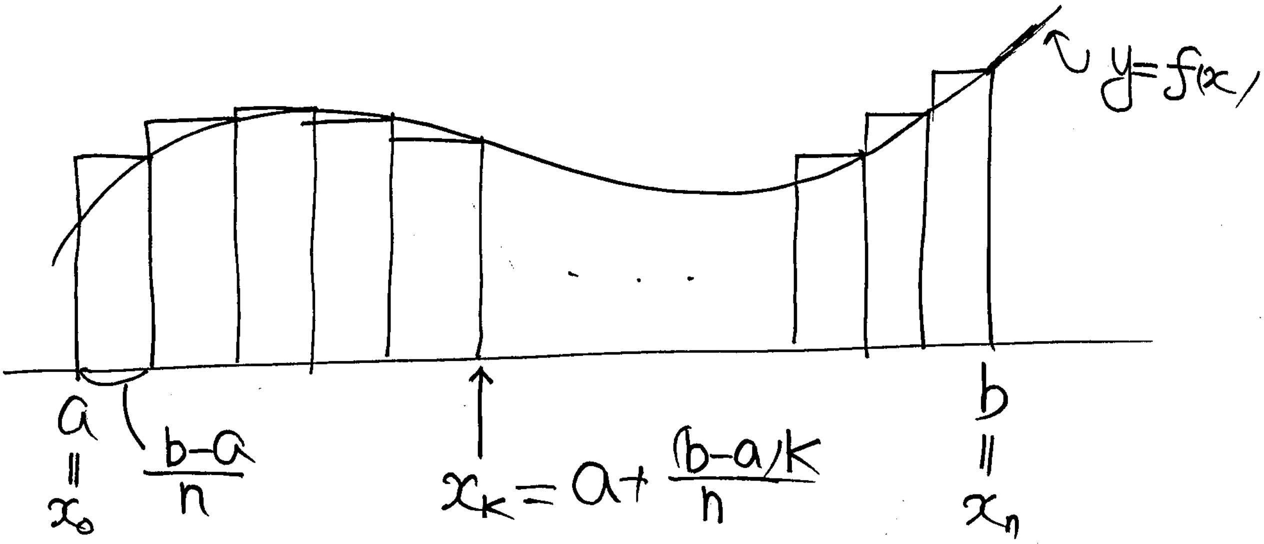
○ 정적분의 정의

$$F'(x) = f(x) \iff \int f(x)\,dx = F(x) + C$$



$$\lim_{n\to\infty}\sum_{k=1}^{n} f\left(a+\frac{(b-a)k}{n}\right)\cdot\frac{b-a}{n} = \int_a^b f(x)\,dx = F(b)-F(a)$$

○ 단위화

$$\begin{cases}\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n} \\ \lim\limits_{n\to\infty}\sum\limits_{k=0}^{n-1}\end{cases} \Rightarrow \int_0^1 \quad , \quad \frac{k}{n} \Rightarrow x \quad , \quad \frac{1}{n} \Rightarrow dx$$

P246

EX)

(1) $\int_0^2 x\,dx = \left[\frac{x^2}{2}\right]_0^2 = \frac{4-0}{2} = 2$

(2) $\int_1^4 x^2\,dx = \left[\frac{x^3}{3}\right]_1^4 = \frac{64-1}{3} = 21$

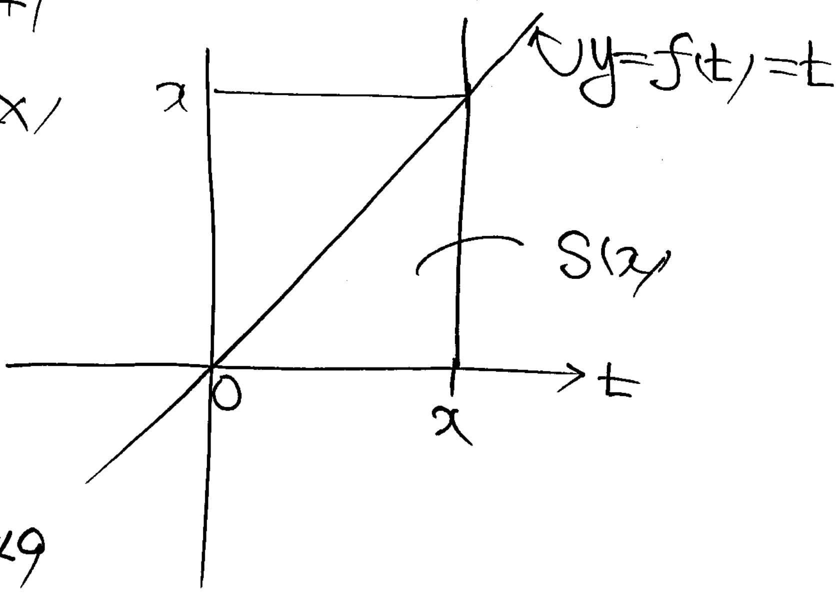
(3) $\int_0^2 t\,dt = \left[\frac{t^2}{2}\right]_0^2 = \frac{4-0}{2} = 2$

(4) $\int_{-1}^3 (2t+4)\,dt = \left[t^2+4t\right]_{-1}^3$

$= (9-1) + 4(3-(-1)) = 8+16 = 24$

P247

EX)



