$

6-21 ⑤



$y=f(x)$

$$g(x)=x^2-4x+K$$
$$=(x^2-4x+4)+K-4$$
$$=(x-2)^2+K-4$$



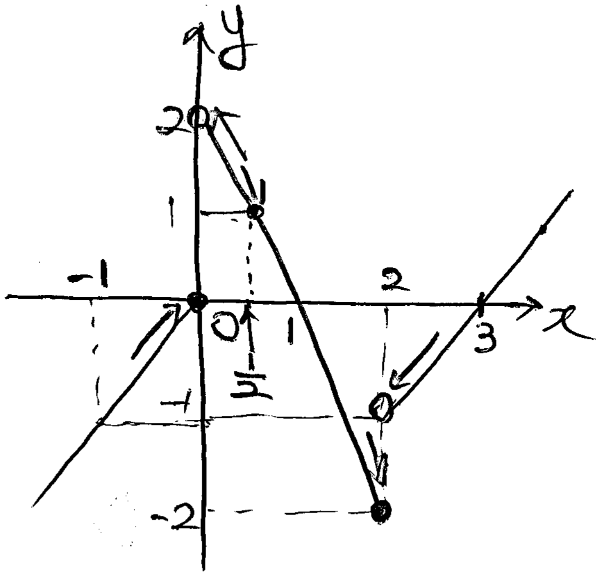
$$f(g(2)) = f(K-4)$$

$$\lim_{x\to 2} f(g(x)) \qquad g(x)=t$$

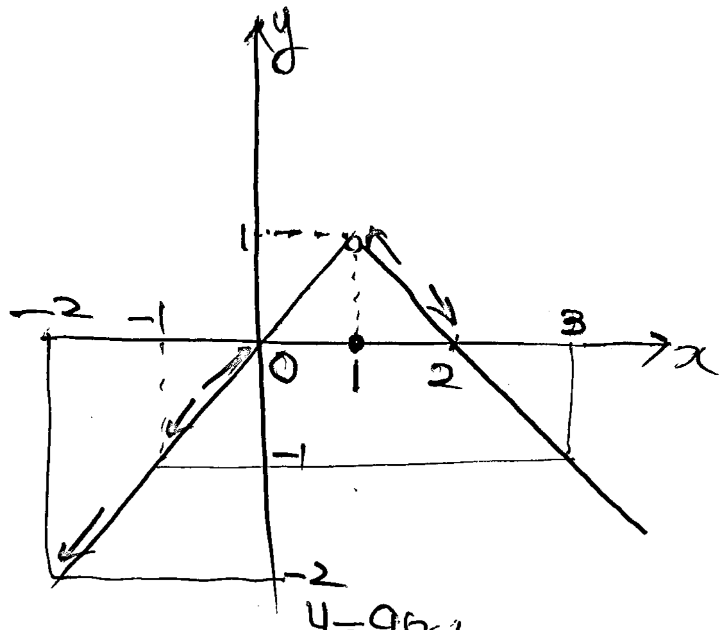
$$=\lim_{t\to K-4+} f(t) \qquad f(K-4)\neq \lim_{t\to K-4+} f(t) \qquad K-4=1$$

$$\boxed{K=5}$$

6-31



$y=f(x)$



$y=g(x)$

$y=(g\circ f)(x)=g(f(x))$

$\boxed{x=0}$, $\boxed{x=2}$ $f(x)=1$

$\boxed{x=\frac{1}{2}}$

㉠ $x=0$ 일때

$g(f(0))=g(0)=0$

$\lim_{x\to 0-} g(f(x))=\lim_{t\to 0-} g(t)=0$ | $f(x)=t$

$\lim_{x\to 0+} g(f(x))=\lim_{t\to 2-} g(t)=0$

$x=0$ 에서 연속

㉡ $x=2$ 일때

$g(f(2))=g(-2)=-2$

$\lim_{x\to 2-} g(f(x))=\lim_{t\to -2+} g(t)=-2$ | $f(x)=t$

$\lim_{x\to 2+} g(f(x))=\lim_{t\to -1+} g(t)=-1$ $\boxed{x=2\text{ 에서 불연속}}$

㉢ $x=\frac{1}{2}$ 일때

$g(f(\frac{1}{2}))=g(1)=0$

$\lim_{x\to \frac{1}{2}-} g(f(x))=\lim_{t\to 1+} g(t)=1$ $f(x)=t$

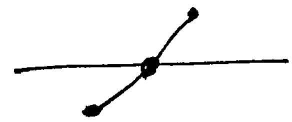
$\boxed{x=\frac{1}{2}\text{ 에서 불연속}}$

7-1 1

(1) $f(x)=x^4+x^3-7x+1 \quad (1,2)$

$f(1)=1+1-7+1=-4<0$

$f(2)=16+8-14+1=11>0$



(2) $f(x)=(x^3+2x+4)(2x^2-1) \quad (-1,1)$

$f(-1)=(-1-2+4)(2-1)=1>0$

$f(1)=(1+2+4)(2-1)=7>0$

$f\left(\frac{\sqrt{2}}{2}\right)=0, \quad f\left(-\frac{\sqrt{2}}{2}\right)=0$

7-2 1 ③

$f(x)=x^3+3x^2+4x-6$

$f(-2)=-8+12-8-6=-10<0$

$f(-1)=-1+3-4-6=-8<0$

$f(0)=-6<0$

$f(1)=1+3+4-6=2>0$

7-3 1 2

	$f(-1)$	$f(1)$	$f(2)$
ㄱ	+	−	+
ㄴ	−	+	−

2

2-11-4

$$4+2+a=2+b$$

$$a-b=-4$$

2-21

(1)

