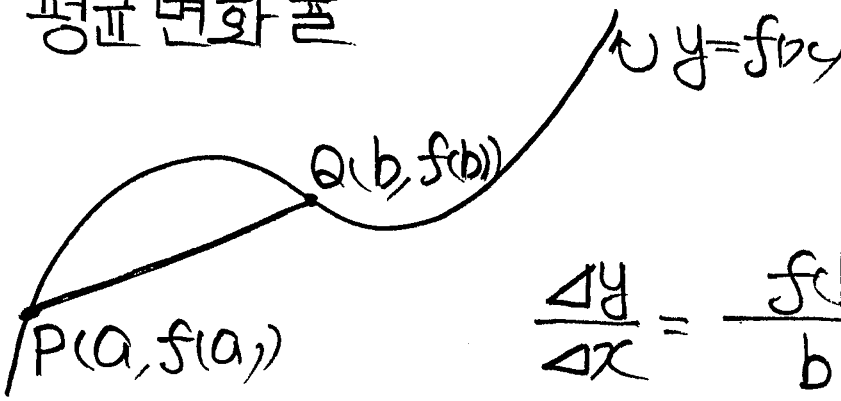
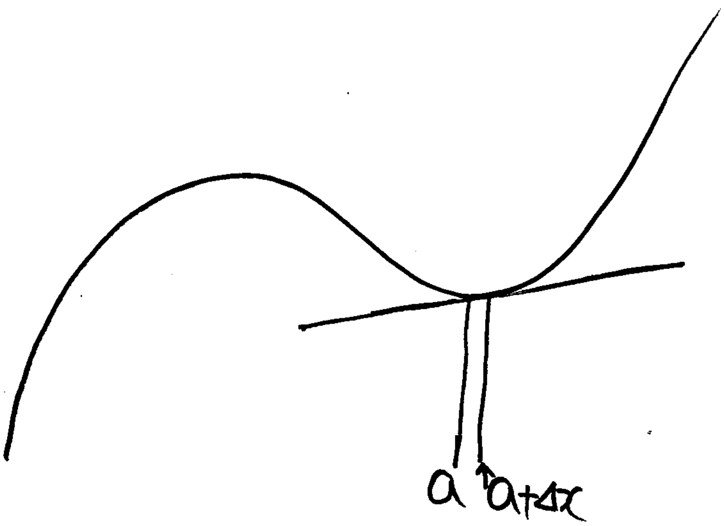
* 미 분 *

1 평균변화율



$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$

2 미분계수



$$f(a) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(a+\Delta x)-f(a)}{\Delta x}$$

$$\boxed{= \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}}$$

$$\boxed{= \lim_{x \to a} \frac{f(x)-f(a)}{x-a}}$$

① 기하학적의미 : $x=a$ 에서의 접선의 기울기

② 물리학적의미 : $x=a$ 에서의 순간 변화율

③ 해석학적의미 : $x=a$ 에서의 미분계수

cf) $$\lim_{h \to 0} \frac{f(a+nh)-f(a)}{mh} = \frac{n}{m} f(a)$$

3 미분가능

$y=f(x)$가 $x=a$에서 미분가능

① $x=a$에서 연속

② $f'(a)$가 존재

4 도함수

$y=f(x) \xrightarrow{\text{미분}} y'=f'(x)$

└ f 프라임 x

$\frac{dy}{dx}=\frac{d}{dx}f(x)$

└ 읽기: dy-dx

$$\boxed{f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}}$$

◦ 공식

$y=c \rightarrow y'=0$

$y=x \rightarrow y'=1$

$y=x^n \rightarrow y'=nx^{n-1}$

$y=(f(x))^n \rightarrow y'=n(f(x))^{n-1}f'(x)$

$y=f(x)\cdot g(x) \rightarrow y'=f'(x)g(x)+f(x)\cdot g'(x)$

$y=f(g(x)) \rightarrow y'=f'(g(x))\cdot g'(x)$

5 로피탈의 정리 (객관식 풀이용)

$$\lim_{x \to a} \frac{g(x)}{f(x)} = K$$

「$f(x)$, $g(x)$: 미분가능

$f(a) = 0$, $g(a) = 0$」

$$\boxed{\lim_{x \to a} \frac{g(x)}{f(x)} = \lim_{x \to a} \frac{g'(x)}{f'(x)} = K}$$

P93

EX) $f(x)=2x^2+1$

$$\frac{\Delta y}{\Delta x}=\frac{f(3)-f(1)}{3-1}=\frac{(18+1)-(2+1)}{2}=\frac{16}{2}=8$$

P94 EX)