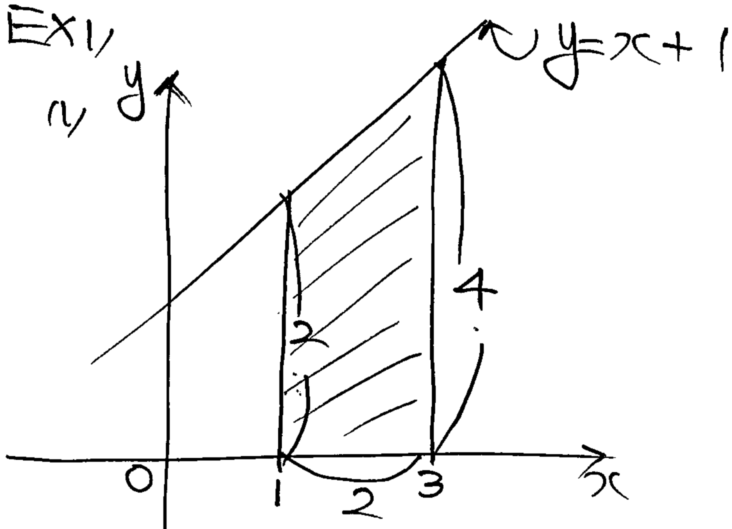
$S(x) = \frac{1}{2}x^2$

$S'(x) = x = f(x)$

P249

EX1)

(1)

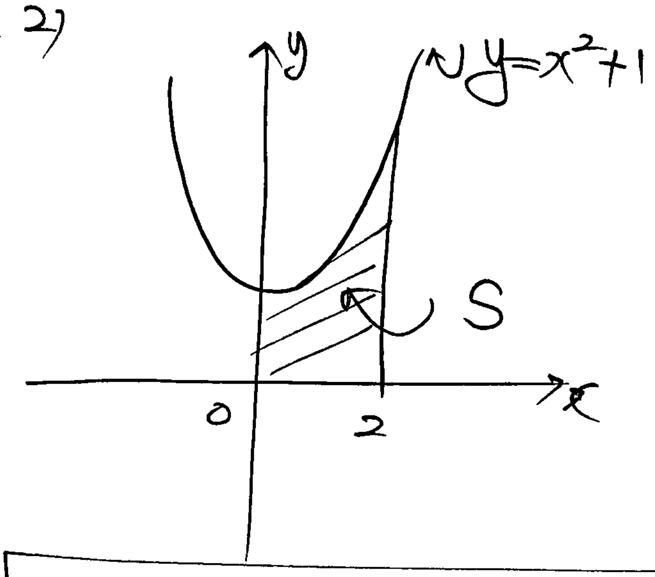


$S = \int_1^3 (x+1)\,dx$

$= \frac{1}{2}\cdot 6\cdot 2$

$= 6$

(2)



$$S=\int_0^2 (x^2+1)\,dx$$
$$=\left[\frac{x^3}{3}+x\right]_0^2$$
$$=\frac{8}{3}+2=\frac{14}{3}$$

$$\boxed{\int f(t)\,dt = F(t)+C \iff f(t)=F'(t)}$$

$$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt} = \frac{d}{dx}\left(F(x)-F(a)\right) = F'(x) = \boxed{f(x)}$$

EX2)

(1) $\dfrac{d}{dx}\displaystyle\int_1^x (3t^2+4t-5)\,dt = 3x^2+4x-5$

(2) $\dfrac{d}{dx}\displaystyle\int_0^x (y^3-2y)\,dy = x^3-2x$

P253

EX)

(1) $\displaystyle\int_3^3 x^2\,dx = 0$

(2) $\displaystyle\int_2^1 4x^3\,dx = -\int_1^2 4x^3\,dx = -\left[x^4\right]_1^2$

$= -[16-1]$

$= -15$

P254

EX)
$$\int_0^2 (4x^2+3x-3)\,dx$$
$$=\left[\frac{4x^3}{3}+\frac{3x^2}{2}-3x\right]_0^2$$
$$=\frac{32}{3}+6-6=\frac{32}{3}$$

P256

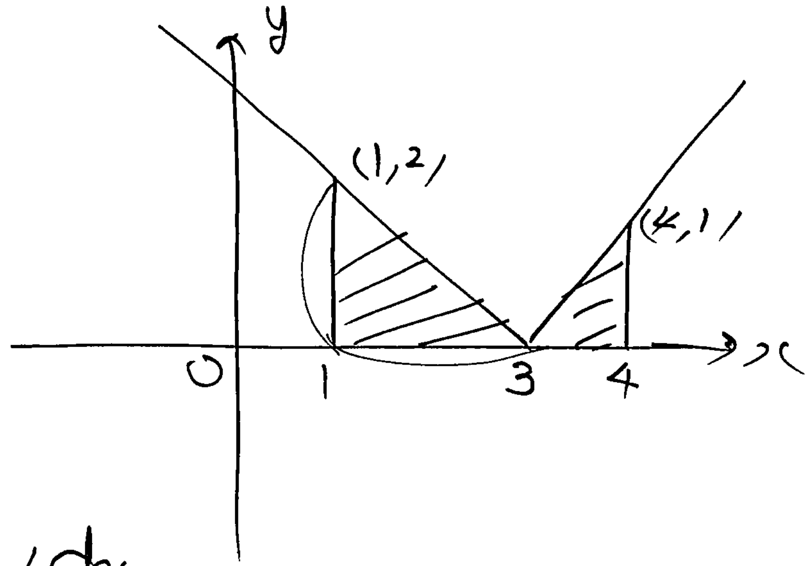
EX1)

