9. 정적분과 함수

P279

EX) $f(x) = 3x^2 - 2x + \int_0^2 f(t)dt$

$\int_0^2 f(t)dt = a$ $f(x) = 3x^2 - 2x + a$

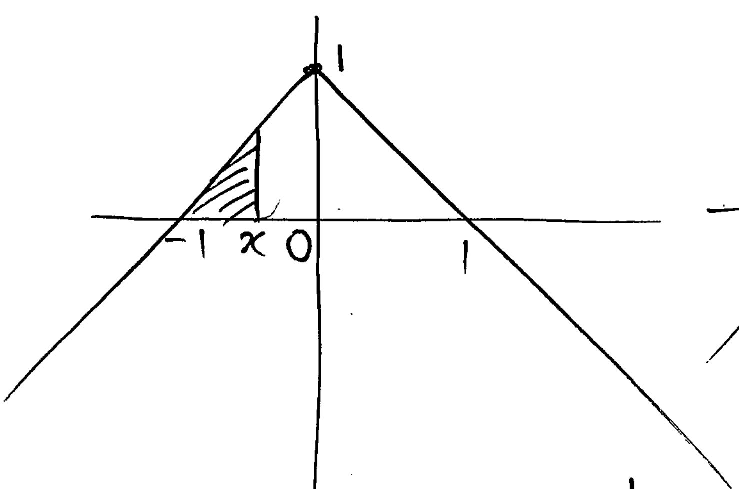
$\int_0^2 (3t^2 - 2t + a)dt = a$

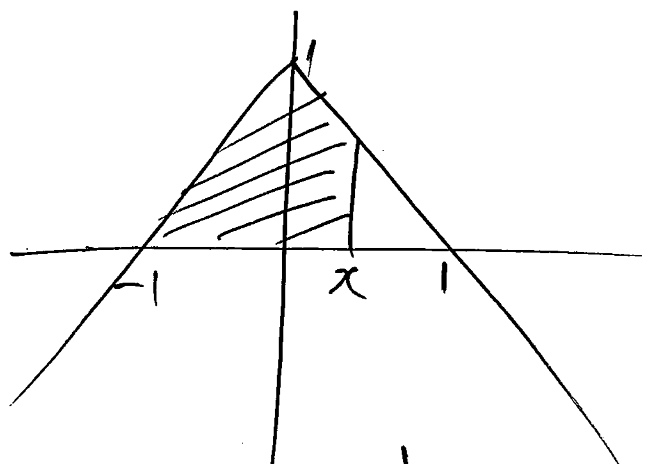
$[t^3 - t^2 + at]_0^2 = 8 - 4 + 2a = 2a + 4 = a$

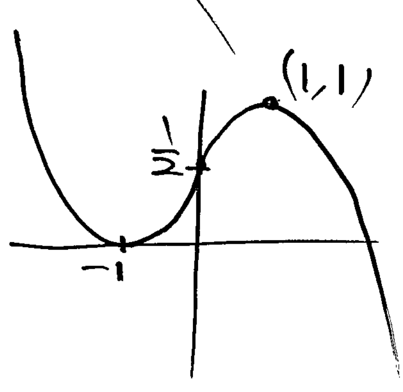
$a = -4$ $\boxed{\therefore f(x) = 3x^2 - 2x - 4}$

P281

EX) $F(x) = \int_{-1}^{x} (1 - |t|)dt$







㉠ $x < 0$; $F(x) = \frac{1}{2}(x+1)^2$

㉡ $x \ge 0$; $F(x) = 1 - \frac{1}{2}(1-x)^2$

P283

EX) $f(x) = \int_0^x (t^2+2t-6)\,dt$

$f'(x) = x^2+2x-6$

P286

EX) $\int (t^2+4t-2)\,dt = F(t)$

$\Leftrightarrow t^2+4t-2 = F'(t)$

(준식) $= \lim_{x\to 1} \frac{F(x)-F(1)}{x-1} = F'(1) = 1+4-2=3$

P287

EX) $\int_2^x f(t)\,dt = x^3 - kx + 4$

㉠ $x=2$; $0 = 8-2k+4$ $\quad k=6$

㉡ $f(x) = 3x^2-6$

EX2)

