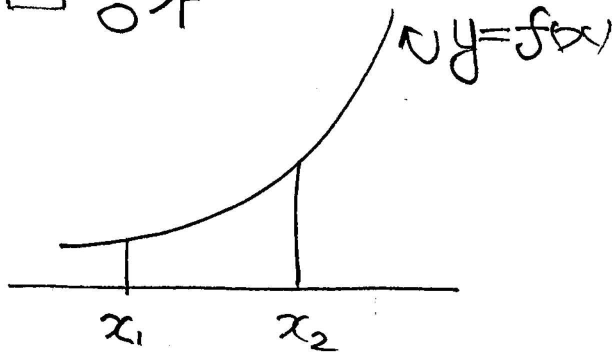
*극대·극소의 미분

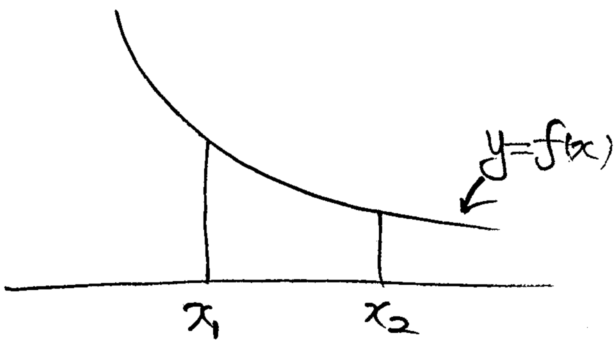
1 증가



$x_1 < x_2$ 이면 $f(x_1) < f(x_2)$ 이다

$\Leftrightarrow f'(x) > 0$

2 감소

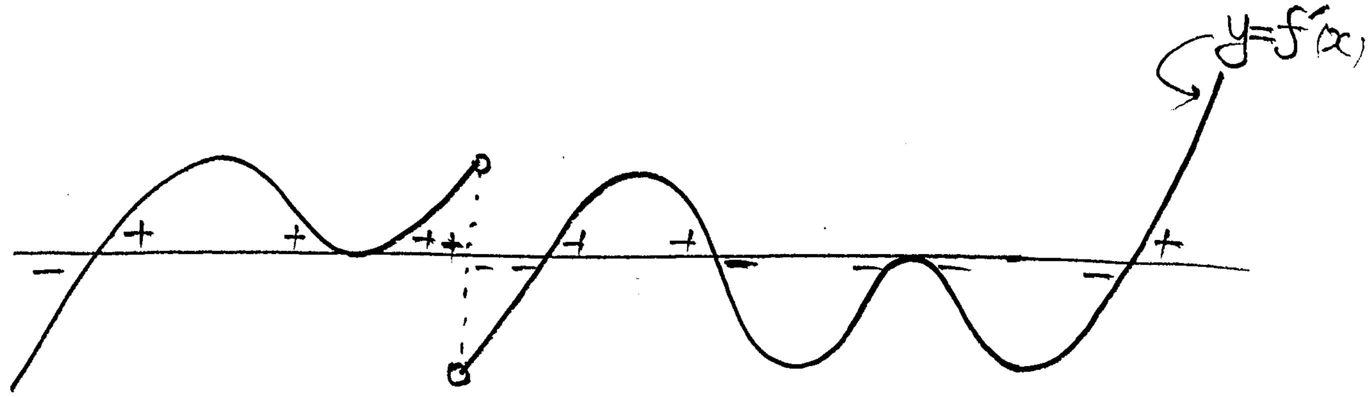


$x_1 < x_2$ 이면 $f(x_1) > f(x_2)$ 이다

$\Leftrightarrow f'(x) < 0$

cf) $f(x)$가 다항함수
$f'(x) \geq 0$: 증가함수
$f'(x) \leq 0$: 감소함수

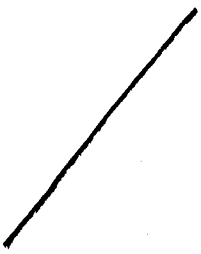
3 극대/극소



4 함수의 그래프

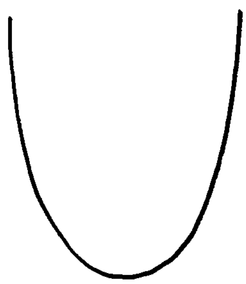
$f(x) = ax^n + \cdots \quad (a>0,\ n$은 자연수$)$

