7. 부정 적분

1 정의

$$F'(x) = f(x) \iff \int f(x)\,dx = F(x) + C$$

2 공식

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \quad (n \neq -1)$$

(cf) (치환)

P226

EX) (1) $\int 4\,dx = 4x + C$

(2) $\int x\,dx = \frac{x^2}{2} + C$

P227

EX) (1) $\frac{d}{dx}\left(\int x^2\,dx\right) = x^2$

(2) $\int \left(\frac{d}{dx}x^2\right)dx = x^2 + C$

P228

EX1) (1) $\int 1\,dx = x + C$

(2) $\int x^2\,dx = \frac{1}{3}x^3 + C$

(3) $\int x^3\,dx = \frac{1}{4}x^4 + C$

EX2) $\int (x+1)^2\,dx$

$= \int (x^2 + 2x + 1)\,dx = \frac{x^3}{3} + x^2 + x + C$

P231

1)

(1) $f(x) = 4x + 3$

(2) $f(x) = -x^2 + 2$

2)

(1) $\int x\,dx = \frac{x^2}{2} + C$

(2) $\int x^5 dx = \frac{1}{6}x^6 + C$

(3) $\int 7 dx = 7x + C$

(4) $\int (2x+3)^4 dx$

$$\boxed{\begin{array}{c} 2x+3=t \\ 2dx = dt \end{array}}$$

$= \int t^4 \cdot \frac{1}{2} dt$

$= \frac{1}{2} \cdot \frac{t^5}{5} + C = \frac{1}{10}(2x+3)^5 + C$

<u>31</u>

$\int (2x^2 - 3x + 5) dx = \frac{2x^3}{3} - \frac{3x^2}{2} + 5x + C$

P233

<u>1-11</u>

(1) $\int (x^6 - 3x^4 + 3x^2 - 1) dx = \frac{1}{7}x^7 - \frac{3x^5}{5} + x^3 - x + C$

(2) $\int (y^3 + 3y^2 + 2y) dy = \frac{1}{4}y^4 + y^3 + y^2 + C$

(3) $\int (6x^2 + 2) dx = 2x^3 + 2x + C$

(4) $\int x(x-2)^3 dx$

$x - 2 = t$
$dx = dt$

$= \int (t+2) \cdot t^3 dt$

$= \int (t^4 + 2t^3) dt$

$= \frac{1}{5}t^5 + \frac{t^4}{2} + C$

$= \frac{1}{5}(x-2)^5 + \frac{1}{2}(x-2)^4 + C$

1-21

(1) $\dfrac{x^3+1}{x+1} = \dfrac{(x+1)(x^2-x+1)}{(x+1)} = x^2-x+1$

$$\int (x^2-x+1)\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 + x + C$$

(2) $\dfrac{x^4+x^2+1}{x^2+x+1} = \dfrac{(x^2+x+1)(x^2-x+1)}{x^2+x+1}$

$= x^2-x+1$

$$\boxed{\begin{aligned} x^4+x^2+1 &= (x^4+2x^2+1)-x^2 \\ &= (x^2+1)^2-x^2 \\ &= (x^2+x+1)(x^2-x+1) \end{aligned}}$$

$$\int (x^2-x+1)\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 + x + C$$

1-31 46

$F'(x) = f(x) \iff \int f(x)\,dx = F(x) + C$

$\cancel{F'(x)} = \cancel{f(x)} + x \cdot f'(x) - 3x^2$

$f'(x) = 3x$

$f(x) = \dfrac{3x^2}{2} + C \qquad f(1) = \dfrac{3}{2} + C = 10 \quad C = \dfrac{17}{2}$

$f(5) = \dfrac{75}{2} + \dfrac{17}{2} = \dfrac{92}{2} = 46$

2-1)

$f(x)+g(x)=x^2+x$

$f(x)\cdot g(x)=x^3-x^2+x-1$

$= x^2(x-1)+(x-1)=(x-1)(x^2+1)$

$f(x)=x^2+1,\quad g(x)=x-1$

2-2)

(1) $f(x)=\int(3x^2-6x+5)dx$

$= x^3-3x^2+5x+2$

$f(1)=1-3+5+2=5$

(2) $f(x)=-\frac{1}{5}x^3-\frac{3}{4}x^2+2x+C$

$f(1)=-\frac{1}{5}-\frac{3}{4}+2+C=0$

$C=-2+\frac{3}{4}+\frac{1}{5}$

$f(-1)=\frac{1}{5}-\cancel{\frac{3}{4}}-2-2+\cancel{\frac{3}{4}}+\frac{1}{5}$

$=\frac{2-20}{5}=-\frac{18}{5}$

2-3) 3

$f(x)=\begin{cases} x^3+C_1 & (x\le 1) \\ x^2+x+C_2 & (x>1) \end{cases}$

$f(0)=C_1=-2$

$1-2=1+1+C_2$

$C_2=-3$

$f(2)=4+2-3=3$

3-11

㉠ $F'(x) = f(x)$

㉡ $\cancel{f(x)} + x \cdot f'(x) - \cancel{F'(x)} = 3x^2 - 8x$

$$f'(x) = 3x - 8$$

$$f(x) = \frac{3x^2}{2} - 8x + C$$

$$f(1) = \frac{3}{2} - 8 + C = C - \frac{13}{2} = -\frac{25}{2}$$

$$C = \frac{-12}{2} = -6 \qquad \boxed{\therefore f(x) = \frac{3x^2}{2} - 8x - 6}$$

3-214

$$\cancel{f(x)} - 2x = \cancel{f(x)} + xf'(x) - 6x^2 + 10x$$

$$xf'(x) = 6x^2 - 12x$$

$$f'(x) = 6x - 12$$

$$f(x) = 3x^2 - 12x + C$$

$$f(1) = 3 - 12 + C = C - 9 = 7 \qquad C = 16$$

$$f(x) = 3x^2 - 12x + 16$$

$$= 3(x-2)^2 + 4$$

$$\boxed{\therefore m = 4}$$

3-3 | 18

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{f(x)} + f(h) + 2 - \cancel{f(x)}}{h}$$

$$= f'(0) = 2$$

$$f(x) = 2x - 2$$

$$f(10) = 18$$

$x=0,\ y=0$

$$f(0) = f(0) + f(0) + 2$$

$$f(0) = -2$$