$\underline{x\int_1^x f(t)\,dt} - \int_1^x t f(t)\,dt = x^3-3x+2$

$\int_1^x f(t)\,dt + \cancel{xf(x)} - \cancel{xf(x)} = 3x^2-3$

$f(x) = 6x$

P289

1)

$f(x) = [t^2+3t]_0^x = x^2+3x$

2)

$f'(x) = ((x+1)^2 - (x+1) + 5) - (x^2 - x + 5)$

$= 2x+1-1 = 2x$

3)

$\int (t^2-3t+1)\,dt = F(t) \iff t^2-3t+1 = F'(t)$

$(준식) = \lim_{h\to 0} \frac{F(1+h)-F(1)}{h} = F'(1) = 1-3+1 = -1$

4)

(1) $f(x) = 3x^2-4x$

(2) $x\cdot\int_0^x f(t)\,dt - \int_0^x tf(t)\,dt = x^3-2x^2$

$\int_0^x f(t)\,dt + \cancel{xf(x)} - \cancel{xf(x)} = 3x^2-4x$

$f(x) = 6x-4$

1-1 5

$\int_0^1 f(t)dt = a \qquad f(x) = 1+ax$

$a = \int_0^1 (1+at)dt = \left[t + \frac{at^2}{2}\right]_0^1 = 1 + \frac{a}{2}$

$\frac{a}{2} = 1 \qquad a = 2 \qquad f(x) = 1+2x \quad \boxed{\therefore f(2) = 5}$

1-2 0

$f(x) = 3x^2 + (2x-1)\int_0^1 f(t)dt$

$\int_0^1 f(t)dt = a \qquad \therefore f(x) = 3x^2 + 2ax - a$

$a = \int_0^1 (3t^2 + 2at - a)dt = \left[t^3 + at^2 - at\right]_0^1 = 1+a-a = 1$

$f(x) = 3x^2 + 2x - 1$

$\int_{-1}^1 (3x^2 + 2x - 1)dx = 2\int_0^1 (3x^2 - 1)dx$

$= 2\left[x^3 - x\right]_0^1 = 2(1-1) = 0$

1-3 6

$f(x) = 1 + x\int_0^1 f(t)dt + \int_0^1 tf(t)dt$

$\int_0^1 f(t)dt = a \qquad \int_0^1 tf(t)dt = b$

$f(x) = 1 + ax + b = ax + b + 1$

㉠ $a=\int_0^1 (at+b+1)\,dt=\left[\frac{at^2}{2}+(b+1)t\right]_0^1$

$a=\frac{a}{2}+b+1 \qquad \therefore \frac{a}{2}-b=1$

㉡ $b=\int_0^1 (at^2+(b+1)t)\,dt=\left[\frac{at^3}{3}+\frac{(b+1)t^2}{2}\right]_0^1$

$b=\frac{a}{3}+\frac{b+1}{2} \qquad \therefore -\frac{a}{3}+\frac{b}{2}=\frac{1}{2}$

$$\begin{array}{rl} & \frac{a}{2}-b=1 \\ + & -\frac{2a}{3}+b=1 \\ \hline & -\frac{a}{6}=2 \end{array}$$

$a=-12$

$b=-7$

$\therefore f(x)=-12x-6$

$f(-1)=12-6=6$

2-11

(1) $\int (x^3+2x^2+3x+4)\,dx=F(x)$

$\Longleftrightarrow x^3+2x^2+3x+4=F'(x)$

$$\lim_{h\to 0}\frac{F(1+h)-F(1-h)}{h}$$

$$=\lim_{h\to 0}\frac{F'(1+h)\cdot 1-F'(1-h)\cdot(-1)}{1}$$

$$=F'(1)+F'(1)=2F'(1)=20$$

(2) $\int (t^2+3t+16)\,dt = F(t)$

$\iff t^2+3t+16 = F'(t)$

$$\lim_{x\to 1} \frac{F(x)-F(1)}{x^2-1} = \lim_{x\to 1} \frac{F'(x)}{2x} = \frac{F'(1)}{2} = 10$$