(1)
$$\int_0^2 (4x^3+3x^2-2)\,dx+\int_2^3 (4x^3+3x^2-2)\,dx$$
$$=\int_0^3 (4x^3+3x^2-2)\,dx$$
$$=\left[x^4+x^3-2x\right]_0^3=81+27-6=102$$

(2)
$$\int_2^{-1} (-6x+4)\,dx+\int_{-1}^1 (-6x+4)\,dx$$
$$=\int_2^1 (-6x+4)\,dx$$
$$=\int_1^2 (6x-4)\,dx$$
$$=\left[3x^2-4x\right]_1^2$$
$$=(12-3)-(8-4)$$
$$=9-4=5$$

EX2) $\int_1^4 |x-3|\,dx$

$= \frac{5}{2}$



P258

EX)

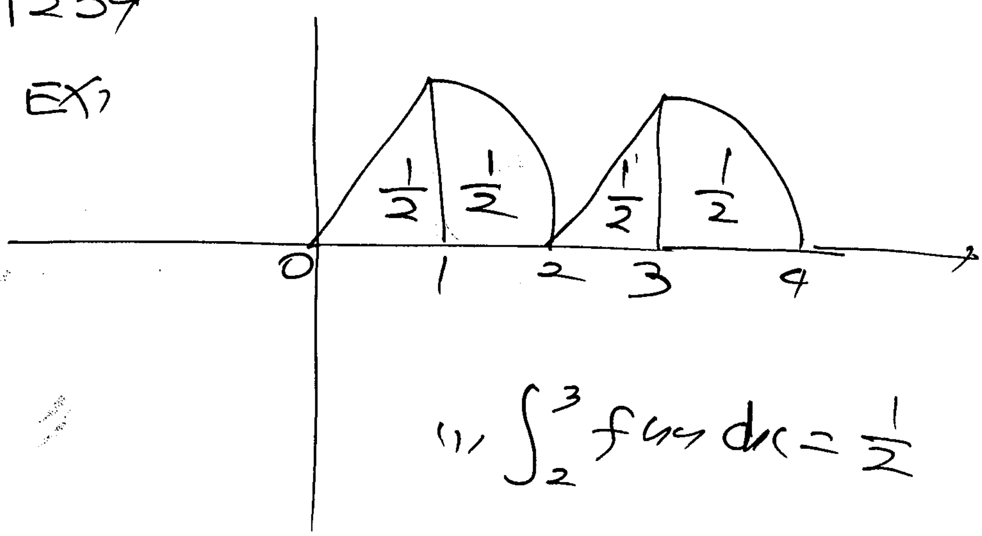
$\int_{-2}^{2}(4x^3+3x^2+2x+1)\,dx$

$= 2\int_0^2 (3x^2+1)\,dx = 2[x^3+x]_0^2$

$= 2(8+2) = 20$

P259

EX)



(1) $\int_2^3 f(x)\,dx = \frac{1}{2}$

(2) $\int_1^3 f(x)\,dx = 1$

P260

EX) (1) $\int_5^9 (x-7)^2 dx$ $x-7=t$, $dx=dt$

$= \int_{-2}^{2} t^2 dt$

$= 2\int_0^2 t^2 dt = 2\left[\frac{t^3}{3}\right]_0^2 = 2\cdot\frac{8}{3} = \frac{16}{3}$

(2) $\int_0^4 (4-x)^3 dx$ $4-x=t$, $-dx=dt$

$= \int_4^0 t^3(-dt)$

$= \int_0^4 t^3 dt = \left[\frac{t^4}{4}\right]_0^4 = 64$

1)

(1) $F'(x) = x^2+2$ (2) $F'(x) = x^3+4x^2-1$

2)

(1) (준식) $= \left[x+x^2-\frac{x^3}{3}\right]_0^{\sqrt{2}} = \sqrt{2}+2-\frac{2\sqrt{2}}{3} = 2+\frac{\sqrt{2}}{3}$

(2) (준식) $= \int_{-1}^{2}(y^2-y-2)dy$

$= \left[\frac{y^3}{3}-\frac{y^2}{2}-2y\right]_{-1}^{2}$

$= \frac{1}{3}(8-(-1)) - \frac{1}{2}(4-1) - 2(2-(-1))$

$= 3-\frac{3}{2}-6 = -\frac{9}{2}$

(3) (준식) $= \left[\frac{4x^3}{3} - x^2 + x\right]_0^{\sqrt{3}}$

$= 4\sqrt{3} - 3 + \sqrt{3} = 5\sqrt{3} - 3$

(4) (준식) $= \int_{-2}^{1} (y^2 + y - 2)\, dy$

$= \left[\frac{y^3}{3} + \frac{y^2}{2} - 2y\right]_{-2}^{1}$

$= \frac{1}{3}(1-(-8)) + \frac{1}{2}(1-4) - 2(1-(-2))$

$= 3 - \frac{3}{2} - 6 = -\frac{9}{2}$

3)