$f(x)=x^2+2x$

$f'(x)=2x+2 \qquad f'(2)=6$

P95

EX) ㄱ ㄴ $\cancel{ㄷ}$ ㄹ

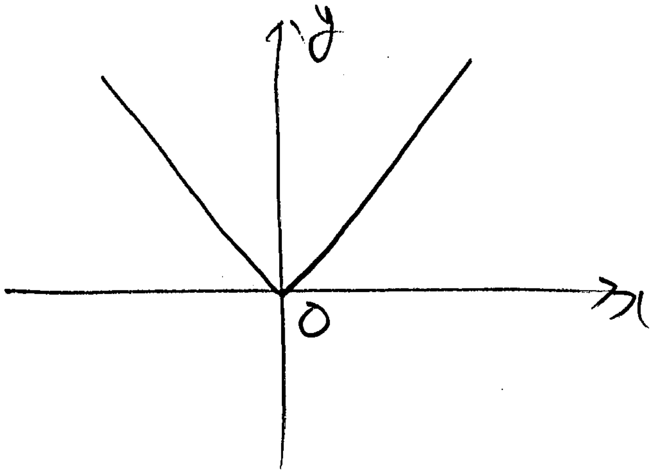
P96

EX) $f(x)=x^2-2x$

$f'(x)=2x-2 \qquad f'(1)=0$

P98

EX) $f(x)=|x|$



$x=0$ 에서 연속

$x=0$ 에서 미분불능

p99

1)

$f'(x) = -2x + 3$ $f'(2) = -1$

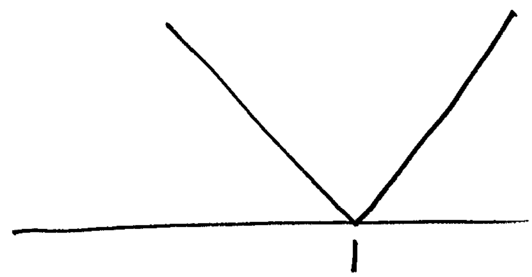
2)

$f(x) = x^2 - 2x + 5$ $f'(x) = 2x - 2$

$f'(-1) = -4$

3)

$$f(x) = \begin{cases} -x+1 & (x<1) \\ x-1 & (x \geq 1) \end{cases}$$



$$f'(x) = \begin{cases} -1 & (x<1) \\ 1 & (x \geq 1) \end{cases}$$

$f'(1)$: 존재 X

$x=1$ 에서 연속

$x=1$ 에서 미분불능

P101

1-1)

$f(x) = x^2 + 5x + 3$

(1) $\dfrac{\Delta y}{\Delta x} = \dfrac{f(5) - f(1)}{5-1} = \dfrac{44}{4} = 11$

$f(5) = 25 + 25 + 3$
$= 53$
$f(1) = 1 + 5 + 3 = 9$

(2) $f'(x) = 2x + 5$

$f'(c) = 2c + 5 = 11$ $\boxed{c = 3}$

1-2) 3

$f(x) = x^2 + 2x$ $f'(x) = 2x + 2$

$f'(1) = 4$

$f'(1) = \dfrac{\Delta y}{\Delta x} = \dfrac{f(k) - f(-1)}{k - (-1)}$

$4 = \dfrac{k^2 + 2k + 1}{k+1}$

$4 = k + 1$ $(\because k > -1)$

$k = 3$

1-3) $\frac{5}{2}$

$f(x) = x^2 + x + 1$, $f'(x) = 2x + 1$

$k = \dfrac{\Delta y}{\Delta x} = \dfrac{f(4) - f(1)}{4-1} = \dfrac{21 - 3}{3} = 6$

$f'(c) = k$ $2c + 1 = 6$ $c = \frac{5}{2}$

2-11

(1) $\lim_{x\to\sqrt{a}} \frac{f(x^2)-f(a)}{x-\sqrt{a}} = \lim_{x\to\sqrt{a}} \frac{f'(x^2)\cdot 2x}{1} = 2\sqrt{a}f'(a)$

(2) $\lim_{x\to a} \frac{a^2f(x)-x^2f(a)}{x-a} = \lim_{x\to a} \frac{a^2f'(x)-2xf(a)}{1} =$

$= a^2f'(a)-2af(a)$

2-21

$f(1)=0,\ f'(1)=6$

(1) $\lim_{x\to 1} \frac{f(x)}{x^2-1} = \lim_{x\to 1} \frac{f'(x)}{2x} = \frac{f'(1)}{2} = \frac{6}{2} = 3$

(2) $\lim_{x\to 1} \frac{f(x^3)}{x-1} = \lim_{x\to 1} \frac{f'(x^3)\cdot 3x^2}{1} = f'(1)\cdot 3 = 18$

(3) $\lim_{x\to 1} \frac{x^3-1}{f(x^2)} = \lim_{x\to 1} \frac{3x^2}{f'(x^2)\cdot 2x} = \frac{3}{2f'(1)} = \frac{1}{4}$

2-31 17

$f(3)-5=0 \qquad f(3)=5$

$\lim_{x\to 2} \frac{f'(x+1)\cdot 1}{2x} = \frac{f'(3)}{4} = 3$

$f'(3)=12 \qquad f(3)+f'(3)=17$

P105

3-1|

(1) (준식) $= \lim_{h\to 0} \dfrac{f'(a+3h)\cdot 3 - f'(a-2h)(-2)}{1}$

$= 3f'(a) + 2f'(a) = 5f'(a)$

(2) (준식) $= \lim_{h\to 0} \dfrac{f'(a+m^2h)\cdot m^2 - f'(a+n^2h)\cdot n^2}{(m-n)}$