2-21 ③

$\int |t-4|\,dt = F(t) \iff |t-4| = F'(t)$

$$\lim_{x\to 1} \frac{F(x)-F(1)}{x^2-1} = \lim_{x\to 1} \frac{F'(x)}{2x}$$

$$= \frac{F'(1)}{2} = \frac{3}{2}$$

2-31 6

$\int f(t)\,dt = F(t) \iff f(t) = F'(t)$

$$\lim_{x\to 1} \frac{F(x^2)-F(1)}{x-1} = \lim_{x\to 1} \frac{F'(x^2)\cdot 2x}{1}$$

$= 2F'(1) = 2f(1) = 6$

3-11

1) $f(x) = \frac{d}{dx}\int_1^x (-5t^5-3t^3)\,dt = -5x^5-3x^3$

$f(-1) = 5+3 = 8$

(2) $\int_a^x f(t)\,dt = x^3+x-2$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

㋐ $x=a$; $0 = a^3+a-2$

$= (a-1)(a^2+a+2)$

$D = 1-8<0$

$a=1$

㋑ $f(x) = 3x^2+1$

$f(a) = f(1) = 4$

3-2)

$\int_1^x (t-1)f(t)\,dt = x^3-x^2-x+a$

㋐

$x=1$; $0 = 1-1-1+a$ $\quad a=1$

㋑ $(x-1)f(x) = 3x^2-2x-1 = (x-1)(3x+1)$

$f(x) = 3x+1$

$f(a) = f(1) = 4$

3-3)

$\int_0^x (t+1)f(t)\,dt = 2x^3-x^2+f(x)$

㋐ $x=0$; $0 = f(0)$

㋑ $(x+1)f'(x) = 6x^2-2x+f'(x)$

$xf'(x) = 6x^2-2x$

$f'(x) = 6x-2$ $\quad f(x) = 3x^2-2x$

4-11

$$x\int_1^x f(t)\,dt - \int_1^x tf(t)\,dt = x^3 + ax^2 - b$$

㉠ $x=1$; $0=1+a-b$

㉡ 미분 : $\int_1^x f(t)\,dt + \cancel{xf(x)} - \cancel{xf(x)} = 3x^2+2ax$

㉢ $x=1$; $0 = 3+2a$ $\quad a=-\frac{3}{2},\ b=-\frac{1}{2}$

㉣ $f(x) = 6x-3$

4-21

(1) $$x\int_0^x f(t)\,dt - \int_0^x tf(t)\,dt = \frac{3}{4}x^4 - 2x^2$$

$$\int_0^x f(t)\,dt + \cancel{xf(x)} - \cancel{xf(x)} = 3x^3 - 4x$$

$$f(x) = 9x^2 - 4$$

$\therefore \underline{\underline{-4}}$

(2) $$x\int_0^x f'(t)\,dt - \int_0^x tf'(t)\,dt = \frac{2}{3}x^3$$

$$\int_0^x f'(t)\,dt + \cancel{xf'(x)} - \cancel{xf'(x)} = 2x^2$$

$$f'(x) = 4x$$

$$\boxed{f(x) = 2x^2+4}$$

4-31

$$g(x) = x\int_0^x f(t)\,dt - \int_0^x t f(t)\,dt$$

$$g'(x) = \int_0^x f(t)\,dt + \cancel{xf(x)} - \cancel{xf(x)}$$

ア $g'(0) = 0$

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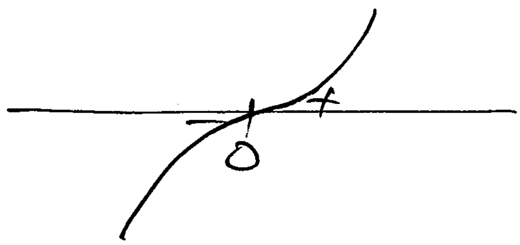
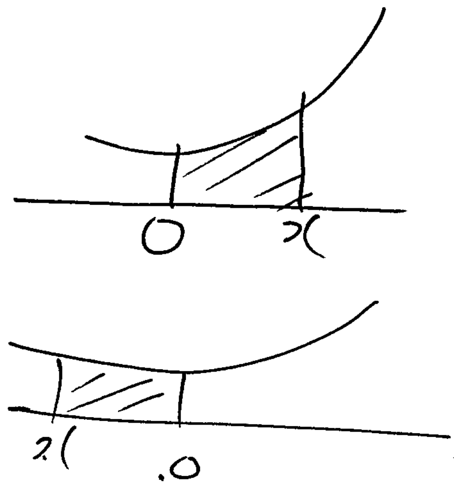