(1) (준식) $= \int_0^2 (x^3 + x - 1)\, dx$

$= \left[\frac{x^4}{4} + \frac{x^2}{2} - x\right]_0^2 = 4 + 2 - 2 = 4$

(2) $\int_0^1 (3x^2 + 2x)\, dx + \int_{-1}^{0} (3x^2 + 2x)\, dx$

$= \int_{-1}^{1} (3x^2 + \underline{2x})\, dx = 2\int_0^1 3x^2\, dx$

$= 2[x^3]_0^1 = 2$

(3) $\int_0^1 (x+1)^2\, dx + \int_1^3 (x+1)^2\, dx$

$= \int_0^3 (x^2 + 2x + 1)\, dx$

$= \left[\frac{x^3}{3} + x^2 + x\right]_0^3 = 9 + 9 + 3 = 21$

(4) $\int_1^2 (x^3-2x^2+1)dx - \int_1^2 (x^3-x^2+2)dx$

$= \int_1^2 (-x^2-1)dx = \left[-\frac{x^3}{3} - x\right]_1^2$

$= -\frac{1}{3}(8-1) - (2-1) = -\frac{7}{3} - 1 = -\frac{10}{3}$

41

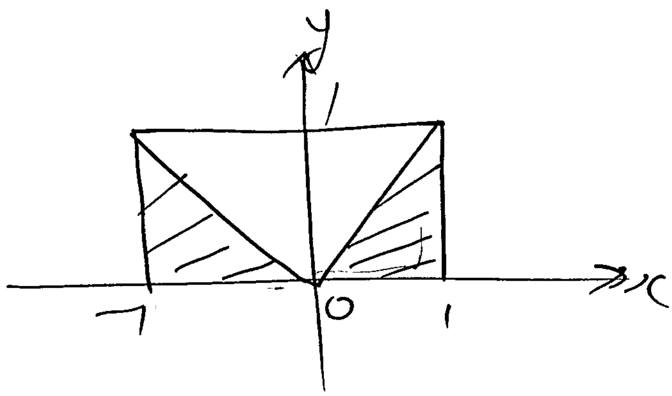
(1) $\int_{-2}^{2} (x^3+4x^2+7x-5)dx$

$= 2\int_0^2 (4x^2-5)dx = 2\left[\frac{4x^3}{3} - 5x\right]_0^2$

$= 2\left(\frac{32}{3} - 10\right) = \frac{4}{3}$

(2) $\int_{-1}^{1} |x| dx$

$= 1$



1-11

(1) $\int_{-1}^{2} \frac{3x^3}{x+2} dx + \int_{-1}^{2} \frac{6x^2}{x+2} dx$

$= \int_{-1}^{2} 3x^2 dx$

$= \left[x^3\right]_{-1}^{2} = 8-(-1)$

$= 9$

$3x^3+6x^2$
$= 3x^2(x+2)$

(2) $\displaystyle\int_0^2 \frac{x^4+x^3}{x+1}\,dx - \int_0^2 \frac{x^3+1}{x+1}\,dx$

$\displaystyle = \int_0^2 \frac{(x^4-1)}{x+1}\,dx$

$\displaystyle = \int_0^2 \frac{(x^2+1)(x-1)(x+1)}{x+1}\,dx$

$\displaystyle = \int_0^2 (x^3-x^2+x-1)\,dx$

$\displaystyle = \left[\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x\right]_0^2 = 4 - \frac{8}{3} + 2 - 2 = \frac{4}{3}$

(3) $\displaystyle\int_{-2}^{2} (3x^2-2)\,dx + \int_2^4 (3x^2-2)\,dx$

$\displaystyle = \int_{-2}^{4} (3x^2-2)\,dx = \left[x^3-2x\right]_{-2}^{4}$

$= (64-(-8)) - 2(4-(-2))$

$= 72 - 12 = 60$

(4) $\displaystyle\int_2^{-1} (3x^2-2x+1)\,dx + \int_{-1}^{1} (3x^2-2x+1)\,dx$

$\displaystyle = \int_2^1 (3x^2-2x+1)\,dx$

$= \left[x^3-x^2+x\right]_2^1$

$= (1-8) - (1-4) + (1-2)$

$= -7+3-1 = -5$

1-21

(1) (준식) $= \int_0^1 (3x^2+2x)\,dx + \int_{-1}^0 (3x^2+2x)\,dx$

$= \int_{-1}^1 (3x^2+2x)\,dx = 2\int_0^1 (3x^2)\,dx = 2[x^3]_0^1 = 2$

(2) (준식) $= \int_{-2}^4 \frac{x^2}{x+1}\,dx + \int_{-2}^4 \frac{2x}{x+1}\,dx - \int_{-2}^4 \frac{2x+1}{x+1}\,dx$