$$f(1)=\lim_{x\to 1}f(x)$$

$$5=\lim_{x\to 1}\frac{x^2+ax+b}{x-1}=\lim_{x\to 1}\frac{(x-1)(x+4)}{x-1}$$

$a=3,\ b=-4$ $\qquad ab=-12$

(2)

$$f(1)=\lim_{x\to 1}f(x)$$

$$\frac{1}{2}=\lim_{x\to 1}\frac{\sqrt{x^2+a}-b}{x-1}$$

$$\sqrt{1+a}-b=0$$

$$b=\sqrt{1+a}$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2+a}-\sqrt{1+a}}{x-1}$$

$$=\lim_{x\to 1}\frac{(x+1)(x-1)}{(x-1)(\sqrt{x^2+a}+\sqrt{1+a})}$$

$$=\frac{2}{2\sqrt{1+a}}=\frac{1}{\sqrt{1+a}}$$

$a=3,\ b=2$

$$10a+b=32$$

2-3) $\frac{1}{4}$

$$f(4) = \lim_{x\to 4} f(x)$$
$$= \lim_{x\to 4} \frac{1}{2(x-2)}$$
$$= \frac{1}{4}$$

$$\frac{1}{2} - \frac{1}{x-2}$$
$$= \frac{(x-2)-2}{2(x-2)}$$
$$= \frac{x-4}{2(x-2)}$$

2-4)

(1) $x=1$; $0 = 1 - a^3$ $a=1$

$$f(a) = f(1) = \lim_{x\to 1} f(x)$$
$$= \lim_{x\to 1} \frac{x^3-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1}$$
$$= 3$$

(2)

$x=0$; $\boxed{0=b}$

$x=1$; $0 = 1-a+b$ $\boxed{a=1}$

$$x(x-1)f(x) = x^3 - x = x(x^2-1)$$

$$f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0}(x+1) = 1$$
$$f(1) = \lim_{x\to 1} f(x) = \lim_{x\to 1}(x+1) = 2$$

(合計) $= 3$

2-51 ②

$n \le x < n+1 \iff [x] = n$ (n은 정수)

$$f(n) = \frac{[n]^2+n}{[n]} = \frac{n^2+n}{n} = n+1$$

$$\lim_{x\to n-} \frac{[x]^2+x}{[x]} = \frac{(n-1)^2+n}{n-1} = \frac{n^2-n+1}{n-1}$$

$$\lim_{x\to n+} \frac{[x]^2+x}{[x]} = \frac{n^2+n}{n} = n+1$$

$$n+1 = \frac{n^2-n+1}{n-1}$$

$$n^2-1 = n^2-n+1 \qquad n=2$$

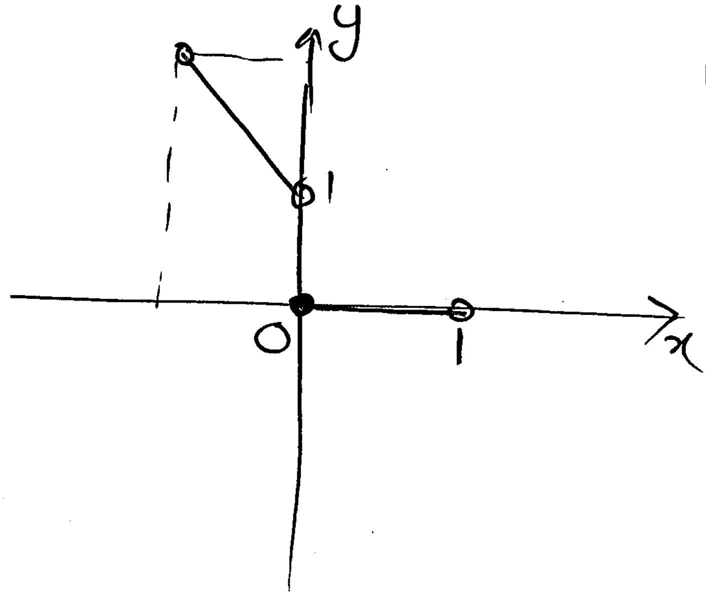
2-61

$(-1,1) \iff -1 < x < 1$

(1) $-1 < x < 0$; $[x] = -1$

$0 \le x < 1$; $[x] = 0$

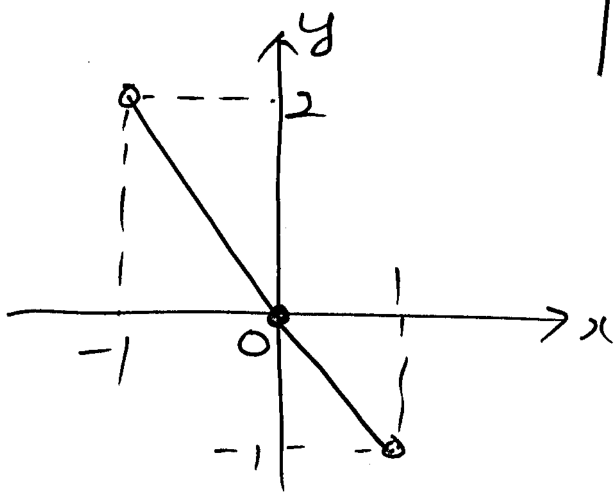
$$f(x) = (x-1)[x] = \begin{cases} -x+1 & (-1<x<0) \\ 0 & (0 \le x < 1) \end{cases}$$