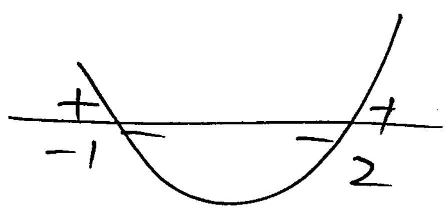
5-1

$$f'(x) = \begin{array}{l} (x+1)^3 - 3(x+1)^2 - 4(x+1) + \frac{1}{4} \\ \underline{x^3 - 3x^2 - 4x + \frac{1}{4}} \end{array}$$

$= 3x^2+3x+1-6x-3-4$

$= 3x^2-3x-6$

$= 3(x^2-x-2)$



$f(x) = x^3 - \frac{3}{2}x^2 - 6x + C$

$f(0) = \int_0^1 (t^3 - 3t^2 - 4t + \frac{1}{4})\,dt$

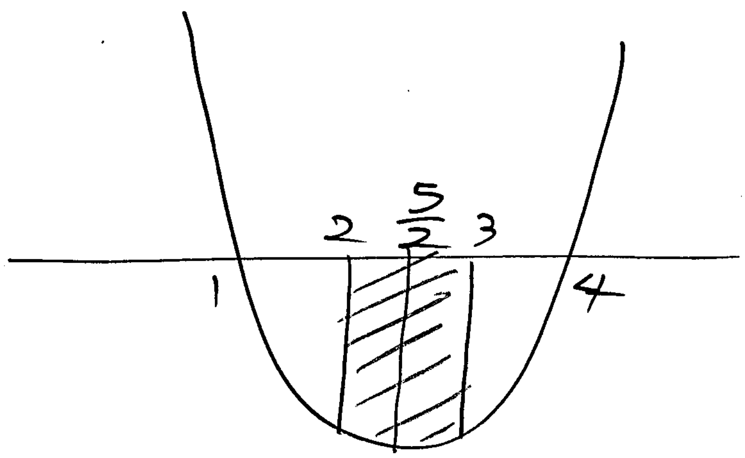
$= \left[\frac{t^4}{4} - t^3 - 2t^2 + \frac{t}{4}\right]_0^1 = \frac{1}{4} - 1 - 2 + \frac{1}{4}$

$= -\frac{5}{2}$

$$M = f(-1) = -1 - \frac{3}{2} + 6 - \frac{5}{2} = 1$$

$$m = f(2) = \underline{8 - 6 - 12} - \frac{5}{2} = \frac{-25}{2}$$

5-2) ③



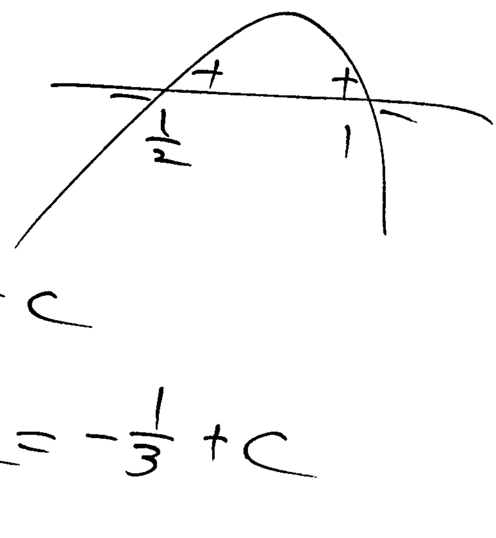
최솟값
g(2)

5-3) $\frac{1}{12}$

$$\cancel{f(x)} = 4x^3 - 6x^2 + 2x + \cancel{f(x)} + x f'(x)$$

$$f'(x) = -4x^2 + 6x - 2$$

$$= -2(2x^2 - 3x + 1)$$



$$f(x) = -\frac{4x^3}{3} + 3x^2 - 2x + C$$

$$M = f(1) = -\frac{4}{3} + 3 - 2 + C = -\frac{1}{3} + C$$

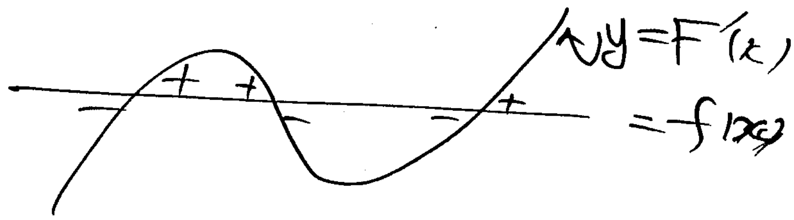
$$m = f\left(\frac{1}{2}\right) = -\frac{1}{6} + \frac{3}{4} - 1 + C = -\frac{1}{6} - \frac{1}{4} + C$$