$= \int_{-2}^4 \frac{x^2-1}{x+1}\,dx = \int_{-2}^4 (x-1)\,dx$

$= [\frac{x^2}{2} - x]_{-2}^4 = \frac{1}{2}(16-4) - (4-(-2))$

$= 6-6=0$

(3) (준식) $= \int_0^1 9(x^8-1)\,dx = \int_0^1 (9x^8-9)\,dx$

$= [x^9 - 9x]_0^1 = 1-9=-8$

(4) $t-1=u$

$dt=du$

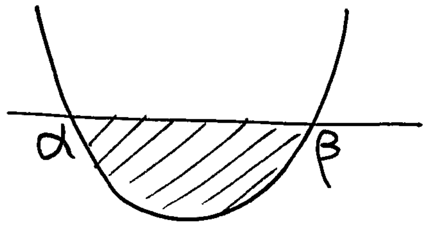
(준식) $= \int_0^1 u^3(u+3)\,du$

$= \int_0^1 (u^4+3u^3)\,du$

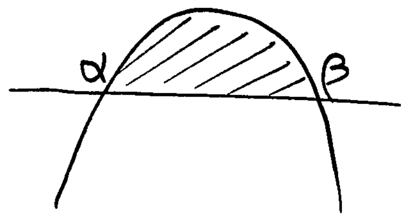
$= [\frac{u^5}{5} + \frac{3u^4}{4}]_0^1 = \frac{1}{5} + \frac{3}{4} = \frac{19}{20}$

1-31

$ax^2+bx+c=0$ 두실근 α, β $(\alpha<\beta)$



$(a>0)$



$(a<0)$

$$\Leftrightarrow ax^2+bx+c = a(x-\alpha)(x-\beta)$$

$$\int_\alpha^\beta (ax^2+bx+c)\,dx = \int_\alpha^\beta a(x-\alpha)(x-\beta)\,dx$$

$$= a\int_\alpha^\beta (x^2-(\alpha+\beta)x+\alpha\beta)\,dx$$

$$= a\left[\frac{x^3}{3} - \frac{(\alpha+\beta)x^2}{2} + \alpha\beta x\right]_\alpha^\beta$$

$$= a\left[\frac{(\beta^3-\alpha^3)}{3} - \frac{(\alpha+\beta)(\beta^2-\alpha^2)}{2} + \alpha\beta(\beta-\alpha)\right]$$

$$= a(\beta-\alpha)\left(\frac{\beta^2+\beta\alpha+\alpha^2}{3} - \frac{(\alpha+\beta)(\beta+\alpha)}{2} + \alpha\beta\right)$$

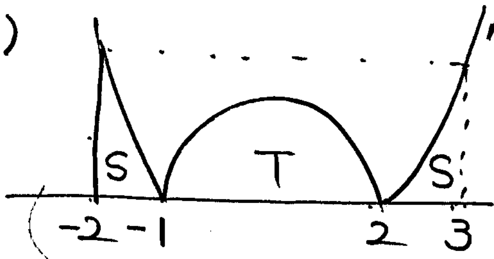
$$= a(\beta-\alpha)\left(\frac{2\beta^2+2\alpha\beta+2\alpha^2-3\alpha^2-6\alpha\beta-3\beta^2+6\alpha\beta}{6}\right)$$

$$= a(\beta-\alpha)\cdot\frac{-(\alpha^2-2\alpha\beta+\beta^2)}{6}$$

$$= -\frac{a}{6}(\beta-\alpha)\cdot(\beta-\alpha)^2$$

$$= -\frac{a}{6}(\beta-\alpha)^3$$

2-11

(1) 

$y=|x^2-x-2|=|(x+1)(x-2)|$

(준식) $=2S+T$

$$S=\int_2^3 (x^2-x-2)\,dx=\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_2^3$$

$$=\frac{1}{3}(27-8)-\frac{1}{2}(9-4)-2(3-2)=\frac{19}{3}-\frac{5}{2}-2$$

$$T=\frac{3^3}{6}=\frac{9}{2}$$

$$2S+T=\frac{38}{3}-5-4+\frac{9}{2}=\frac{38}{3}-\frac{9}{2}=\frac{76-27}{6}=\frac{49}{6}$$

(2) $$[x]=\begin{cases}-1 & (-1\leq x<0)\\ 0 & (0\leq x<1)\end{cases}$$

(준식) $$=\int_{-1}^{0}\{-(x-1)(x+2)\}\,dx$$

$$=\int_0^{-1}(x^2+x-2)\,dx$$

$$=\left[\frac{x^3}{3}+\frac{x^2}{2}-2x\right]_0^{-1}$$

$$=-\frac{1}{3}+\frac{1}{2}+2=\frac{13}{6}$$

2-21

(1) $y=(1+|x|)^2=\begin{cases}(1-x)^2 & (-1\leq x<0)\\ (1+x)^2 & (0\leq x<2)\end{cases}$

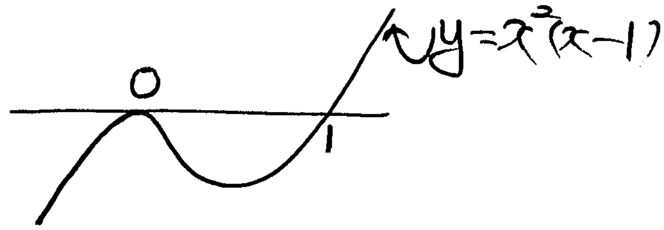
$$(\text{준식})=\underbrace{\int_{-1}^{0}(x^2-2x+1)\,dx}_{=A}+\underbrace{\int_{0}^{2}(x^2+2x+1)\,dx}_{=B}$$

$$A=\left[\frac{x^3}{3}-x^2+x\right]_{-1}^{0}=0-\left(-\frac{1}{3}-1-1\right)=\frac{7}{3}$$

$$B=\left[\frac{x^3}{3}+x^2+x\right]_{0}^{2}=\frac{8}{3}+4+2$$

$$\therefore\ A+B=11$$

(2)