①



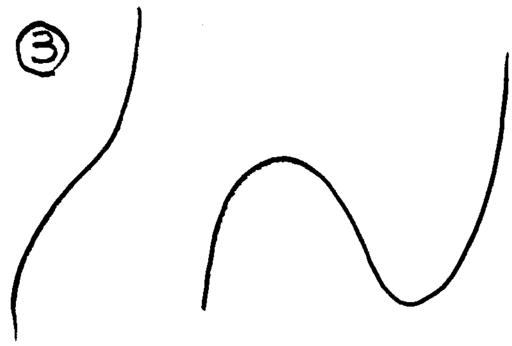
$n=1$

②



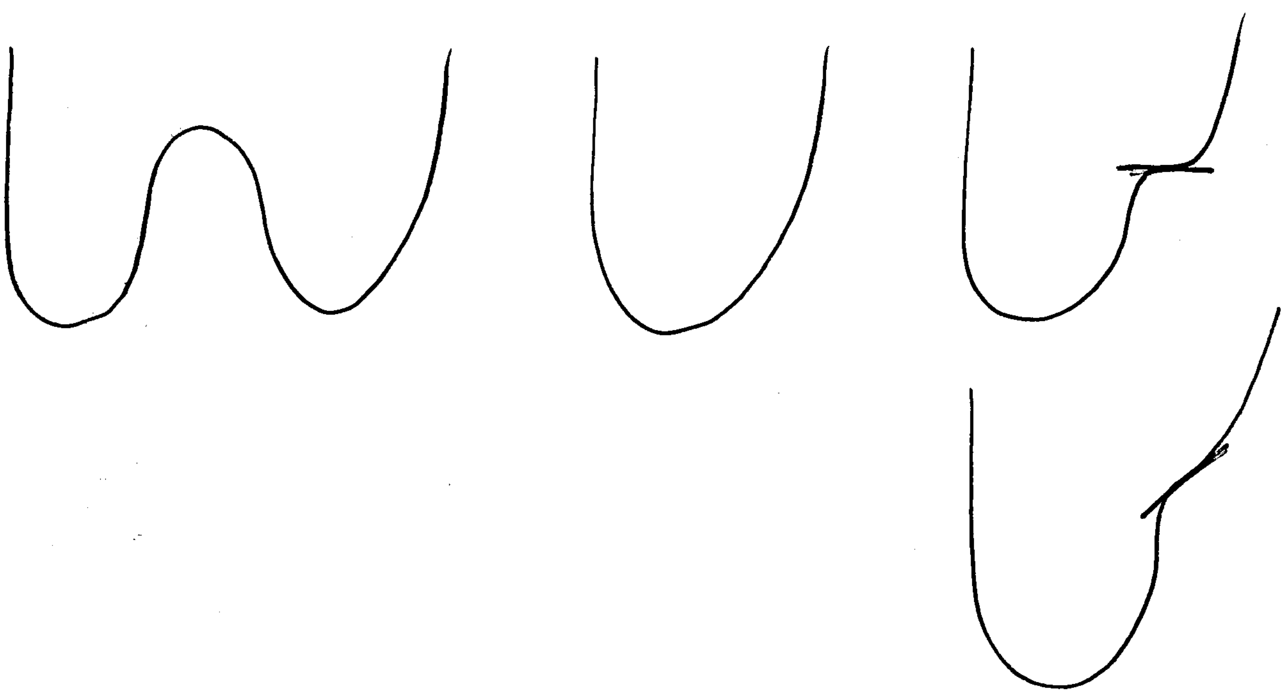
$n=2$

③



$n=3$

④



P159

EX) $f(x)=x^2$

$f'(x)=2x\geq 0 \quad \therefore x\geq 0$

구간 $[0, \infty)$ 에서 증가

P160

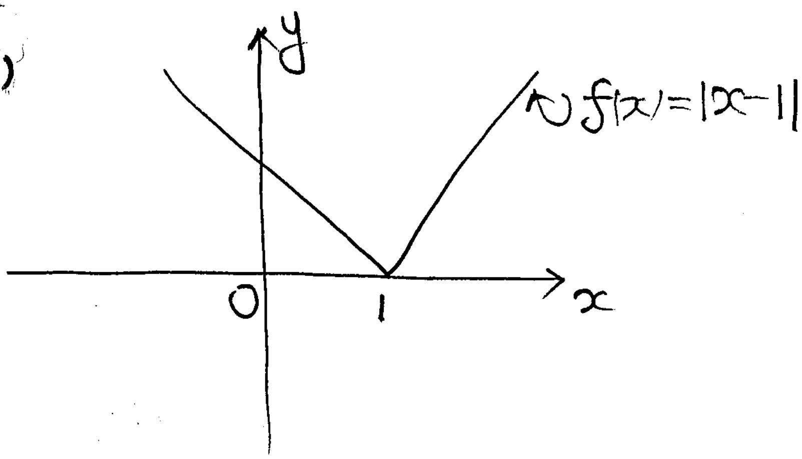
EX) $f(x)=x^2-2x+2$

$f'(x)=2x-2\geq 0 \quad \therefore x\geq 1$

구간 $[1, \infty)$ 에서 증가

P162

EX)



$x=1$ 에서

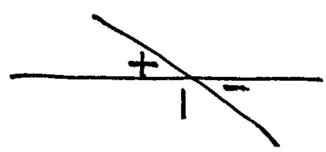
극솟값 : $f(1)=0$

극댓값 : X

P163

EX) $f(x)=-x^2+2x$

$f'(x)=-2x+2$



$x=1$ 에서

극댓값 : $f(1)=-1+2=1$

P165

1|

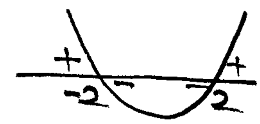
$f(x)=x^2$ $\quad f'(x)=2x\le 0$ $\quad \therefore x\le 0$