$= \dfrac{m^2f'(a) - n^2f'(a)}{m-n} = (m+n)f'(a)$

3-2|

$f'(a) = 2$

$\lim_{h\to 0}\dfrac{g(h)}{h} = \lim_{h\to 0}\dfrac{f(a+2h)-f(a)}{h} = 2f'(a) = 4$

3-3|

ㄱ. $\lim_{h\to 0} h = 0$ 이므로 $\lim_{h\to 0} f(1+h) = f(1) = 0$

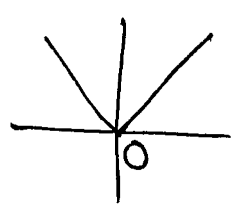
$f'(1) = \lim_{h\to 0}\dfrac{f(1+h)-f(1)}{h} = 0$

ㄴ (반례) $f(x) = |x|$ $f(h) = |h|$

$f(-h) = |-h| = |h|$

$f(h) - f(-h) = 0$

$\lim_{h\to 0}\dfrac{f(h)-f(-h)}{2h} = 0$



$f'(0)$: 존재 X

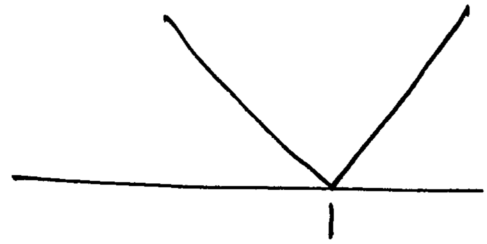
ㄷ $f(x)=|x-1|$

$f(1+h)=|1+h-1|=|h|$

$f(1-h)=|1-h-1|=|-h|=|h|$

$f(1+h)-f(1-h)=0$

$\lim_{h\to 0}\frac{f(1+h)-f(1-h)}{2h}=0$



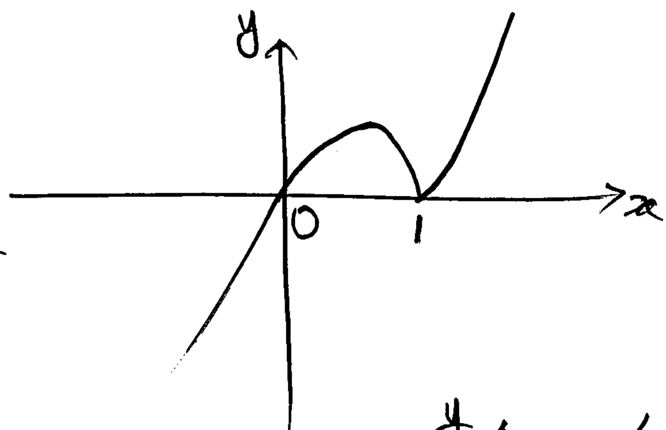
4-1)

$f(x)=\begin{cases}-x^2+x & (x<1)\\ x^2-x & (x\geq 1)\end{cases} \Rightarrow f'(x)=\begin{cases}-2x+1 & (x<1)\\ 2x-1 & (x\geq 1)\end{cases}$

$x=1$에서 연속

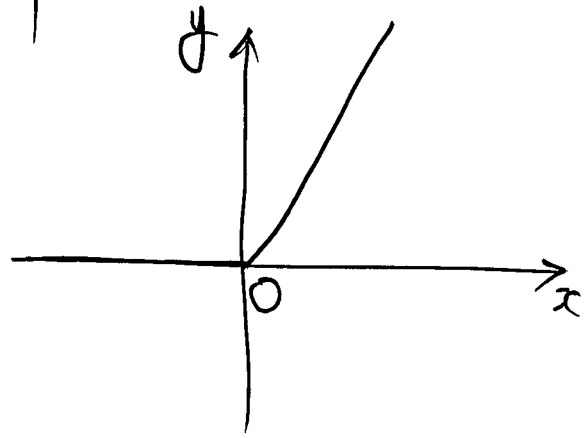
$x=1$에서

미분가능하지 않다



4-2)

$f(x)=\begin{cases}0 & (x<0)\\ 2x & (x\geq 0)\end{cases}$



(가) : 0 (나) : 2 (다) : 0

$x=0$에서 미분가능 ×

4-31

$f(0) = \lim_{x \to 0} f(x) = k$, $f'(0)$: 존재 X

ㄱ. $g(x) = xf(x)$

① $g(0) = 0$

② $\lim_{x \to 0} g(x) = \lim_{x \to 0} xf(x) = 0$

$g(0) = \lim_{x \to 0} g(x) = 0$ 이므로

$x = 0$ 에서 연속

③ $g'(0) = \lim_{h \to 0} \frac{g(h) - g(0)}{h} = \lim_{h \to 0} \frac{h f(h)}{h} = k$

$x = 0$ 에서 미분가능

~~ㄴ~~. $g(x) = x^2 + f(x)$

① $g(0) = f(0) = k$

② $\lim_{x \to 0} g(x) = \lim_{x \to 0} (x^2 + f(x)) = k$

$g(0) = \lim_{x \to 0} g(x) = k$ 이므로 $x = 0$ 에서 연속

③ $g'(0) = \lim_{h \to 0} \frac{g(h) - g(0)}{h} = \lim_{h \to 0} \frac{h^2 + f(h) - f(0)}{h}$