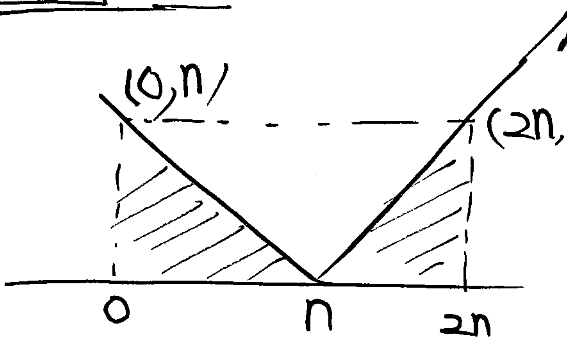
$$S=\underbrace{\int_{0}^{1}(-x^3+x^2)\,dx}_{=A}+\underbrace{\int_{1}^{2}(x^3-x^2)\,dx}_{=B}$$

$$A=\left[-\frac{x^4}{4}+\frac{x^3}{3}\right]_{0}^{1}=-\frac{1}{4}+\frac{1}{3}$$

$$B=\left[\frac{x^4}{4}-\frac{x^3}{3}\right]_{1}^{2}=\frac{1}{4}(16-1)-\frac{1}{3}(8-1)=\frac{15}{4}-\frac{7}{3}$$

$$S=A+B=\frac{7}{2}-2=\frac{3}{2}$$

2-3) 385

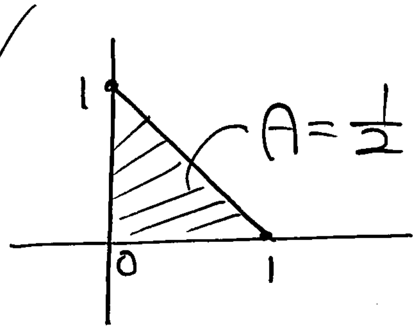


$$y=|x-n|=\begin{cases}-x+n & (x<n)\\ x-n & (x\geqq n)\end{cases}$$

$$f(n)=n^2 \qquad \sum_{k=1}^{10}k^2=\frac{10\cdot 11\cdot 21}{6}=385$$

3-1)

(1) $$\underbrace{\int_0^1(-x+1)\,dx}_{=A}+\underbrace{\int_1^2(x-1)^2\,dx}_{=B}=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$$



$x-1=t$

$dx=dt$

$$B=\int_0^1 t^2\,dt=\left[\frac{t^3}{3}\right]_0^1=\frac{1}{3}$$

(2) $$\int_{-2}^{0}xf(x+1)\,dx$$

$x+1=t$

$dx=dt$

$$=\int_{-1}^{1}(t-1)f(t)\,dt$$

$$=\underbrace{\int_{-1}^{0}(t-1)(t+1)\,dt}_{=A}+\underbrace{\int_0^1(t-1)(-t+1)\,dt}_{=B}=-1$$

$$A=\int_{-1}^{0}(t^2-1)\,dt=\left[\frac{t^3}{3}-t\right]_{-1}^{0}=0-\left(-\frac{1}{3}+1\right)=-\frac{2}{3}$$

$$B=\int_0^1(-t^2+2t-1)\,dt=\left[-\frac{t^3}{3}+t^2-t\right]_0^1=-\frac{1}{3}+1-1=-\frac{1}{3}$$

3-2)

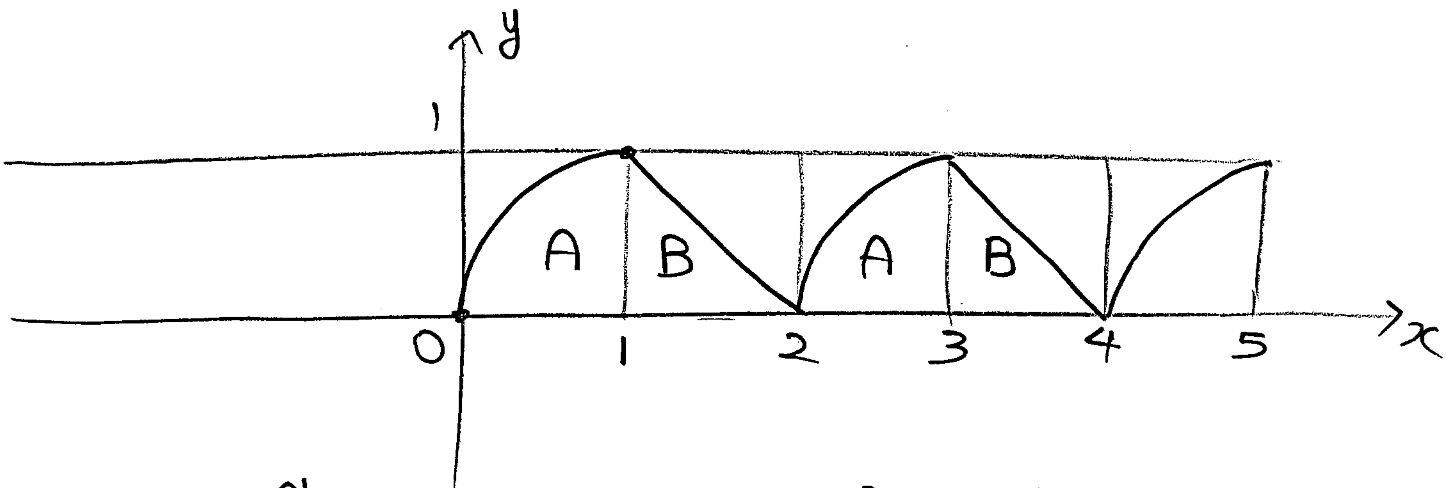
$$f(x) = \begin{cases} 4 & (x \leq 0) \\ -2x+4 & (x \geq 0) \end{cases}$$