$$M - m = \frac{-4 + 2 + 3}{12} = \frac{1}{12}$$

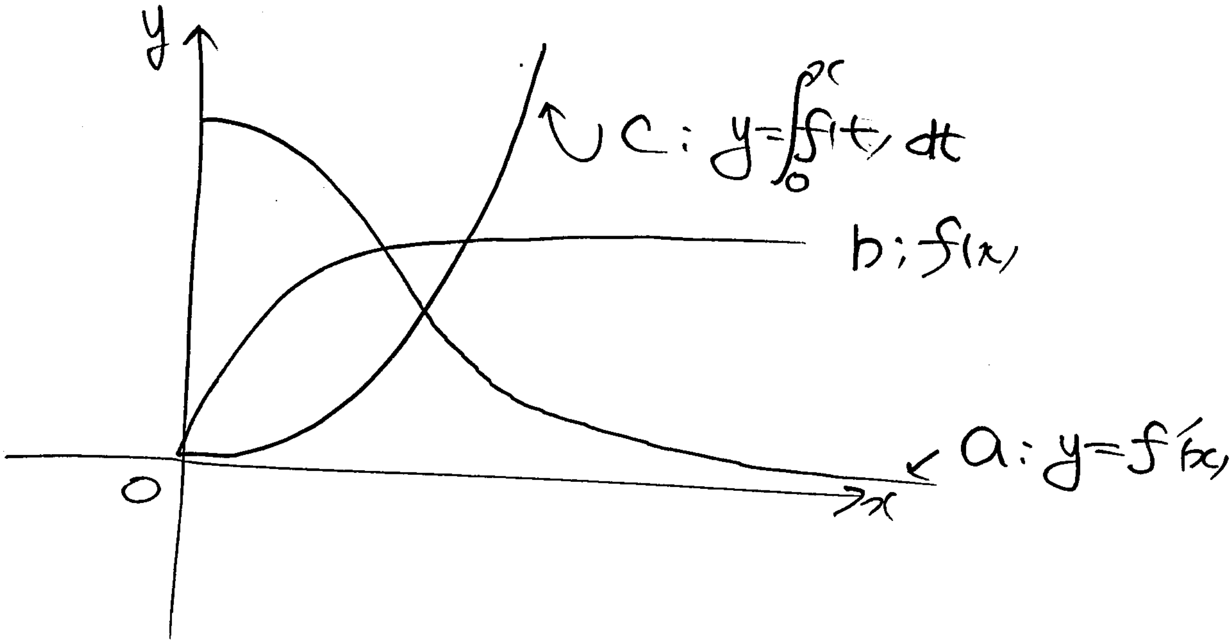
6-1 ⑤

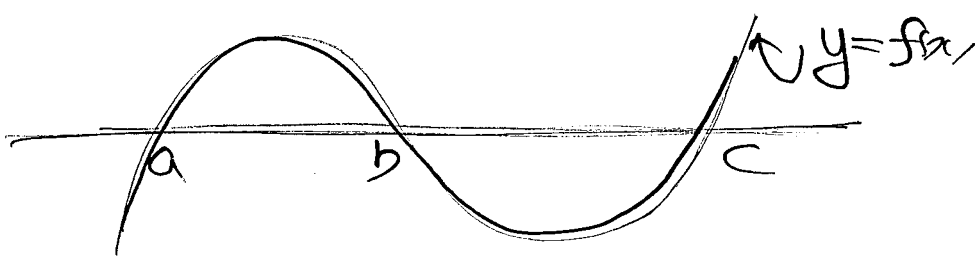
$F'(x) = f(x)$

$F(0) = 0$



6-2 ③





$$\int_a^b f(x)\,dx + \int_b^c (-f(x))\,dx$$

$$= \int_a^c |f(x)|\,dx$$