$= \lim_{h \to 0} \left(h + \frac{f(h) - f(0)}{h}\right)$: 존재 X

$x = 0$ 에서 미분가능 X

ㄷ $g(x) = \dfrac{1}{1+xf(x)}$

① $g(0) = 1$

② $\lim_{x \to 0} g(x) = \lim_{x \to 0} \dfrac{1}{1+xf(x)} = 1$

$g(0) = \lim_{x \to 0} g(x) = 1$ 이므로

$x=0$ 에서 연속

③ $g'(0) = \lim_{h \to 0} \dfrac{g(h) - g(0)}{h} = \lim_{h \to 0} \dfrac{\frac{1}{1+hf(h)} - 1}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{1} - \cancel{1} - hf(h)}{h(1+hf(h))}$

$= \lim_{h \to 0} \dfrac{-f(h)}{1+hf(h)} = -k$

$x=0$ 에서 미분가능

P108

EX) $f(x) = 3x^2+2$ $\quad f'(x) = 6x$

P109

$f(x) = x^2 + x$

$f'(x) = 2x+1$

$f'(-1) = -2+1 = -1$ $\quad f'(2) = 5$

P110

EX) (1) $y = x^3$ $\quad y' = 3x^2$

(2) $y = 10^5$ $\quad y' = 0$

P112

EX) (1) $y = x^3 + 4x$ $\quad y' = 3x^2 + 4$

(2) $y = (x^2+1)(x^2+x+1)$

$y' = 2x \cdot (x^2+x+1) + (x^2+1) \cdot (2x+1)$

P113

<u>1)</u>

$f'(x) = 2x - 3$

<u>2)</u>

$f'(x) = 3$ $\quad f'(3) = 3$

<u>3)</u>

(1) $y' = 12x^2 + 4x - 5$

(2) $y' = (6x+2)(x^2-x) + (3x^2+2x)(2x-1)$

P115

<u>5-1)</u>

(1) $f'(x) = (4x)(3x - x^3) + (2x^2-1) \cdot (3 - 3x^2)$

$f'(1) = 4 \cdot 2 + 1 \cdot 0$

$= 8$

(2) $f'(x) = 3(x^2-1)^2 \cdot 2x \qquad f'(1) = 0$

(3) $f'(x) = 2x(x+3)(x+4) + (x^2+2) \cdot 1 \cdot (x+4)$

$+ (x^2+2)(x+3) \cdot 1$

$f'(1) = 2 \cdot 4 \cdot 5 + 3 \cdot 1 \cdot 5 + 3 \cdot 4 \cdot 1$

$= 40 + 15 + 12 = 67$

<u>5-2)</u>

(1) $f'(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$

$f'(1) = 1 + 2 + 3 + 4 + 5 = 15$

(2) $f'(x) = 1 \cdot (1+2x)(1+3x)(1+4x)(1+5x)$

$+ (1+x) \cdot 2 \cdot (1+3x)(1+4x)(1+5x)$

$+ (1+x)(1+2x) \cdot 3 \cdot (1+4x)(1+5x)$

$+ (1+x)(1+2x)(1+3x) \cdot 4 \cdot (1+5x)$

$+ (1+x)(1+2x)(1+3x)(1+4x) \cdot 5$

$f'(0) = 1 + 2 + 3 + 4 + 5 = 15$

<u>5-3)</u>

$$\lim_{h \to 0} \frac{f'(1+h) \cdot 1 \cdot g(1+2h) + f(1+h) \cdot g'(1+2h) \cdot 2}{1}$$

$= f'(1) \cdot g(1) + 2f(1) \cdot g'(1)$

$f(x) = (x^2+1)(x+1)$ $\qquad$ $f(1) = 4$

$f'(x) = 2x(x+1) + (x^2+1)\cdot 1$ $\qquad$ $f'(1) = 4+2 = 6$

$g(x) = (x+1)(x^3+x)$ $\qquad$ $g(1) = 4$

$g'(x) = 1\cdot(x^3+x) + (x+1)(3x^2+1)$ $\qquad$ $g'(1) = 2+8 = 10$

(준식) $= f'(1)g(1) + 2f(1)g'(1)$

$= 24 + 80 = 104$

6-1 | 26

$f(x) = ax^2 + bx + 3$, $f'(x) = 2ax + b$

$\boxed{f(1) = -1}$, $\lim_{x \to 1} \frac{f'(x)}{3x^2} = \frac{f'(1)}{3} = 2$ $\boxed{f'(1) = 6}$

$f(1) = a + b + 3 = -1$ $\quad a + b = -4$

$f'(1) = 2a + b = 6$ $\quad -\underline{|2a + b = 6}$

$a = 10, \ b = -14$ $\quad -a \quad = -10$

$f'(x) = 20x - 14$ $\quad f'(2) = 40 - 14 = 26$

6-2 | 28

$f(x) = x^3 + x + 2$ $\quad f'(x) = 3x^2 + 1$

$af'(2) = 39$ $\quad f'(2) = 12 + 1 = 13$

$a = 3$

$f'(a) = f'(3) = 27 + 1 = 28$

6-3 | 43

$f(x) = x^4 + ax^2 + bx + 2a$, $f'(x) = 4x^3 + 2ax + b$

$\boxed{\begin{array}{l} f(-1) = -2 \\ f'(-1) = 9 \end{array}}$, $\lim_{x \to -1} \frac{f'(x)}{3x^2} = \frac{f'(-1)}{3} = 3$