$$(\text{준식}) = \underbrace{\int_{-2}^{0} 4x\,dx}_{=A} + \underbrace{\int_{0}^{2} (-2x^2+4x)\,dx}_{=B} = -\frac{16}{3}$$

$$\begin{cases} A = \left[2x^2\right]_{-2}^{0} = 0-(8) = -8 \\ B = \left[-\frac{2}{3}x^3 + 2x^2\right]_{0}^{2} = -\frac{16}{3} + 8 \end{cases}$$

3-3)

$f(x) = f(x+2) \iff$ 주기 : 2



$$A = \int_{0}^{1} (-x^2+2x)\,dx = \left[-\frac{x^3}{3} + x^2\right]_{0}^{1} = -\frac{1}{3} + 1 = \frac{2}{3}$$

$$B = \frac{1}{2}$$

$$\int_{0}^{29} f(x)\,dx = 15A + 14B = 10 + 7 = 17$$

4-1 1

$$2\int_0^1 3xf(x)\,dx = 18$$

4-2 $-\frac{1}{2}$

$$2\int_0^a (3x^2 + a)\,d = 2[x^3 + ax]_0^a = 2(a^3 + a^2)$$

$$2a^2(a+1) = (a+1)^2$$

$$2a^2(a+1) - (a+1)^2 = 0$$

$$(a+1)(2a^2 - a - 1) = 0$$

$$(a+1)(2a+1)(a-1) = 0 \qquad a=-1 \text{ 또는 } a=-\tfrac{1}{2}$$

$$\text{또는 } a=1$$

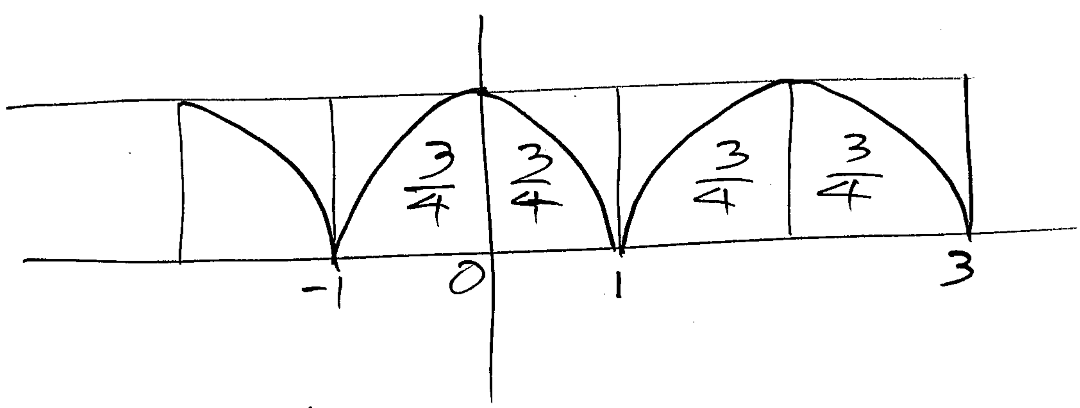
$$\therefore (-1) + \left(-\tfrac{1}{2}\right) + 1$$