$(-\infty, 0]$ 에서 감소

2|

$f(x)=x^3-12x$

$f'(x)=3x^2-12=3(x^2-4)=3(x+2)(x-2)$



$x=-2$ 에서 극댓값 : $f(-2)=-8+24=16$

$x=2$ 에서 극솟값 : $f(2)=8-24=-16$

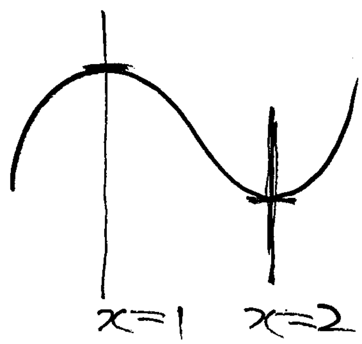
3|

$f(x)=2x^3+ax^2+bx+c$

$f'(x)=6x^2+2ax+b$

$=6(x-1)(x-2)$

$=6x^2-18x+12$



$\boxed{f'(1)=0,\ f'(2)=0}$

$f(1)=0$

$2a=-18$ $\quad \boxed{a=-9}$

$\boxed{b=12}$

$f(x)=2x^3-9x^2+12x+c$

$f(1)=2-9+12+c=c+5=0$

$\boxed{\therefore c=-5}$

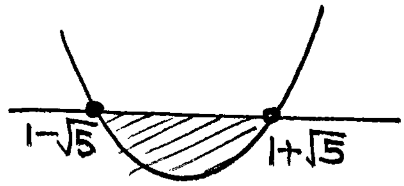
P167

1-1 1

(1) $f'(x) = 4x^3 - 2x$ $\quad f'(1) = 4-2 = 2 > 0$ 증가상태

(2) $g(x) = -3x^2 + 6x + 12 \geq 0$

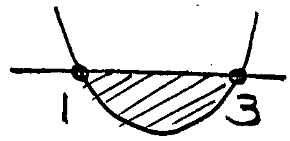
$x^2 - 2x - 4 \leq 0$



$[1-\sqrt{5}, 1+\sqrt{5}]$

1-2 7

$f'(x) = 3(x^2 - 2ax + (2a-1)) \leq 0 \iff$

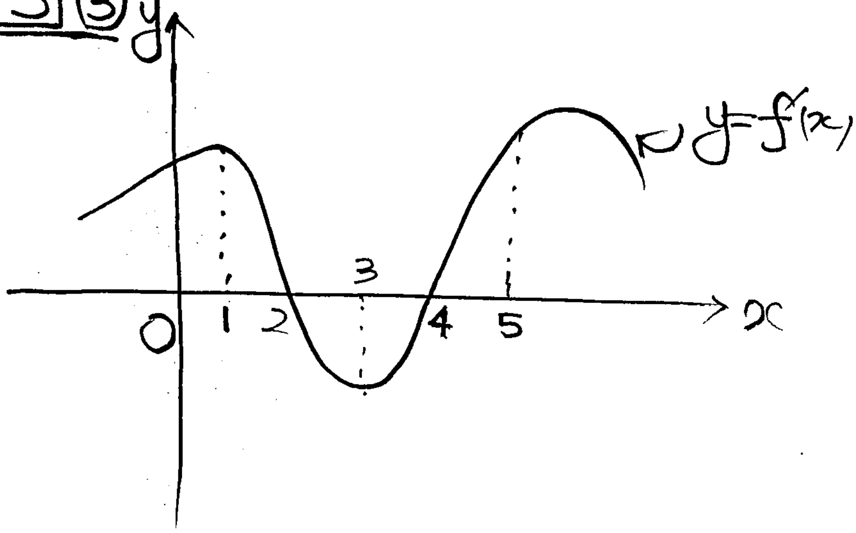


$f'(x) = 3(x-1)(x-3) = 3(x^2 - 4x + 3)$

$a = 2$; $f(x) = x^3 - 6x^2 + 9x + 5$

$f(a) = f(2) = 8 - 24 + 18 + 5 = 7$

1-3 ③



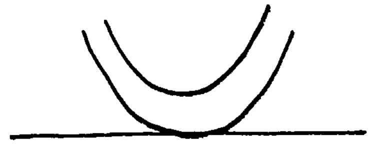
ㄱ $f'(1) > 0$

ㄴ

~~ㄷ~~

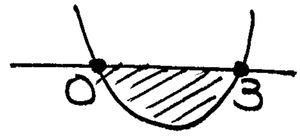
2-1)

$f'(x) = 3x^2 + 2ax + a \geq 0$



$3x^2 + 2ax + a = 0$

$D/4 = a^2 - 3a = a(a-3) \leq 0$

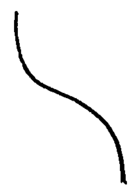


$\boxed{\therefore 0 \leq a \leq 3}$

2-2) 7

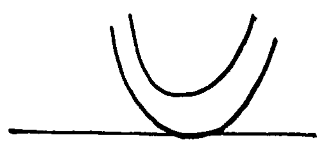
대우: $x_1 \neq x_2$ 이면 $f(x_1) \neq f(x_2)$ 이다

$\Leftrightarrow$ 일대일함수 $\Leftrightarrow$



$\Leftrightarrow$ $f(x)$: 감소함수

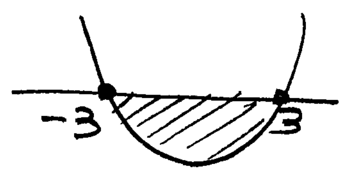
$f'(x) = -3x^2 + 2ax - 3 \leq 0$



$3x^2 - 2ax + 3 \geq 0$

$3x^2 - 2ax + 3 = 0$

$D/4 = a^2 - 9 = (a+3)(a-3) \leq 0$

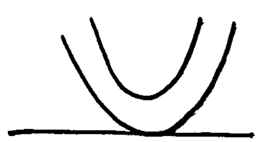


$-3 \leq a \leq 3$ $\therefore$ 7개

2-3)