$f(-1) = 1 + a - b + 2a = -2$ $\quad 3a - b = -3$

$f'(-1) = -4 - 2a + b = 9$ $\quad +\underline{|-2a + b = 13}$

$a = 10$

$a + b = 43$ $\quad b = 33$

7-11

$$f(x)=\begin{cases} ax^2+bx-1 & (x\geq 1) \\ x^3 & (x<1) \end{cases}$$

$a+b-1=1$

$$f'(x)=\begin{cases} 2ax+b & (x\geq 1) \\ 3x^2 & (x<1) \end{cases}$$

$2a+b=3$

$a+b=2$

$2a+b=3$

$\boxed{a=1,\ b=1}$

7-21

$$f(x)=\begin{cases} ax^n & (x\geq 1) \\ bx+c & (x<1) \end{cases}$$

$a=b+c$

$$f'(x)=\begin{cases} anx^{n-1} & (x\geq 1) \\ b & (x<1) \end{cases}$$

$an=b$

$c=a-b=a-an$

$$\frac{a+c}{b}=\frac{2a-an}{an}=\frac{2-n}{n}$$

7-31

$$f(x)=\begin{cases} ax^3-3x+10 & (x<2) \\ x^3+5x+b & (x\geq 2) \end{cases}$$

$8a-6+10=8+10+b$

$8a-b=14$

$$f'(x)=\begin{cases} 3ax^2-3 & (x<2) \\ 3x^2+5 & (x\geq 2) \end{cases}$$

$12a-3=12+5$

$12a=20$

$a=\frac{5}{3}$

$b=8a-14=\frac{40}{3}-14$

$=\frac{-2}{3}$

$\boxed{a+b=1}$

8-11

$f(x)=ax^2+bx+1 \quad (a\neq 0)$

$f'(x)=2ax+b$

$2ax^2+2bx+2+3x-1=(x+1)(2ax+b)$

$2ax^2+\underline{(2b+3)}x+\underline{1}=2ax^2+\underline{(b+2a)}x+\underline{b}$

$b=1$, $5=1+2a$ $\quad a=2$

$\boxed{\therefore f(x)=2x^2+x+1}$

8-21

$\boxed{f(x) = ax^n + \cdots}$ $(a \neq 0)$

$f'(x) = anx^{n-1}$

(좌변) $= \underline{3a}x^n + \cdots$ $n=3$

(우변) $= \underline{an}\,x^n + \cdots$

$f(x) = ax^3 + bx^2 + cx + 1$

$f'(x) = 3ax^2 + 2bx + c$

$3(ax^3 + bx^2 + cx + 1) = (x-1)(3ax^2 + 2bx + c)$

$3ax^3 + 3bx^2 + 3cx + 3 = 3ax^3 + (2b-3a)x^2$
$+ (c-2b)x - c$

$3b = 2b - 3a$ $3c = c - 2b$ $3 = -c$

$b = -3a$ $2c = -2b$ $c = -3$

$3 = -3a$ $b = -c = 3$

$a = -1$

$\boxed{f(x) = -x^3 + 3x^2 - 3x + 1}$

8-31

$f(x) = (x-2)^2 Q(x) + ax + b$ ⋯⋯ ㉠

$f'(x) = 2(x-2)\cdot Q(x) + (x-2)^2 Q'(x) + a$ ⋯ ㉡

$f(2) = 2a + b = 1$

$f'(2) = a = 3$　　　$b = -5$　　　$\boxed{\therefore 3x - 5}$

9-11

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\cancel{f(x)} + f(h) + 4xh + 2 - \cancel{f(x)}}{h}$

$= \lim_{h \to 0} \left(\frac{f(h) + 2}{h} + 4x \right)$

$= \lim_{h \to 0} \left(\frac{f(h) - f(0)}{h} + 4x \right)$

$= f'(0) + 4x$

$= 4x + 3$

$x = 0,\ y = 0$

$f(0) = f(0) + f(0) + 2$

$\boxed{f(0) = -2}$

9-2) 4

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{f(x)} + f(h) + 2x^2h + 2xh^2 - \cancel{f(x)}}{h}$$

$$= \lim_{h\to 0} \left(\frac{f(h)}{h} + 2x^2 + 2xh\right)$$

$$= f'(0) + 2x^2$$

$$f'(1) = f'(0) + 2 = 6$$

$$f'(0) = 4$$

$x=0, y=0$

$f(0) = f(0) + f(0) + 0$

$f(0) = 0$

9-3) 4

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{f(x)} + f(h) + 2xh - 1 - \cancel{f(x)}}{h}$$