$x=0$ 에서 불연속

$[x+m] = [x]+m$ (m은정수)

$$g(x) = x([x]-1) = \begin{cases} -2x & (-1 \le x < 0) \\ -x & (0 \le x < 1) \end{cases}$$



$\boxed{\text{연속}}$

2-가-1

$f(1)g(1) = 4(1+k)$

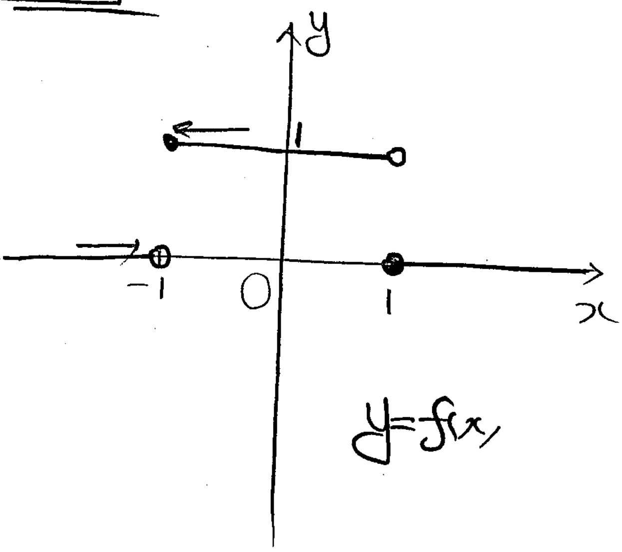
$\lim_{x \to 1-} f(x)g(x) = 1+k$, $\lim_{x \to 1+} f(x)g(x) = 4(1+k)$