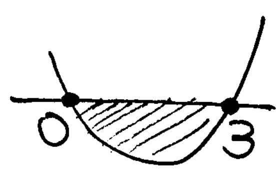
① $f(x)$: 증가함수

$f'(x) = 3x^2 + 2ax + a \geq 0$



$3x^2 + 2ax + a = 0$

$D/4 = a^2 - 3a = a(a-3) \leq 0$

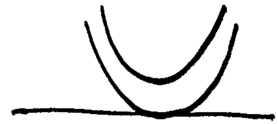


$\therefore 0 \leq a \leq 3$

ⓛ $g(x)$: 감소함수

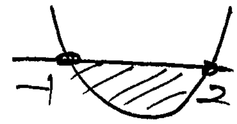
$g'(x) = -3x^2 + 2(a+1)x - (a+1) \le 0$

$3x^2 - 2(a+1)x + (a+1) \ge 0$



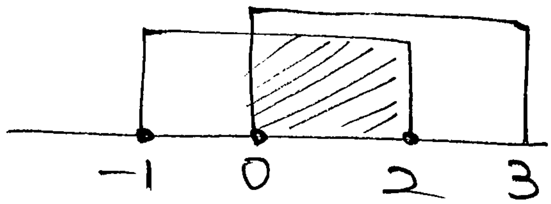
$3x^2 - 2(a+1)x + (a+1) = 0$

$D/4 = (a+1)^2 - 3(a+1) = (a+1)(a-2) \le 0$



$\therefore -1 \le a \le 2$

㉠, ⓛ 에서

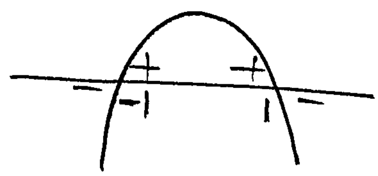


$\boxed{0 \le a \le 2}$

3-11

$f(x) = -2x^3 + 6x + a$

$f'(x) = -6x^2 + 6 = -6(x+1)(x-1)$



$f(-1) = 2 - 6 + a = a - 4 = -7$

$a = -3$

$x = 1$ 에서 극댓값 : $f(1) = -2 + 6 + a = a + 4 = 1$

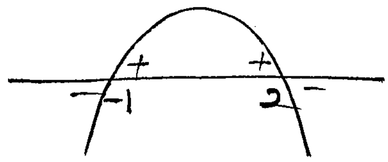
3-2) 12

$f(x) = -x^3 + ax^2 + bx + 2$

$f'(x) = -3x^2 + 2ax + b$

$= -3(x+1)(x-2)$

$= -3x^2 + 3x + 6$

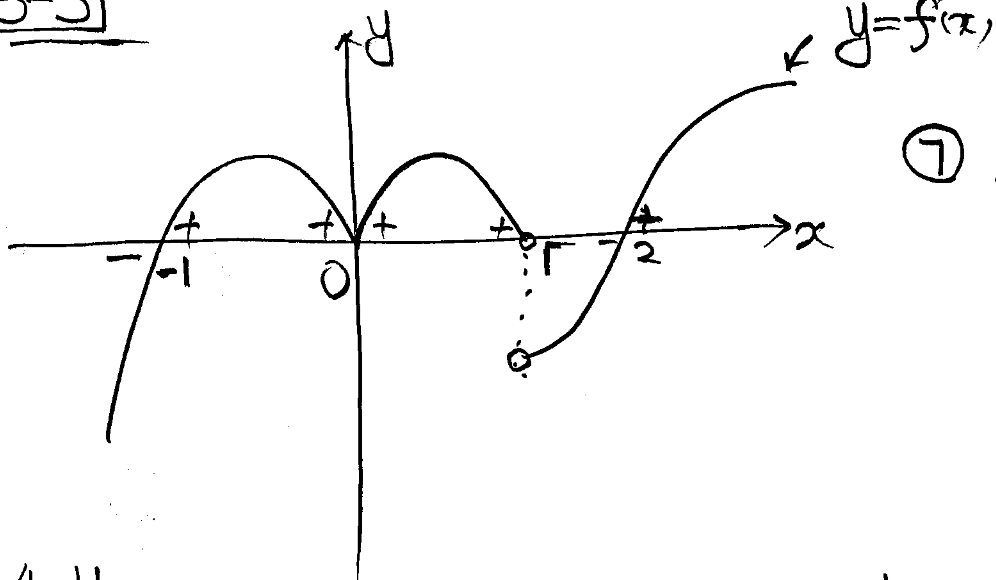


$a = \frac{3}{2}, \ b = 6$

$f(x) = -x^3 + \frac{3}{2}x^2 + 6x + 2$

$x=2$에서 극댓값: $f(2) = -8 + 6 + 12 + 2 = 12$

3-31



㉠ ✓ ㉢

4-11

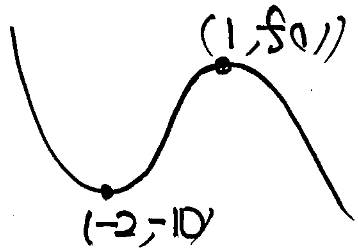
$f(x) = -2x^3 + ax^2 + bx + c$

$f'(x) = -6x^2 + 2ax + b$

$= -6(x+2)(x-1)$

$= -6x^2 - 6x + 12$

$\boxed{a = -3, \ b = 12}$



$f'(-2) = 0, \ f'(1) = 0$

$f(-2) = -10$

$f(x) = -2x^3 - 3x^2 + 12x + c$

$f(-2) = 16 - 12 - 24 + c = c - 20 = -10$ $\boxed{c = 10}$

$x=1$에서 극댓값: $f(1) = -2 - 3 + 12 + 10 = 17$

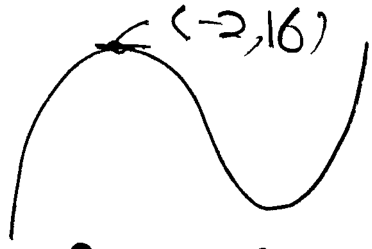
4-2 | 15

$f(x) = 2x^3 + ax^2 + bx - 4$

$f'(x) = 6x^2 + 2ax + b$

$f(-2) = -16 + 4a - 2b - 4 = 16$

$f'(-2) = 24 - 4a + b = 0$



$f(-2) = 16$

$f'(-2) = 0$

$$\begin{array}{r} 4a - 2b = 36 \\ -\underline{\quad 4a - b = 24} \\ -b = 12 \end{array}$$

$b = -12$; $4a + 12 = 24$

$a = 3$

$a - b = 3 - (-12) = 15$

4-31

(가) $f(0) = 1$, $f'(0) = -6$

(나) $f(1) = -3$, $f'(1) = 0$

$f(x) = ax^3 + bx^2 + cx + 1$

$f'(x) = 3ax^2 + 2bx + c$

$f'(0) = c = -6$

$f(1) = a + b + c + 1 = -3$

$f'(1) = 3a + 2b + c = 0$

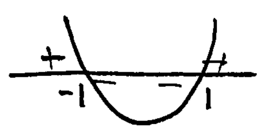
$a + b = 2$

$$\begin{array}{r} 3a + 2b = 6 \\ -\underline{\quad 2a + 2b = 4} \\ a = 2 \end{array}$$