$$= \lim_{h\to 0} \left(\frac{f(h) - 1}{h} + 2x\right)$$

$$= f'(0) + 2x$$

$x=0, y=0$

$f(0) = f(0) + f(0) - 1$

$f(0) = 1$

$$\lim_{x\to 1} \frac{f(x) - 2}{1} = 4$$

$$f'(1) - 2 = 4$$

$f'(1) = 6$

$f'(0) + 2 = 6$

$f'(0) = 4$

3-1) ②

$$\frac{\Delta y}{\Delta x} = \frac{f(a)-f(1)}{a-1} = \frac{(a^3+a^2+a+1)-4}{a-1}$$

$$= \frac{a^3+a^2+a-3}{a-1}$$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & 1 & -3 \\ & & 1 & 2 & 3 \\ \hline & 1 & 2 & 3 & 0 \end{array}$$

$$= \frac{(a-1)(a^2+2a+3)}{a-1}$$

$$= a^2+2a+3 = 18$$

$$a^2+2a-15=0$$

$$(a+5)(a-3)=0$$

$a>1$ 이므로

$a=3$

3-2) ②

$f(x) = x^3 - x + 1$, $f'(x) = 3x^2 - 1$

$$\frac{(a^3-a+1)-(-1+1+1)}{a+1} = 3a^2-1$$

$$\frac{a^3-a}{a+1} = 3a^2-1$$

$$a(a-1) = 3a^2-1$$

$$2a^2+a-1=0$$

$$(2a-1)(a+1)=0$$

$$a=\frac{1}{2}$$

3-3|

(1) $\lim_{h\to 0} \dfrac{f'(1-h)(-1) - f'(1+h)\cdot 1}{1} = -f'(1) - f'(1)$

$= -2f'(1) = -2$

(2) $\lim_{t\to 1} \dfrac{(t-1)(t+1)}{f(t) - f(1)}$ $\qquad \sqrt{x} = t$

$\qquad x = t^2$

$= \dfrac{2}{f'(1)} = 2$

(3) $\lim_{x\to 1} \dfrac{f(x^2) - x^2 f(1)}{x - 1}$

$= \lim_{x\to 1} \dfrac{f'(x^2)\cdot 2x - 2x f(1)}{1}$

$= 2f'(1) - 2f(1) = 2 - 4 = -2$

3-4| 90

$\lim_{x\to 1} \dfrac{f'(x^2)\cdot 2x}{1} = 2f'(1) = 90$

$f'(x) = 1 + 2x + \cdots + 9x^8$

$f'(1) = 1 + 2 + \cdots + 9 = \dfrac{9\cdot 10}{2} = 45$

3-5| ①

$f(2) = 1, \quad f'(2) = -2$

(준식) $= \lim_{h\to 0} \dfrac{f'(2-5h)(-5) - f'(2+3h)\cdot 3}{1} = -8f'(2) = 16$

3-6) 5

$f(3)=2 \qquad g(3)=1$

$f'(3)=1 \qquad g'(3)=2$

$h(x)=f(x)g(x)$

$h'(x)=f'(x)g(x)+f(x)g'(x)$

$h'(3)=f'(3)g(3)+f(3)\cdot g'(3)=1+4=5$

3-7) $\frac{5}{2}$

① $x=1$; $2f(1)=4 \qquad f(1)=2$

② $2x f(x)+(x^2+1)f'(x)=5x^4+3x^2+1$

$x=1$; $2f(1)+2f'(1)=9 \qquad \boxed{f'(1)=\frac{5}{2}}$

3-8) 12

$f(x)=\begin{cases} ax^3 & (x\geq 1) \\ bx+4 & (x<1) \end{cases} \qquad f'(x)=\begin{cases} 3ax^2 & (x\geq 1) \\ b & (x<1) \end{cases}$

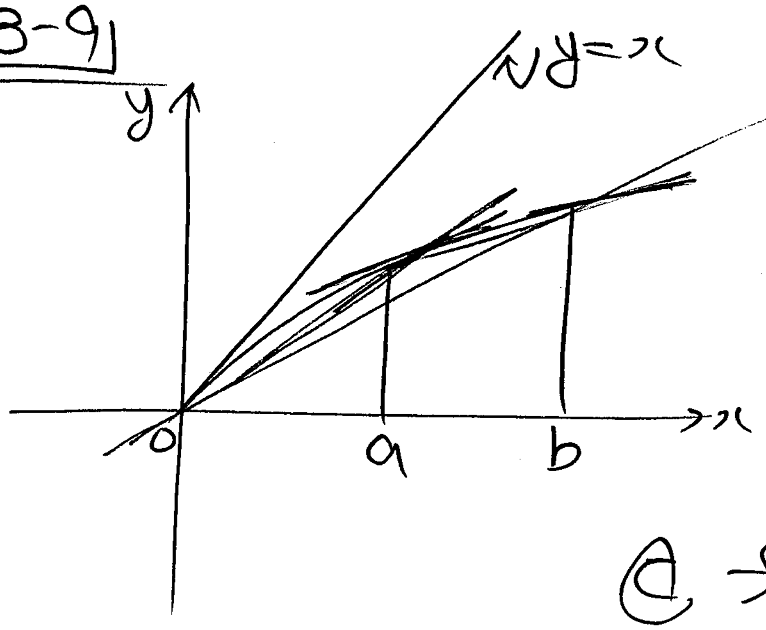
$a=b+4$, $3a=b$

$a=3a+4$

$2a=-4 \qquad a=-2,\ b=-6$

$\boxed{ab=12}$

3-9



ア $\dfrac{f(a)}{a} > \dfrac{f(b)}{b}$

イ $\dfrac{f(b)-f(a)}{b-a} < 1$

$f(b)-f(a) < b-a$

ウ $f'(a) > f'(b)$

3-10

(1) $y = 2f(x)\cdot f'(x)$

(2) $y = (f(x))^{\frac{1}{2}}$ $\qquad y' = \frac{1}{2}(f(x))^{-\frac{1}{2}}\cdot f'(x)$