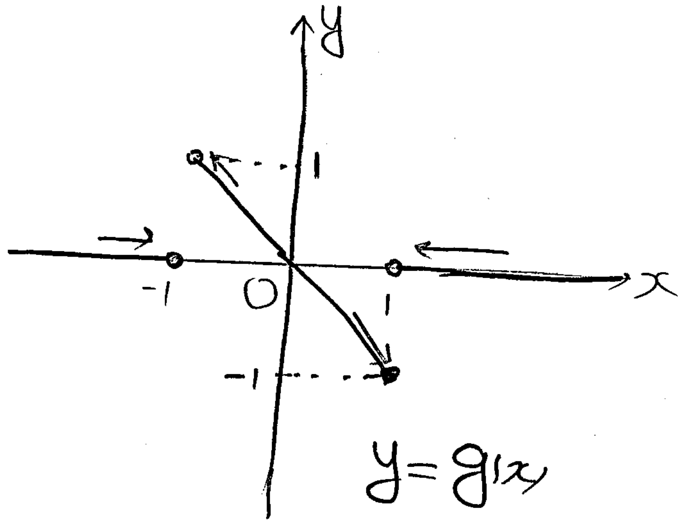
$4(1+k) = 1+k$

$3(1+k) = 0$

$k = -1$

281





(1) $x=1$ 일때

$f(g(1)) = f(-1) = 1$

$\lim_{x\to 1-} f(g(x)) = \lim_{t\to -1+} f(t) = 1$

$\lim_{x\to 1+} f(g(x)) = f(0) = 1$

$x=1$ 에서 연속

(2) $x=-1$ 일때

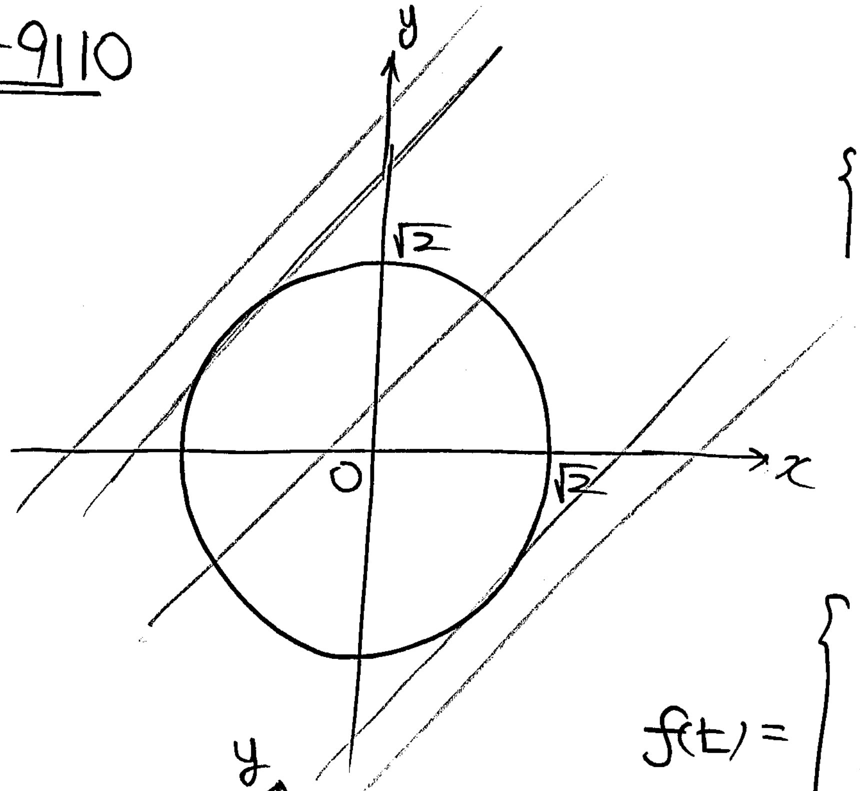
$f(-1)\cdot g(-1) = (-1)\cdot 0 = 0$

$\lim_{x\to -1-} f(x)g(x) = 0$

$\lim_{x\to -1+} f(x)g(x) = 1$

$x=-1$ 에서 불연속

2-9) 10

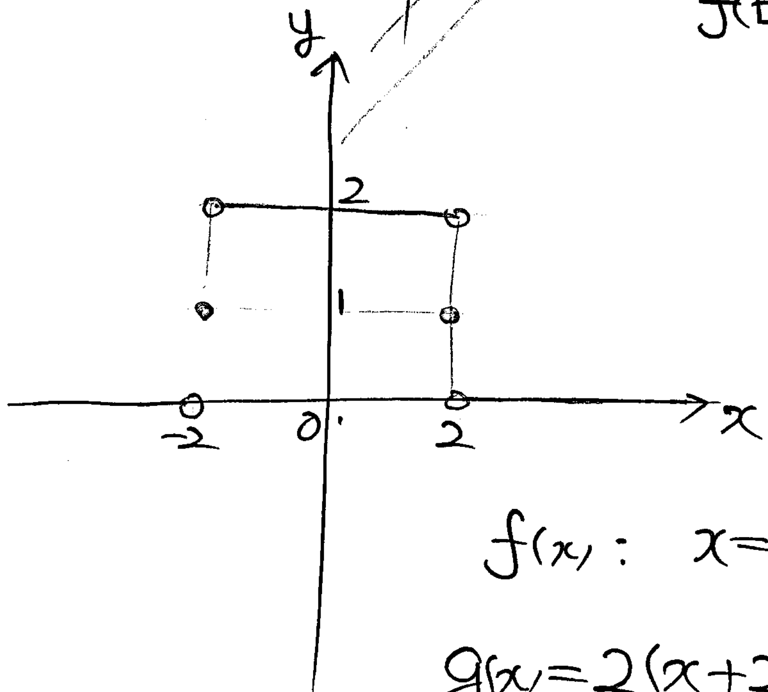


$$\begin{cases} x - y + t = 0 \\ (0,0) \end{cases}$$

$$\frac{|t|}{\sqrt{1+1}} = \sqrt{2}$$

$$t = \pm 2$$

$$f(t) = \begin{cases} 0 & (t < -2) \\ 1 & (t = -2) \\ 2 & (-2 < t < 2) \\ 1 & (t = 2) \\ 0 & (t > 2) \end{cases}$$