$a = 2$, $b = 0$

$f(x) = 2x^3 - 6x + 1$

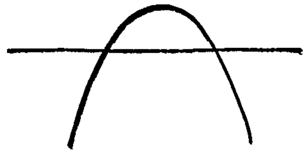
$f'(x) = 6(x^2 - 1) = 6(x+1)(x-1)$



$x = -1$ 에서 극댓값 : $f(-1) = 5$

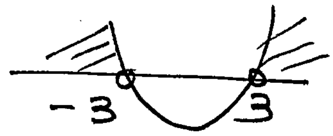
5-11

(1) $f'(x) = -3x^2 + 2ax - 3$



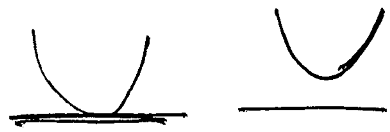
$-3x^2 + 2ax - 3 = 0$

$D/4 = a^2 - 9 = (a+3)(a-3) > 0$



$\boxed{a < -3 \text{ 또는 } a > 3}$

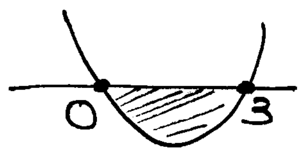
(2) $f(x) = x^3 + ax^2 + ax + 2$



$f'(x) = 3x^2 + 2ax + a$

$3x^2 + 2ax + a = 0$

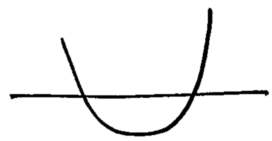
$D/4 = a^2 - 3a = a(a-3) \le 0$



$\boxed{0 \le a \le 3}$

5-21 ②

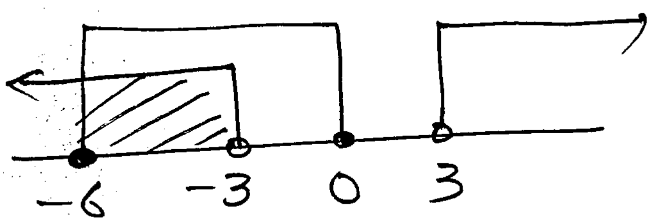
$f'(x) = 3x^2 + 2ax + 3 = 0$



$D/4 = a^2 - 9 > 0$

$g(x) = 3x^2 + 2ax - 2a = 0$

$D/4 = a^2 + 6a \le 0$



$-6 \le a < -3$

∴ 3개

5-31