$$= -\frac{1}{2}$$

4-3 5

$f(x) = f(-x) \iff$ y축 대칭

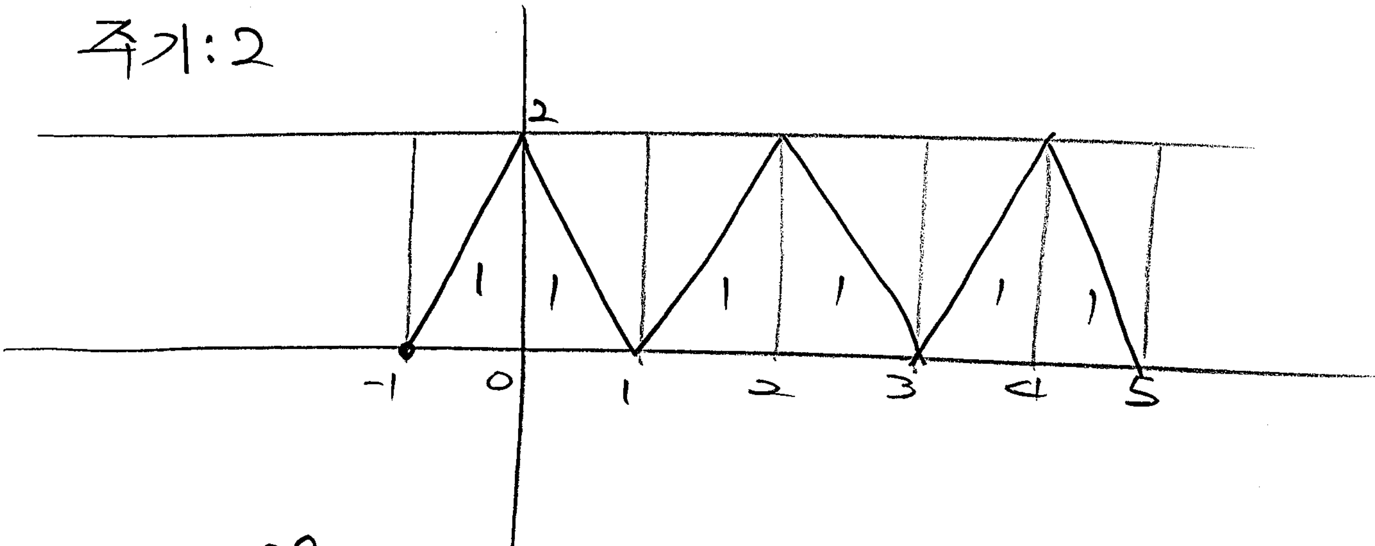
$f(x) = f(x+2) \iff$ 주기: 2



$$\int_0^4 x\,dx - \int_0^4 f(x)\,dx = \left[\frac{x^2}{2}\right]_0^4 - 3 = 5$$

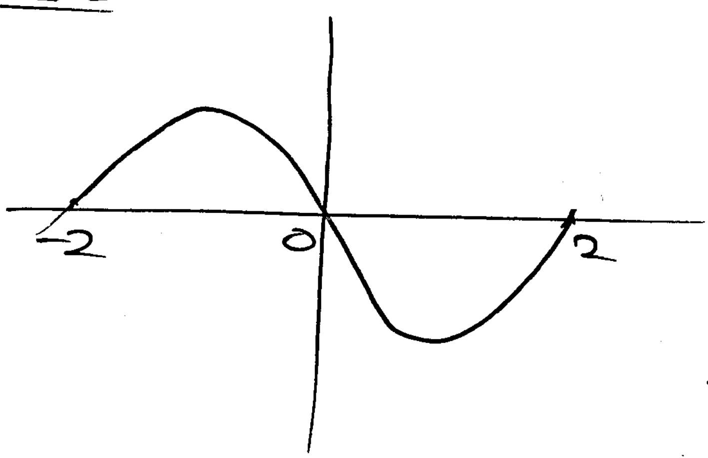
5-1)

주기:2



$$\int_{-1}^{9} f(x)\,dx = 10$$

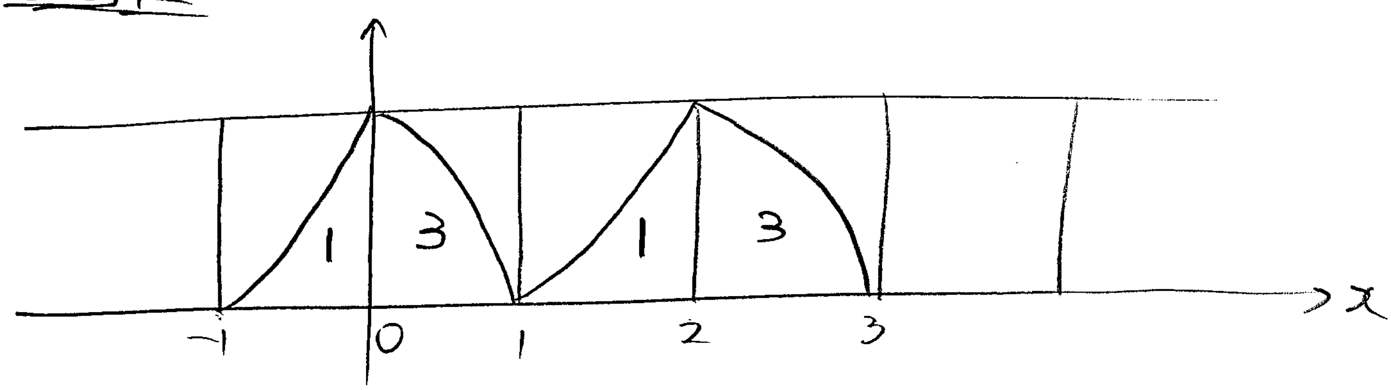
5-2) ③



주기 : 4

$$\int_{1}^{2} f(x)\,dx = \int_{2005}^{2006} f(x)\,dx$$

5-3) 12



$$(\text{준식}) = \int_{1}^{3} (2x + f(x))\,dx = \int_{1}^{3} 2x\,dx + \int_{1}^{3} f(x)\,dx$$

$$= [x^2]_1^3 + 4 = (9-1)+4 = 12$$