$f(x)$: $x = \pm 2$ 에서 불연속

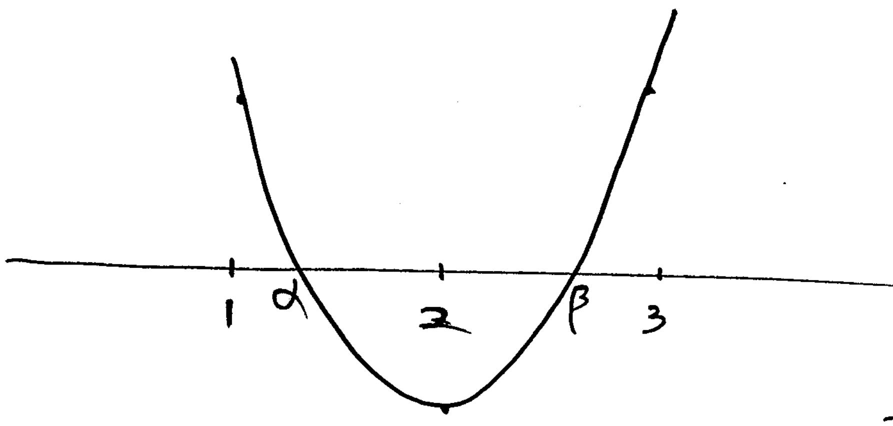
$g(x) = 2(x+2)(x-2)$

$g(3) = 10$

2-10| 3

$f(x) = (x-1)(x-2) + (x-2)(x-3) + (x-1)(x-3)$

$f(1) = 2$, $f(2) = -1$, $f(3) = 2$



$1 < \alpha < 2$

또는 $2 < \beta < 3$

$\boxed{k=1 \text{ 또는 } k=2}$

$1+2=3$

2-11| 15

㉠ $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{f(x)-x^2}{x-1} = 2$

$f(x) = x^2 + 2x + a$

㉡ $g(1) = \lim_{x\to 1} g(x)$

$k = \lim_{x\to 1} \frac{2x+a}{x-1}$

$= \lim_{x\to 1} \frac{2(x-1)}{x-1}$

$= 2$

$a = -2$

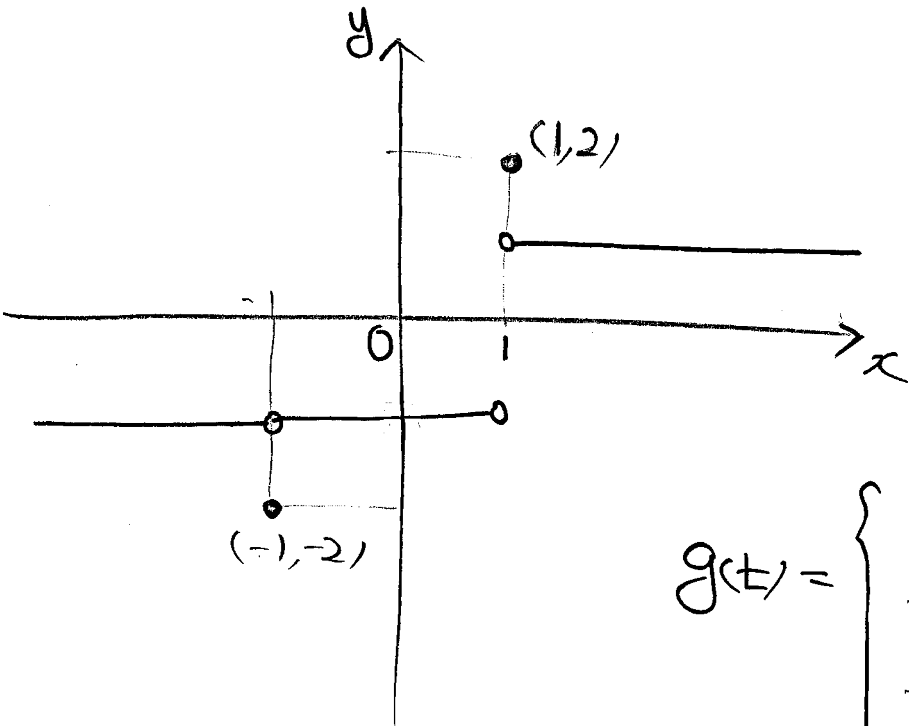
$f(x) = x^2 + 2x - 2$

$f(3) = 9 + 6 - 2 = 13$

$k + f(3) = 15$

2-1213

$$f(x)=\begin{cases} 1 & (x>1) \\ -1 & (x<1,\ x\neq -1) \\ 2x & (x=\pm 1) \end{cases}$$

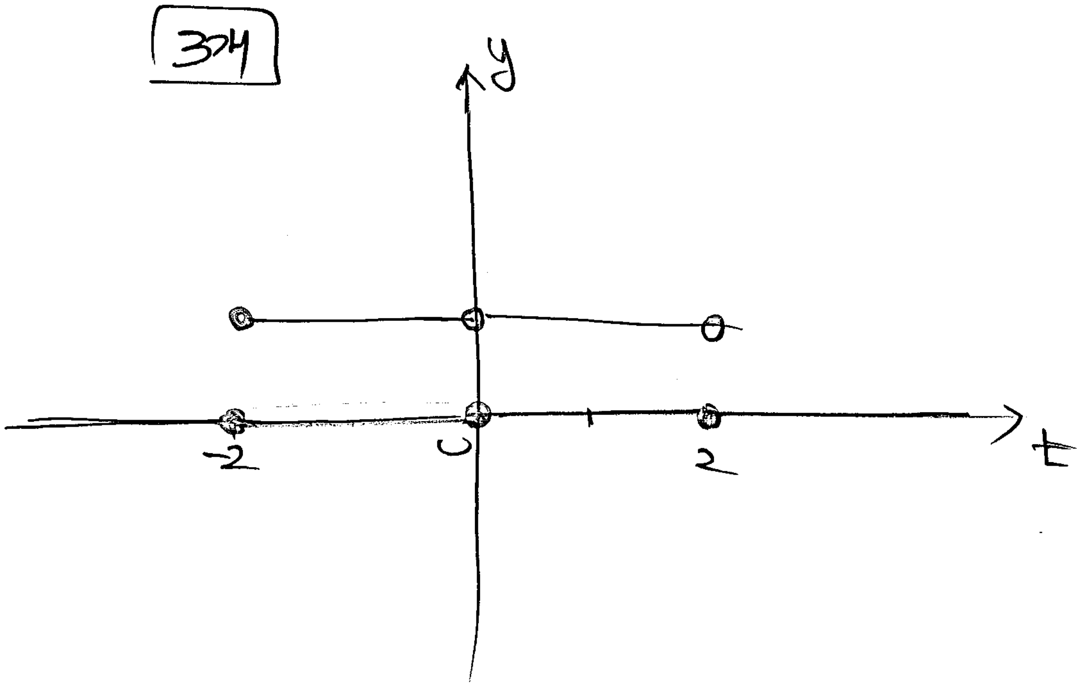


$$g(t)=\begin{cases} 0 & t\le -2 \\ 1 & (-2<t<0) \\ 0 & (t=0) \\ 1 & (0<t<2) \\ 0 & (t\ge 2) \end{cases}$$