$f(x)=\frac{1}{3}x^3-x^2+ax+4$

$f'(x)=x^2-2x+a$

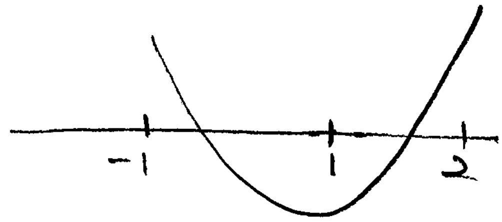
$f'(-1)=1+2+a=a+3>0$

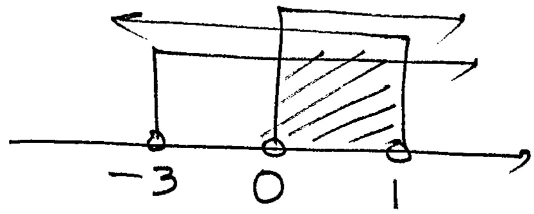
$f'(1)=1-2+a=a-1<0$

$f'(2)=4-4+a=a>0$

$a>-3,\ a<1,\ a>0$

$\boxed{\therefore 0<a<1}$



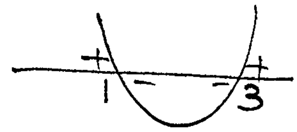


P179

11

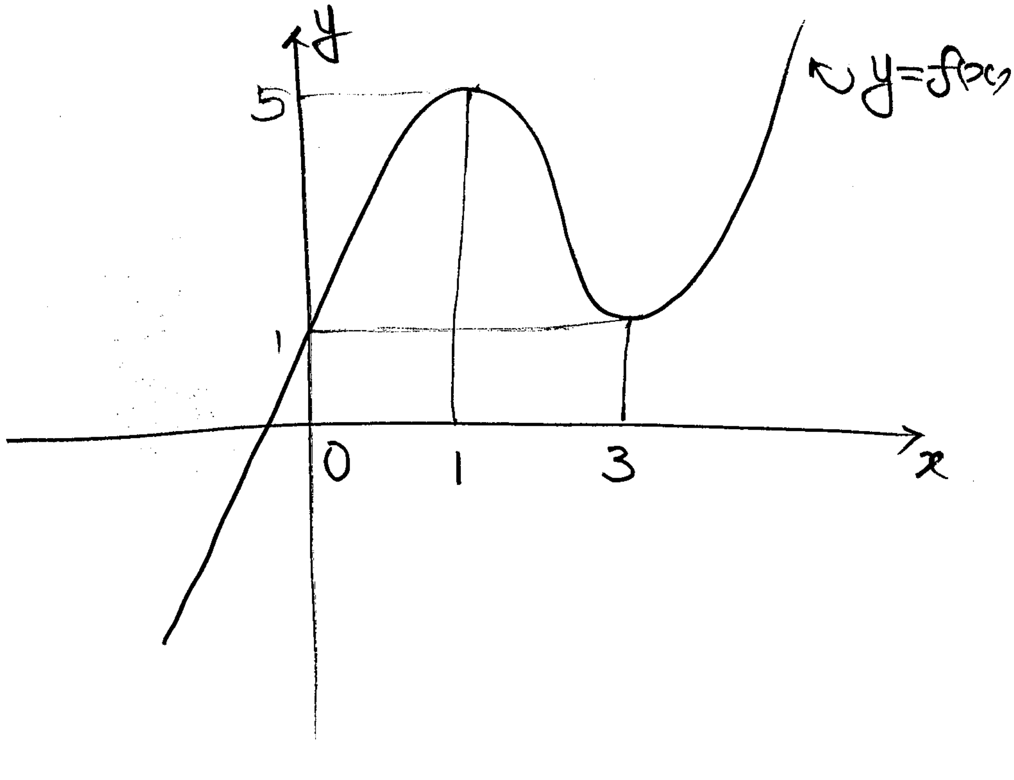
$f(x)=x^3-6x^2+9x+1$

$f'(x)=3x^2-12x+9=3(x-1)(x-3)$



$x=1$ 에서 극댓값 : $f(1)=1-6+9+1=5$

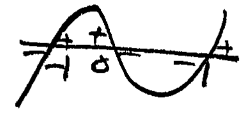
$x=3$에서 극솟값 : $f(3)=27-54+27+1=1$



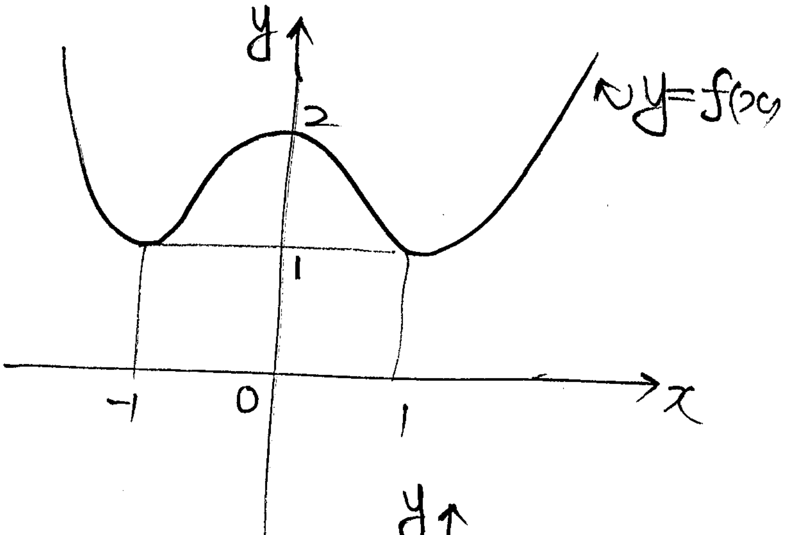
2)

$f(x)=x^4-2x^2+2 \qquad f(-x)=f(x)$

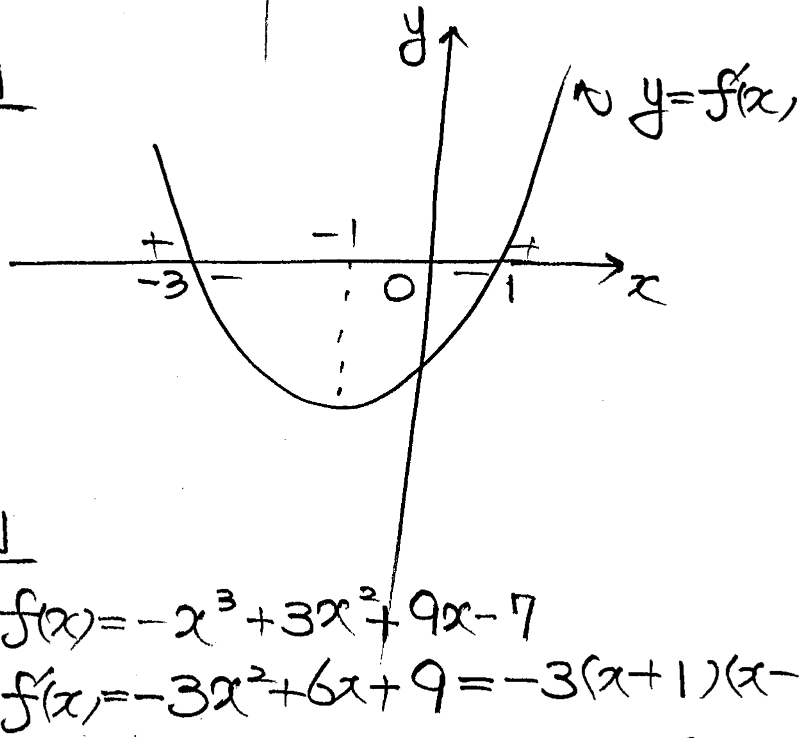
$f'(x)=4x^3-4x=4x(x^2-1)=4x(x-1)(x+1)$



$f(-1)=1-2+2=1, \quad f(0)=2, \quad f(1)=1-2+2=1$



3)



ㄱ

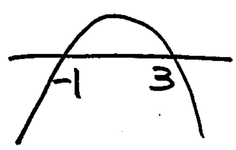
ㄴ

ㄷ

6-1)