$$= \frac{f'(x)}{2\sqrt{f(x)}}$$

3-11 18

$$\lim_{x\to 1}\left(\frac{f(x)-2}{x-1}\cdot\frac{f(x)}{-1}\right) = f'(1)\cdot(-f(1))$$

$$= -2f'(1) = -36$$

$$f'(1) = 18$$

3-12) 32

$$g(1) = \lim_{x\to1} g(x)$$

$$a = \lim_{x\to1} \frac{f(x^2)-f(1)}{x-1} = \lim_{x\to1} \frac{f'(x^2)\cdot 2x}{1}$$

$$= 2f'(1)$$

$$= 32$$

$$f'(x) = 4x^3+6x^2+6x$$

$$f'(1) = 4+6+6 = 16$$

3-13) 12

$$g(2) = \lim_{x\to2} g(x)$$

$$f'(2) = \lim_{x\to2} \frac{f(x)-4}{x-2}$$

$$\therefore \boxed{f(2) = 4}$$

$$f(2x) = 4f(x)$$

$$\boxed{f(4) = 4f(2) = 16}$$

$$f(0) = 4f(0)$$

$$f(0) = 0$$

$$\boxed{f(x) = ax^n + \cdots \quad (a\neq0)}$$

$$f(2x) = a(2x)^n + \cdots$$

$$= a\cdot 2^n\cdot x^n + \cdots$$

$$n=2$$

$$f(x) = ax^2+bx$$

$$f(2) = 4a+2b = 4$$

$$f(4) = 16a+4b = 16$$

$$b=0$$

$$a=1$$

$$f(x) = x^2$$

$$g(10) = \frac{100-4}{8} = 12$$

3-14) $\frac{45}{2}$

$(x^2+1)f(x) = x + x^2 + \cdots + x^{10}$

$x=1$; $2f(1)=10$ $\quad f(1)=5$

미분; $2xf(x) + (x^2+1)f'(x) = 1+2x+3x^2+\cdots+10x^9$

$x=1$; $2f(1)+2f'(1)=55$

$f'(1)=\frac{45}{2}$

3-15) ①

$f(x) = ax^{n+1}+bx^n+1 = (x-1)^2Q(x) \quad \cdots$ ㉠

$f'(x) = a(n+1)x^n + bnx^{n-1} = 2(x-1)\cdot 1\cdot Q(x) + (x-1)^2Q'(x) \cdots$ ㉡

㉠ $x=1$; $a+b+1=0$

㉡ $x=1$; $a(n+1)+bn=0$

$(a+b)n+a=0$

$-n+a=0 \quad a=n$

$\boxed{b=-1-n}$

3-16) 2

㉠ $k=1$

$$g(x)=xf(x)=\begin{cases} x-x^2 & (x<0) \\ x^3-x & (0\le x<1) \end{cases}$$

$$g'(x)=\begin{cases} 1-2x & (x<0) \\ 3x^2-1 & (0\le x<1) \end{cases}$$

$g'(0)$: 존재 X

㉡ $k=2$

$$g(x)=x^2f(x)=\begin{cases} x^2-x^3 & (x<0) \\ x^4-x^2 & (0\le x<1) \end{cases}$$

$$g'(x)=\begin{cases} 2x-3x^2 & (x<0) \\ 4x^3-2x & (0\le x<1) \end{cases}$$

$g'(0)=0$

미분가능

$$\boxed{k=2}$$

3-17)

(1) $g(x)=xf(x)$

$g(0)=0$, $\lim_{x\to 0} g(x)=\lim_{x\to 0} xf(x)=0$

$x=0$ 에서 연속

$$g'(0)=\lim_{x\to 0}\frac{g(x)-g(0)}{x-0}=\lim_{x\to 0}\frac{xf(x)}{x}$$

$$=\lim_{x\to 0}f(x)=a$$

$x=0$ 에서 미분가능

(2) $g(x)=(x-1)f(x)$

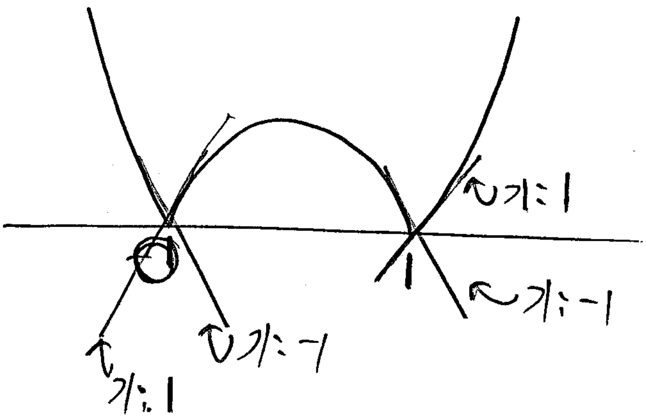
$g(1)=0$, $\lim_{x\to1} g(x)=\lim_{x\to1}(x-1)f(x)=0$