$t=-2,\ 0,\ 2$

에서 불연속

$\boxed{3개}$



2-13 | 7

$x=a$ 에서 연속

$f(a)\cdot g(a)=(a+2)(2a-8)$

$\lim_{x\to a-} f(x)g(x) = (a^2-2a)(2a-8)$

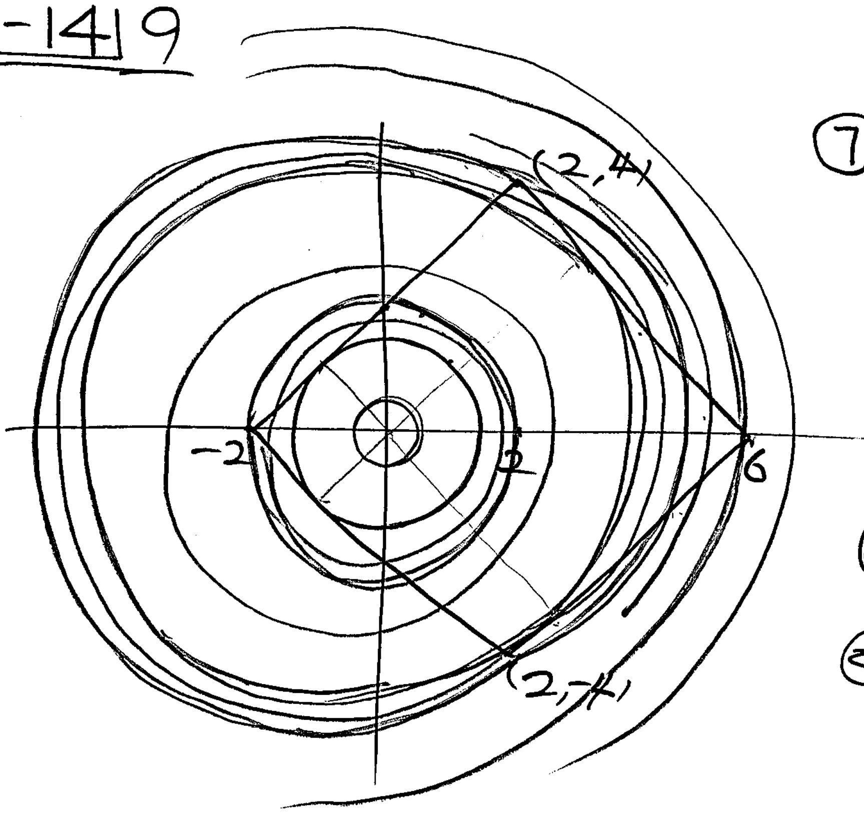
$\lim_{x\to a+} f(x)g(x) = (a+2)(2a-8)$

$(a+2)(2a-8)=(a^2-2a)(2a-8)$

$(a^2-3a-2)(2a-8)=0$

$3+4=7$

2-14 | 9



㉠ $y=x+2$

$x-y+2=0$
$(0,0)$

$\frac{2}{\sqrt{1+1}}=\sqrt{2}$

㉡ 2

㉢ $y=-x+6$

$x+y-6=0$

$(0,0)$ $\frac{|-6|}{\sqrt{1+1}}=3\sqrt{2}$

㉣ $2\sqrt{5}$

㉤ 6

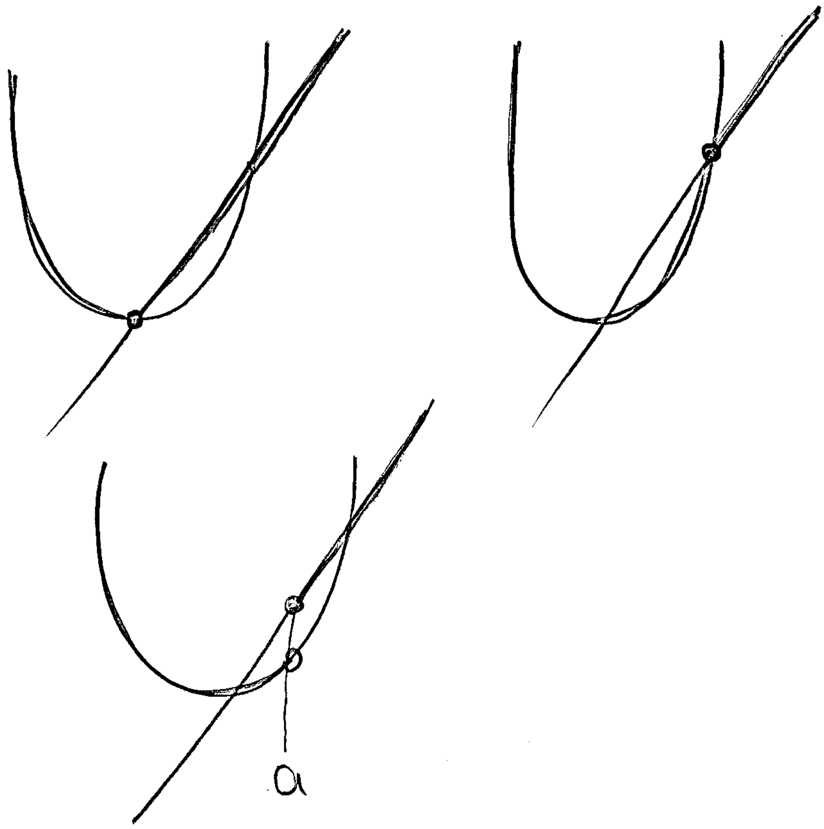
$$f(t)=\begin{cases} 0 & (0<t<\sqrt{2}) \\ \boxed{2 \quad (t=\sqrt{2})} \\ 4 & (\sqrt{2}<t<2) \\ \boxed{3 \quad (t=2)} \\ 2 & (2<t<3\sqrt{2}) \\ \boxed{4 \quad (t=3\sqrt{2})} \\ 6 & (3\sqrt{2}<t<2\sqrt{5}) \\ \boxed{4 \quad (t=2\sqrt{5})} \\ 2 & (2\sqrt{5}<t<6) \\ \boxed{1 \quad (t=6)} \\ 0 & (t>6) \end{cases}$$

$m=5 \qquad f(a_{m-1})=f(a_4)=4$

$5+4=9$

2-15)