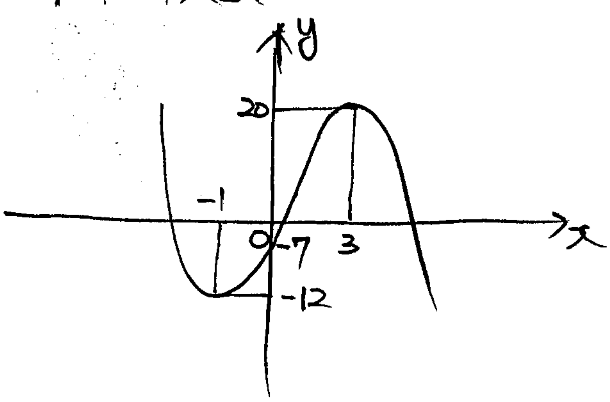
(1) $f(x)=-x^3+3x^2+9x-7$

$f'(x)=-3x^2+6x+9=-3(x+1)(x-3)$



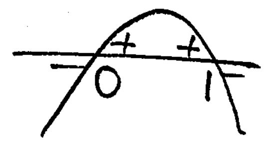
$x=-1$ 에서 극솟값 : $f(-1)=1+3-9-7=-12$

$x=3$ 에서 극댓값 : $f(3)=-27+27+27-7=20$



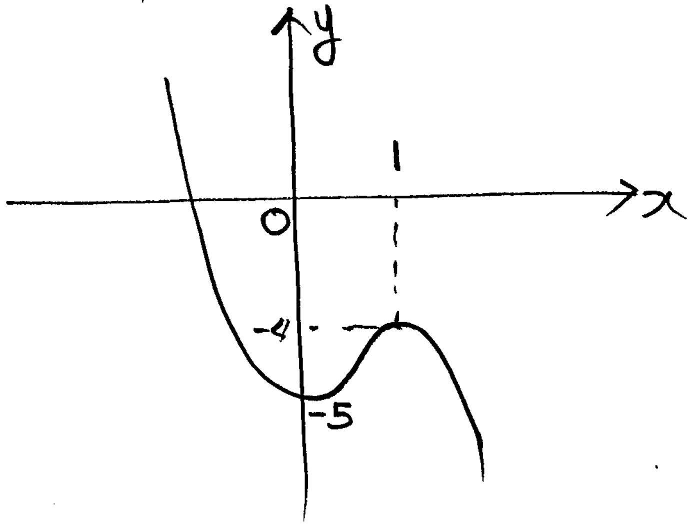
(2) $f(x) = -2x^3 + 3x^2 - 5$

$f'(x) = -6x^2 + 6x = -6x(x-1)$



$x=0$ 에서 극솟값: $f(0) = -5$

$x=1$ 에서 극댓값: $f(1) = -2+3-5 = -4$



6-21

$f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

y절편: $c > 0$

$3x^2 + 2ax + b = 0$ 두근 α, β

$\alpha + \beta = -\frac{2a}{3} > 0$ $\qquad a < 0$

$\alpha\beta = \frac{b}{3} < 0$ $\qquad b < 0$

$\alpha < 0 < \beta$

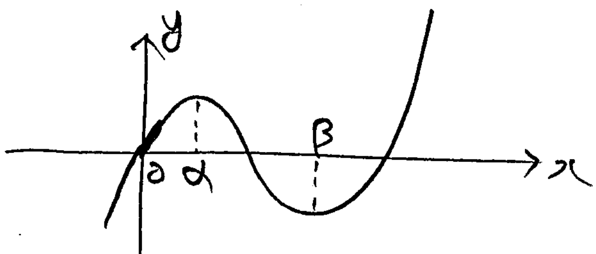
$|\alpha| < |\beta|$

$-\alpha < \beta$

$\alpha + \beta > 0$

㉠ $ab > 0$, ㉡ $bc < 0$ ~~ㄷ~~ $ac < 0$

6-31 ②



$f(x) = x^3 + ax^2 + bx$

$f'(x) = 3x^2 + 2ax + b = 0$

두근 α, β $(0 < \alpha < \beta)$

$\alpha + \beta = -\frac{2a}{3} > 0$ $a < 0$

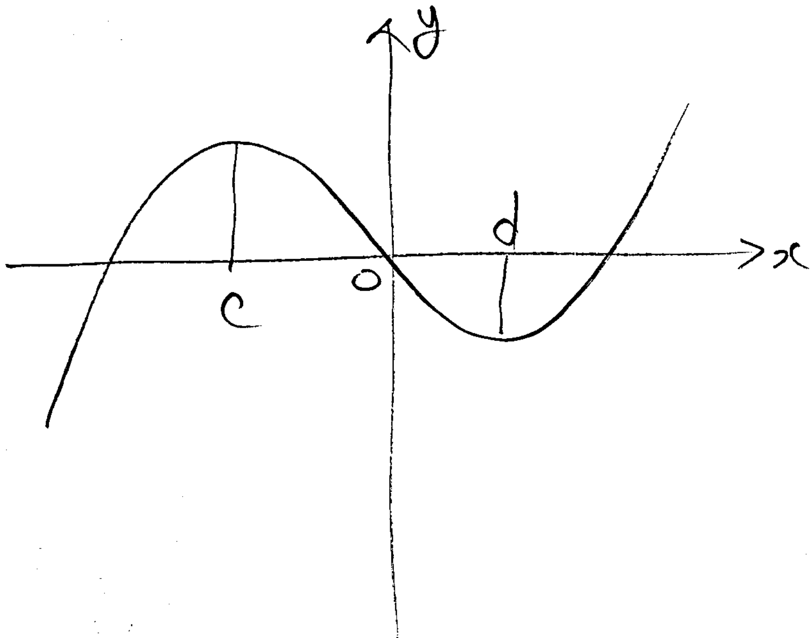
$\alpha\beta = \frac{b}{3} > 0$ $b > 0$

$g(x) = x^3 + bx^2 + ax$

$g'(x) = 3x^2 + 2bx + a = 0$ 두근 c, d $(c < d)$