$x=1$ 에서 연속

$$g'(1)=\lim_{x\to1}\frac{g(x)-g(1)}{x-1}=\lim_{x\to1}\frac{(x-1)f(x)}{(x-1)}$$

$=0$ $\quad x=1$ 에서 미분가능

3-181



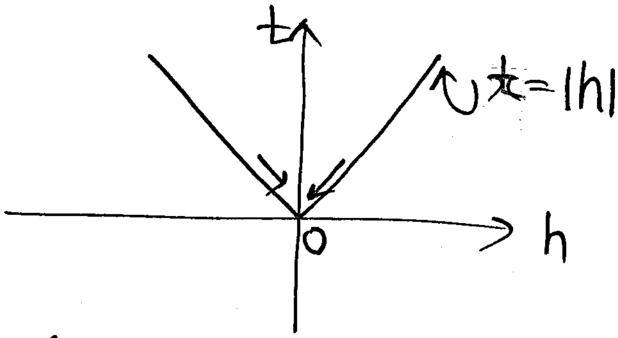
$f(x)=x^2-x$

$f'(x)=2x-1$

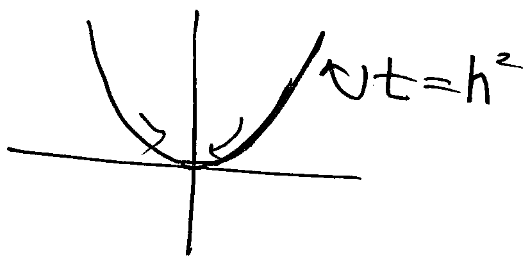
$f'(1)=1$

$f'(0)=-1$

㉠ $\lim_{t\to0+}\frac{g(t)}{t}=1$



㉡ $\lim_{t\to0+}\frac{g(1+t)-g(1)}{t}=f'(1)$

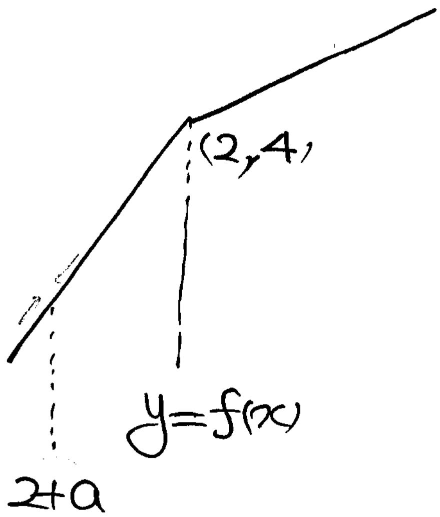


㉢ $\lim_{h\to0+}\frac{2h^{\cancel{2}}}{\cancel{h}}=0$

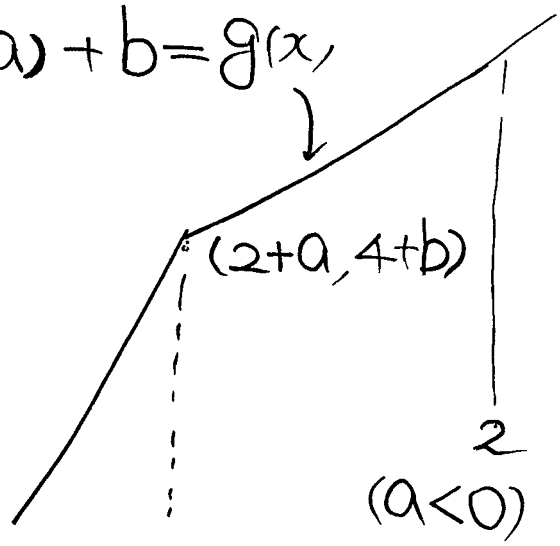
$g(1+h)=(1+h)^2-(1+h)=h^2+h$

$g(1-h)=-(1-h)^2+(1-h)=-1+2h-h^2+1-h=h-h^2$

3-19| 29



$f(x-a)+b=g(x)$



$h(x)=f(x)g(x)$

$h'(x)=f(x)g(x)+f(x)\cdot g(x)$

㉠ $x=a+2$ 일때

$f(a+2)=0$ $\quad f(a+2)=2a+4=0$ $\quad \boxed{a=-2}$

㉡ $x=2$ 일때

$g(2)=0$

$g(2)=f(2-a)+b$

$=f(4)+b=5+b=0$ $\quad \boxed{b=-5}$

$a^2+b^2=29$

3-201

$$f'(x) = \lim_{h \to 0} \frac{f(x-h) - f(x)}{-h}$$

$$= \lim_{h \to 0} \frac{f(x) - f(h) + xh(x-h) - f(x)}{-h}$$

$$= \lim_{h \to 0} \left(\frac{f(h)}{h} - x^2 + xh \right)$$

$$= f'(0) - x^2 = \boxed{-x^2 + 8}$$