ㄱ

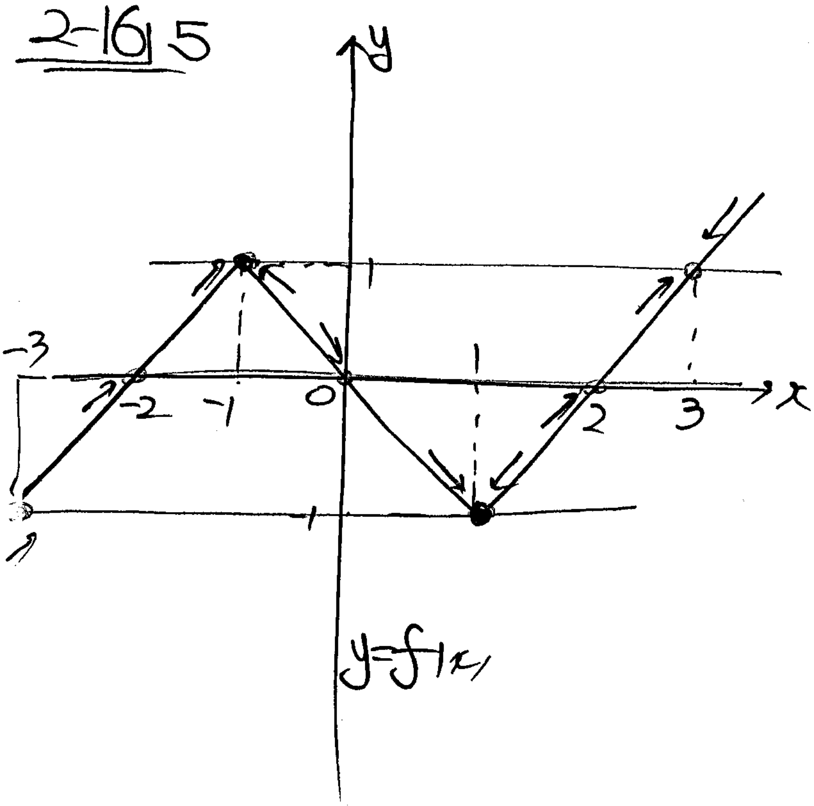


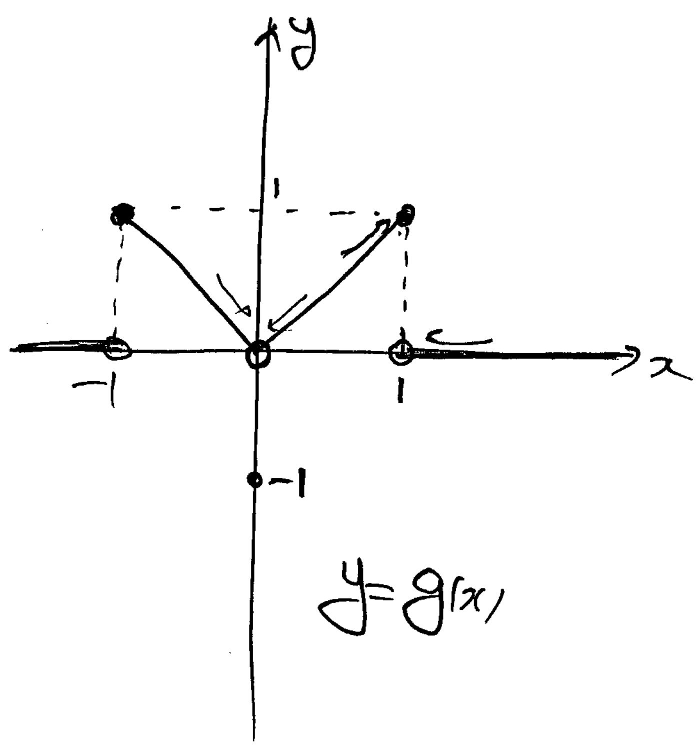
ㄴ

~~ㄷ~~ $N(f, g)$; $f(x) = g(x)$ 의 교점의 개수

$N(h\circ f, h\circ g)$; $\{f(x)\}^2 = \{g(x)\}^2$ 의 교점의 개수

2-16) 5





$y=g(f(x))$ $\quad$ $y=g(t)$

$f(x)=t=-1,\ 0,\ 1 \longrightarrow x=-1,\ 3$

$-1 \downarrow\ x=-3, 1$ $\quad$ $0 \downarrow\ x=-2, 0, 2$

① $x=-3$

$g(f(-3))=g(-1)=1$

$\displaystyle\lim_{x\to-3-} g(f(x)) = \lim_{t\to-1-} g(t)=0$

$x=-3$ 에서 불연속

② $x=1$

$g(f(1))=g(-1)=1$

$\displaystyle\lim_{x\to1-} g(f(x)) = \lim_{t\to-1+} g(t)=1,\ \lim_{x\to1+} g(f(x)) = \lim_{t\to-1+} g(t)=1$

$x=1$ 에서 연속

③ $x=-2$

$g(f(-2))=g(0)=-1$

$\displaystyle\lim_{x\to-2-} g(f(x)) = \lim_{t\to0-} g(t)=0$

$x=-2$ 에서 불연속

④ $x=0$

$g(f(0))=g(0)=-1$

$\displaystyle\lim_{x\to0-} g(f(x)) = \lim_{t\to0+} g(t)=0$

$x=0$ 에서 불연속

⑤ $x=2$

$$g(f(2))=g(0)=-1$$

$$\lim_{x\to 2-} g(f(x))=\lim_{t\to 0-} g(t)=0$$

$x=2$에서 불연속

⑥ $x=-1$

$$g(f(-1))=g(1)=1$$

$$\lim_{x\to -1-} g(f(x))=\lim_{t\to 1-} g(t)=1$$

$$\lim_{x\to -1+} g(f(x))=\lim_{t\to 1-} g(t)=1$$