(두근의 합) $= -\frac{2b}{3} < 0$ $c < 0 < d$

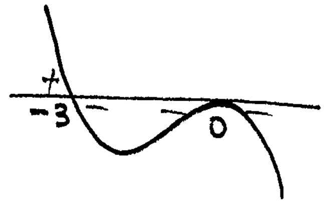
(두근의 곱) $= \frac{a}{3} < 0$



7-1

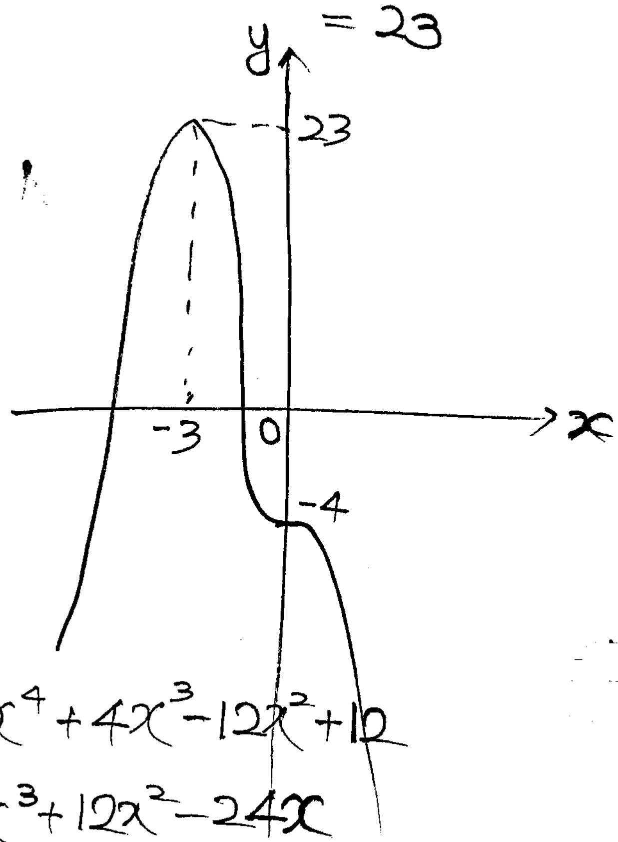
(1) $f(x) = -x^4 - 4x^3 - 4$

$f'(x) = -4x^3 - 12x^2 = -4x^2(x+3)$



$x=-3$에서 극댓값 : $f(-3) = -81+108-4$
$= 23$

$f(0) = -4$

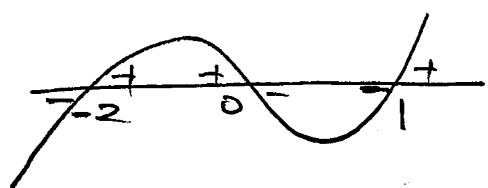


(2) $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

$f'(x) = 12x^3 + 12x^2 - 24x$

$= 12(x^3 + x^2 - 2x)$

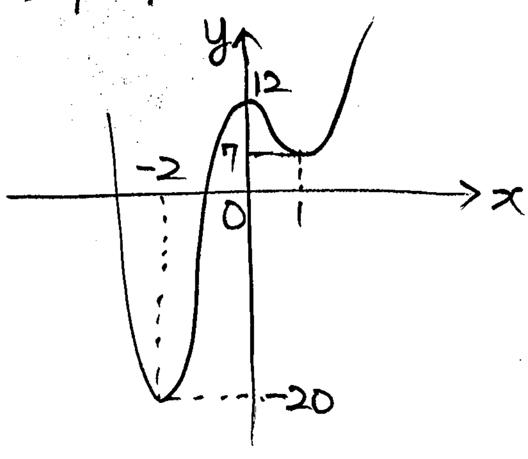
$= 12(x+2)x(x-1)$



$x=-2$에서 극솟값 : $f(-2) = 48-32-48+12 = -20$

$x=0$에서 극댓값 : $f(0) = 12$

$x=1$ 에서 극솟값 : $f(1) = 3+4-12+12 = 7$



7-2) $\frac{9}{4}$

$f(x)=x^4-4x^3+2ax^2$ 이 극댓값을 가질때

$\Longleftrightarrow$  $\Longleftrightarrow$ $f'(x)=0$ 서로다른 세실근

$f'(x)=4x^3-12x^2+4ax$

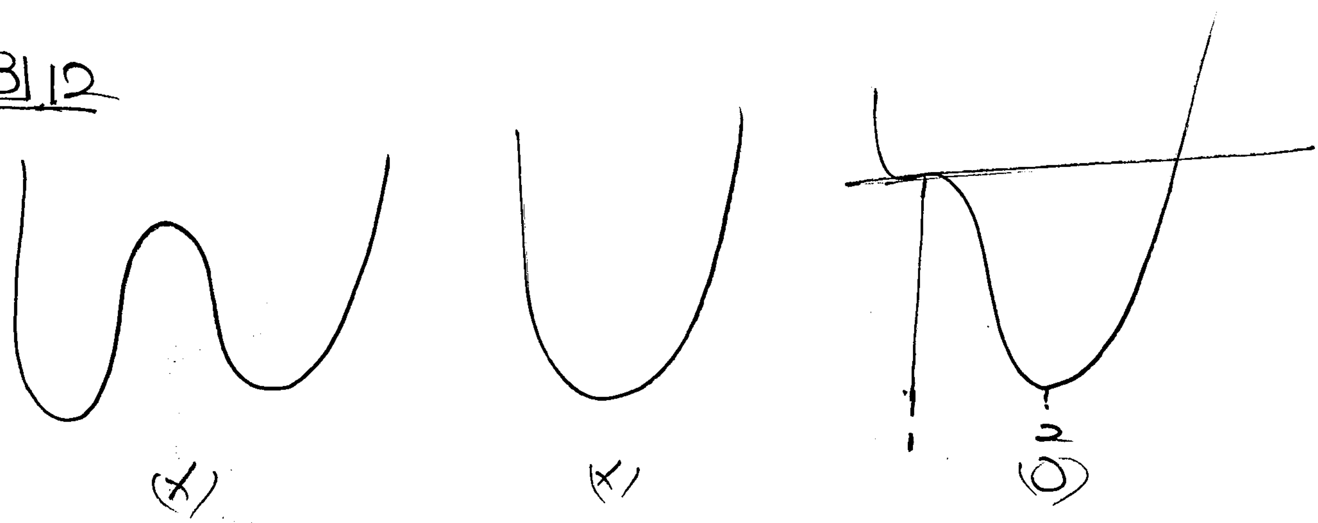
$=4x(\underbrace{x^2-3x+a}_{㉠})=0$

㉠ $a\neq 0$, $D=9-4a>0$

$a<\frac{9}{4}$

극댓값을 가지지 않을때 $a=0$ 또는 $a\geq\frac{9}{4}$

7-3) 12



$f'(x)=p(x-1)^2(x-2)$ $(p\neq 0)$

$\frac{f'(5)}{f'(3)}=\frac{p\cdot 16\cdot 3}{p\cdot 4\cdot 1}=12$