$x=-1$ 에서연속

⑦ $x=3$

$$g(f(3))=g(1)=1$$

$$\lim_{x\to 3-} g(f(x))=\lim_{t\to 1-} g(t)=1$$

$$\lim_{x\to 3+} g(f(x))=\lim_{t\to 1+} g(t)=0$$

$x=3$ 에서불연속

5개

2-17) 3

$g(x) = f(x) - x$

$g(0) = 1 - 0 = 1 > 0$

$g(\frac{1}{4}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} > 0$

①

$g(\frac{1}{2}) = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6} < 0$

②

$g(\frac{3}{4}) = \frac{3}{2} - \frac{3}{4} = \frac{3}{4} > 0$

③

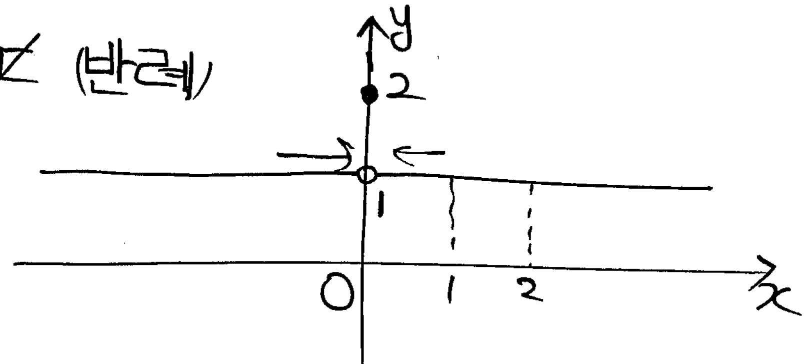
$g(1) = 0 - 1 < 0$

2-18)

ㄱ

$\boxed{f(0) = \lim_{x \to 0} f(x) = a}$ $x = a$ 연속 ?

(반례)



$y = f(x)$

$f(f(0)) = f(2) = 1$

$\lim_{x \to 0-} f(f(x)) = f(1) = 1$

$\lim_{x \to 0+} f(f(x)) = f(1) = 1$

$x = 0$ 에서 연속

2-191

ㄱ. $g(f(1))=g(0)=-1$

$\lim_{x\to1-} g(f(x)) = \lim_{t\to0+} g(t)=-1$, $\lim_{x\to1+} g(f(x))=\lim_{t\to0+} g(t)=-1$

~~ㄴ~~. $g(f(-1))=g(0)=-1$

$\lim_{x\to-1-} g(f(x)) = \lim_{t\to0-} g(t)=0$ $\quad x=-1$ 에서 불연속

ㄷ. $0<x<2 \rightarrow 0\le f(x)<2$

$y=g(f(x))$ $\quad 0<x<2$ 에서 연속

$g(f(0))=g(1)<0$

$g(f(2))=g(2)>0$